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THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XLIX JANUARY 1919 NUMBER 1

ON THE DARK MARKINGS OF THE SKY

WITH A CATALOGUE OF 182 SUCH OBJECTS

By E. E. BARNARD

It would be unwise to assume that all the dark places shown on photographs of the sky are due to intervening opaque masses between us and the stars. In a considerable number of cases no other explanation seems possible, but some of them are doubtless only vacancies.

I do not think it necessary to urge the fact that there are obscuring masses of matter in space. This has been quite definitely proved in my former papers on this subject. If any doubt remains of this it will perhaps be readily dispelled by a close examination of the photographs previously printed. The conclusive ones I think are:

1. The photograph of the nebula about Nu Scorpii[1] which clearly shows partial and complete obscuration by the great wing-like nebula that covers much of the immediate region of Nu Scorpii and extends southward to the great nebula of Rho Ophiuchi.

2. The region of Rho Ophiuchi, where a large space of sky is blotted out by a great and beautiful nebula.[2] The fact of obscuration is clearly evident here, for wherever a trace of the nebula

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[1] *Astrophysical Journal*, 31, 1910, Plate I, facing p. 8.

[2] *Ibid.*, Plate II*a*, facing p. 10.

extends, especially to the west, the general background of small stars is sharply blotted out.

3. Especially conclusive is the object (No. 7) which is shown in an article in this *Journal*[1] on a nebulous background in Taurus, where a nebula, only partly luminous, seems to fit in a hole in the sky. Even a casual inspection shows that this nebula can be feebly seen over the entire spot where all the stars are blotted out sharply, and that the absence of stars is due to the obscuring presence of the nebula. This object is really the key to the explanation of most of the dark regions of the sky.

4. The small black spot (No. 92) shown in the photographs in this *Journal* for December 1913,[2] where visual observations prove the existence of a material object.

To me these are all conclusive evidence that masses of obscuring matter exist in space and are readily shown on photographs with the ordinary portrait lenses. What the nature of this matter may be is quite another thing. Slipher has shown spectroscopically[3] that the great nebula about Rho Ophiuchi is probably not gaseous; that is, it does not have the regular spectrum of a gaseous nebula. The word "nebula," nevertheless, remains unchanged by this fact, so that we are free to speak of these objects as nebulae. For our purpose it is immaterial whether they are gaseous or non-gaseous, as we are dealing only with the question of obscuration. In the present paper it is intended to give a catalogue of some of these objects and to show further examples of obscuration and other peculiarities, and to try to emphasize the fact that they are not necessarily confined to the Milky Way but are found in other parts of the sky as well; and also to bring as much evidence as possible to prove that these extra-galactic objects show that space is itself more or less luminous.

Outside of these examples, where the object is partly luminous, there are a number of others which appear to be entirely devoid of light. These are naturally best shown on the bright background of the Milky Way, against which they appear black on the photographs. Fine examples of these Milky Way objects are the black spots in $\alpha = 18^{\mathrm{h}}8^{\mathrm{m}}$, $\delta = -18^\circ 16'$ (No. 92), and in $\alpha = 17^{\mathrm{h}}55^{\mathrm{m}}$, $\delta = -28^\circ$ (No. 86), which are so striking in photographs of the star-clouds in Sagittarius.[1]

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[1] *Astrophysical Journal*, 25, 1907, Plate XI, facing p. 219, and Plate XII, facing p. 221.

[2] *Ibid.*, 38, 1913, Plates XIX and XX, facing p. 496.

[3] *Lowell Observatory Bulletin*, No. 75, 2, 155.

The last of these two is very remarkable when seen in a 5-inch telescope with a low power. In such an instrument it appears like a drop of black ink on the bright background of the Milky Way. It was found in my comet-seeking in the early eighties.[2] On account of its extreme blackness it was one of the most impressive objects in the Milky Way. I examined it with the 36-inch refractor of the Lick Observatory with a power of 350 and a field of 6′, on August 18, 1895. It nearly filled the field of view. The western half was fairly well defined, while the eastern half was more diffused. Considerable nebulosity seemed to be connected with it. A photograph on July 11, 1917, with the Crossley reflector, kindly sent me by Dr. H. D. Curtis, shows this black spot to be very remarkable, having considerable nebulosity connected with it. On his photograph its southwest side is very sharply defined, closely resembling in this respect the east side of the black spot in $\alpha = 18^{\mathrm{h}}8^{\mathrm{m}}$, $\delta = -18^\circ 16'$ shown in Plate XX.[3] Its east side, like the west side of the foregoing object, is more or less diffused. The star C.D. $-27^\circ 12302$ ($7^{\mathrm{M}}\!.4$) is on the northwest border, while $-27^\circ 12310$ ($9^{\mathrm{M}}\!.0$) is close east of the spot. The beautiful cluster of small bright stars, N.G.C. 6520, is also close east.

Visual observations actually show, however, that the object No. 92 is really feebly luminous. All those that are in the Milky Way are not necessarily devoid of light, for they may appear black by contrast with the greater brightness of the Milky Way. There are numerous examples, however, which are not in the Milky Way and which are perhaps entirely devoid of light. It would seem that such a body would be lost in the blackness of space, but they are visible as black objects against space itself. I have previously explained this anomaly[1] by suggesting that space is probably filled with a feeble light which forms a slightly luminous background for these dark bodies. Further investigations have fully convinced me that this is actually the explanation of the phenomenon, for there is no evidence of an ordinary nebulous background in these cases. Furthermore, this feeble illumination is widespread and undoubtedly universal (so far, at least, as our stellar universe is concerned), for these dark objects are found in opposite parts of the sky, where there are few stars, and away from any possible brighter background.

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[1] See various plates in *Publications of the Lick Observatory*, **11**, and especially Plates 49 and 54.

[2] See *Astronomische Nachrichten*, **108**, 370, 1884, where it is described as "a small triangular hole in the Milky Way. Perfectly black, some 2′ diameter, much like a jet black nebula."

[3] *Astrophysical Journal*, **38**, 496, 1913.

One of the finest examples of a dark object seen against the ordinary sky and away from the Milky Way is No. 15, shown on the photographs in $\alpha = 4^h 22^m 50^s$, $\delta = +46°21'$. It is elliptical, 10′ by 15′ in diameter. The background on which the stars shine is uniform over the entire plate. The object is in a region somewhat larger than itself, where there are relatively few stars, and is black by contrast with the sky alone. It clearly shows the presence of a feeble uniform luminosity in space which, from the appearance of similar objects in widely different parts of the sky, leads to the belief that this feeble illumination of distant space is universal. If this object were seen against the star-clouds of the Milky Way it would appear strikingly black.

There is another and very similar black spot (No. 48) in $\alpha = 16^h 53^m$, $\delta = -40°30'$, close west and north of the eighth magnitude star C.D. −40°11088. There can be no doubt that this is a material object. There are a number of other similar black masses in this region, as shown by the catalogue.

Another excellent example of this class of objects is No. 160, in $\alpha = 21^h 34^m 47^s$, $\delta = +55°41'$. The body of this marking is much broken with darker masses. It is convex to the north and passes just below (south of) B.D. +59°2291 ($8^M_.0$). The western end curves to a slender "tail" which ends close southwest of B.D. +59°2283 ($8^M_.2$). The eastern end widens out into a large "head" containing much detail, which is more or less convoluted, with sharply defined projections like the horns of an insect.

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[1] *Astrophysical Journal*, 43, 1, 1916.

There seems to be no question but that this is a real, opaque object seen in projection against space. The stars in this region are too few to serve as a luminous background and there is no nebulosity to show it in dark relief, yet it stands out black and strong against the sky.

In previous papers I have dealt mainly with the larger dark masses and occulting nebulosities. The smaller ones that are now treated of are perhaps more interesting in a way than the larger ones. They are more definite and in a sense more clearly show the effect of obscuration of the smaller stars.

The small scale of the portrait lens accentuates the blackness and definiteness of these objects. This is a valuable asset in such a lens; it draws attention to peculiarities which might be lost by diffusion with a more powerful telescope. They are worthy of a careful study, however, with some of the large photographic reflectors. This has already been done, as I have stated, in the case of the black spot (No. 86) in $\alpha = 17^h 55^m$, $\delta = -28°$, by Dr. H. D. Curtis with the 36-inch Crossley reflector of the Lick Observatory. I am sure that some of the objects shown in the present photographs will give very interesting results when similarly investigated.

There are two regions which can be reached from the northern hemisphere that are specially rich in these dark markings: (1) the region immediately north of Theta Ophiuchi; (2) the region of the great star-cloud in Scutum near the cluster M 11. There are other regions in which black markings occur, but these two contain the most striking ones, striking for their smallness and peculiarities.

Some of the dark objects in the remarkable region north of Theta Ophiuchi are so strange in their forms that we would find it difficult to match them with similar forms among the real nebulae. This in itself would almost discourage the supposition that they are dark nebulae, and one would rather seek some other explanation for them. In other parts of the sky, however, there seems to be no need of hesitation in accepting them as real, obscuring masses, most probably dark nebulae.

Perhaps one of the finest of the large dark regions (No. 78) lies several degrees southeast of Theta Ophiuchi. It is a large, irregular, dark spot some 3° in diameter and less definite on the eastern side. There is considerable detail in it of a more or less nebulous character. This is specially evident near the bright star C.D. $-26°12152$ ($6^{M}.2$). Westward from this region a broken dark lane extends for about 5° to what I have called the "sink hole," because of its peculiar form and outlines. This sink hole (No. 59) is full of rich detail. Similar structural detail shows at frequent intervals along the broken lane (which is about $\frac{3}{4}°$ wide) to its origin in the larger dark region southeast of Theta Ophiuchi. Splendid half-tone reproductions of this remarkable region have been published in *Popular Astronomy*.[1]

The bright nebulae seldom show extraordinary forms. Some of them, however, exhibit structural details and general forms that are very remarkable and that sometimes are very beautiful, such as the zigzag, streaky, or "lace" nebula in Cygnus, the great nebula of Orion, and many of the planetary nebulae. It is possible then that the objects north of Theta Ophiuchi are, after all, only exceptions to the general run of nebulous forms and are similar to such objects as those in Cygnus and elsewhere among the bright nebulae.

A peculiarity of the dark markings in the star-cloud in Scutum is that some of the well-defined spots are uniformly gray, while others are either entirely black or have much blacker, well-defined spaces in them. In nearly every case their outlines are very definite and few have stars in them.

To the east of the $4^{M}.5$ star B.D. $-4°4582$ (6 Aquilae) and north of the cluster M 11 is a beautiful region of dark structures (No. 111). It forms a wide crescent with the convex side to the west, sharply bounded by the neck and head of the splendid star-cloud in which M 11 is placed. It is irregular and broken to the east. Its width north and south is about 2°. This area is full of dark structures, the more conspicuous ones being Nos. 106, 107, 110, and 113. The two stars B.D. $-5°4775$ ($8^{M}.3$) and $-5°4778$ ($8^{M}.4$) are near the middle of this region. The center would be in about $\alpha = 18^{h}44^{m}$, $\delta = -5°\,6'$.

In many cases one side of a dark marking is very definite, while the other side is diffused. This occurs so often that there must be some reason for it. The same peculiarity is sometimes seen in the bright nebulae, the great nebula of Orion being a striking example. Three of these objects, which are very much alike, and which show this feature strongly, are No. 50, $\alpha=16^{h}54^{m}42^{s}$, $\delta=-34^{\circ}12'$; No. 143, $\alpha=19^{h}35^{m}30^{s}$, $\delta=+10^{\circ}43'$; No. 160, $\alpha=21^{h}33^{m}40^{s}$, $\delta=+55^{\circ}40'$. The first two very closely resemble each other in size and form and the third differs from them only in a small degree.

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[1] *Popular Astronomy*, 14, 579 (December), 1906, Plates XIII and XIV.

The dark object (No. 133) close to the $9\overset{\mathrm{M}}{.}2$ star B.D. $-7^{\circ}4852$, which looks like a negative of a comet with a curved tail, more or less fan-shaped, was examined with the 40-inch telescope July 14, 1917 (see Fig. 1). It was clearly evident that there is a faint hazy object at this place. It is dull and feebly luminous compared with the adjacent sky. It is very dark at its south end and curves northward for nearly 10′. The northern part, which is more diffused, is broken by a few faint stars. Obscuring matter of some kind certainly exists at this point. Furthermore, it is not black except by contrast. In this respect it very much resembles the dark object (No. 92) in $\alpha=18^{h}8^{m}$, $\delta=-18^{\circ}16'$, which was examined with the 40-inch telescope in July 1913.[1] Two other objects, Nos. 127 and 129, were carefully examined on this date. The object No. 127 is similar except in form to the one just described. It seems to be a dull, feebly luminous mass as in the case of the other, with no definite outline. Sweeping rapidly over these objects with the telescope

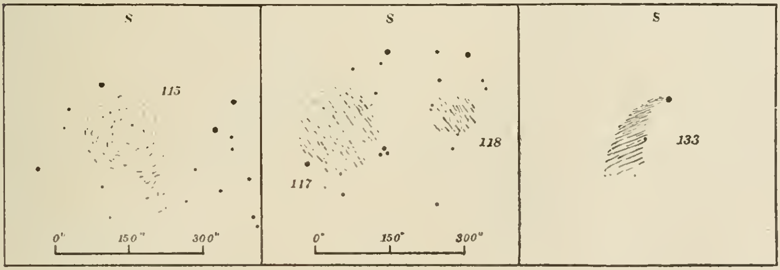

FIG. 1.—Sketched from visual observations of Nos. 115, 117, 118, and 133, with the 40-inch telescope.

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[1] See *Astrophysical Journal*, 38, 496–501, 1913.

there is no hesitation as to their actual presence where the photograph shows them. There is no evidence of ordinary nebulosity at these points except the feeble, dull appearance described.

One remarkable thing in this visual investigation is the conspicuousness of the B.D. stars everywhere, while on the photographs they are difficult to make out. Evidently the sky comes up luminous on the photographs from the myriads of small stars not seen in the telescope. It is this general effect of unseen stars which do not show individually, either in the telescope or on the photograph, that helps to round out the great star-cloud. Doubtless there is not a star on the plate that cannot be seen in the large telescope, but I am not sure of this. It is mainly the light from unseen stars that makes the white background of the photographs against which the dark markings show so conspicuously.

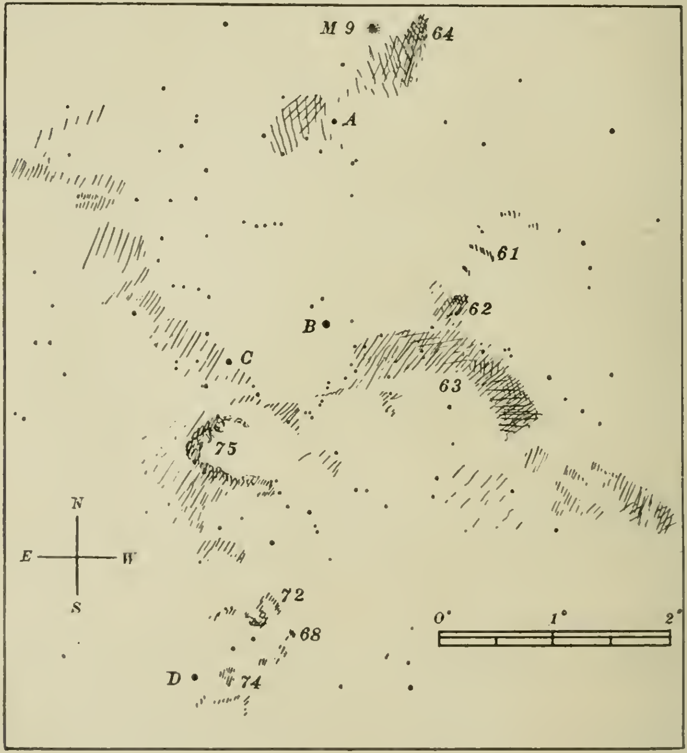

Fig. 2.—Sketch map of Plate I

In these observations the sky was satisfactory and the seeing fairly good. To me the result is conclusive that these markings are real objects seen in dark relief on the bright background of the Milky Way. They are certainly not holes in the star-cloud. The definite outlines which appear so striking on the photographs are not seen in the sky. They are lost through the great magnifying power of the telescope. On July 18, 1917, the small objects Nos. 115, 117, and 118 were similarly examined and sketches made. The spot No. 118 shows some detail, or different degrees of darkness, while No. 117 is uniformly dark or dusky.

The great partially dark nebula (No. 7) in $\alpha=4^h4^m14^s$, $\delta=+28^\circ28'$, reference to which has already been made in the beginning of this paper, is irregularly round, with a wide extension to the west to about $\alpha=4^h1^m$, $\delta=+27\frac{1}{2}^\circ$ or $+28^\circ$. The object is unevenly dark with a brighter condensation in its eastern part. Its diameter along the east and west extension is 2°; north and south its diameter is $1^\circ\!.2$. The bright condensation (No. 10) is 8′ in diameter and is in $\alpha=4^h10^m57^s$, $\delta=+27^\circ58'$. There are several small, round, black spots in the northern part of the dark nebula. Their approximate positions are:

α	δ	
$4^h\ 8^m\ 6^s$	$+28^\circ31'\!.0$	Indefinite; diameter 8′
4 9 26	+28 20.5	Round; diameter 5′
4 10 10	+28 21.6	Round; diameter 5′
4 10 44	+28 9.5	Length 15′ north and south; width 5′; involving B.D. +28°637 ($9^M\!.5$)

The region for some distance about this great dark nebula is very remarkable and suggestive. Beginning at about $\alpha=4^h9^m50^s$, $\delta=+27^\circ25'$, an irregular dark lane about 10′ wide with unequal dark markings in it runs southeast for $2\frac{1}{2}^\circ$ nearly to another large obscure nebula (No. 22). A similar dark lane beginning at a group of large dark spots (No. 18) in $\alpha=4^h23^m40^s$, $\delta=+24^\circ5'$, runs westerly in a curved and irregular manner for about 4°.

These lanes are all definite and dark on the sky, independent of the background of stars.[1] There are indications of considerable areas of feeble nebulosity for several degrees in this region. These will be treated of in a later publication.

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[1] See *Astrophysical Journal*, **25**, 218, 1907, where photographs of the region are given.

No. 144 of the catalogue is a very large, unequally dark area in a very rich region of the Milky Way. Its general outlines are fairly distinct and somewhat abrupt except toward the south, where it widens out and becomes lost in the general groundwork. Over this large area the dense background of small stars is wanting, but there are scattered over it everywhere a great many stars, big and little. Its eastern side near the north end is partly covered by a protrusion over it of the denser part of the Milky Way, which quite obliterates it for a degree or more. The northern end of this area is more or less rounded and is terminated at the northeast side by a sharper projection in $\alpha = 20^h 3^m$, $\delta = +36\frac{1}{2}°$. This great partial vacancy is probably due to an actual thinning out of the small stars in this region, for it does not have the appearance of obscuring matter.

To illustrate the remarkable appearance of some of these objects the accompanying half-tone plates have been prepared. These show the peculiarities as they appear on ordinary portrait-lens photographs. The brief descriptions of the objects shown in the illustrations are based on an examination of the original negatives. The engravings are good, but they fail to show the more delicate structures which are so clear on the photographs and are such important features of these objects. Therefore, if there should be any difficulty in identifying all the features described, it must be attributed to the failure of the half-tones to show them.

DESCRIPTION OF THE PLATES

PLATE I

This shows the region north of Theta Ophiuchi and its rather fantastic markings as a whole. The glow from Theta is seen at the lower edge of the plate. By comparing this picture with the chart (Fig. 2) the relative positions of the various objects shown in some of the other plates can readily be seen.

PLATE II

Two of these queer markings are shown here, the lower being readily recognized as one of those on Plate I. It is a curious, narrow, looped, black marking (No. 75) that covers about a degree in its peculiar windings. It occupies a considerable space and one position will not represent its place. The upper or north part of it is in $\alpha=17^h17^m44^s$, $\delta=-21^\circ51'$. From this point it bends to the east, passing close south of the star B.D. $-21^\circ4598$ ($8^{\mathrm{M}}.3$) and then south and west to an abrupt stop in $\alpha=17^h15^m44^s$, $\delta=-22^\circ21'$. A diffused branch springs from it and curves easterly and southwest again in a very broken manner. At this point there is much dark detail of an irregular nature. Close north and west of this object, just above (and involving) the star B.D. $-21^\circ4591$ ($9^{\mathrm{M}}.5$) is a very dark, irregular figure (No. 67*a*).

This marking is almost exactly duplicated in its scalloped appearance by a similar one (No. 84) in $\alpha=17^h39^m$, $\delta=-20^\circ12'$, which is shown in the upper picture of this plate. It lies between the stars B.D. $-20^\circ4865$ ($8^{\mathrm{M}}.2$) and $-20^\circ4869$ ($9^{\mathrm{M}}.1$) and curves a little to the north. The resemblance of these two strange objects and their peculiar character is very striking.

PLATE III

These are two more of the dark objects north of Theta Ophiuchi. The upper one (No. 63) is the very large definite curved marking. Its southwest end strikingly and abruptly terminates in $\alpha=17^h6^m14^s$, $\delta=-21^\circ41'$. In the original negative there is much dark detail in this object.

The lower picture shows the S-shaped marking (No. 72) in $\alpha=17^h16^m17^s$, $\delta=-23^\circ37'$, and several of the very black spots south of it.

PLATE IV

The upper right-hand object (No. 133) resembles a dark comet with a small definite head and curved fan-shaped tail. The main body seems to be a sharply defined, black, lozenge-shaped object which forms the head and part of the body, from which a diffused extension runs north, forming the tail of the "comet." A description of the visual appearance of this object with the 40-inch telescope is given in the present paper, page 7. The position of the dark head, which lies very close west of the star B.D. $-7^\circ4852$ ($9^{\mathrm{M}}.2$), is $\alpha=18^h59^m29^s$, $\delta=-7^\circ4'.8$.

The picture immediately beneath this one, to the right, is that of a similar "cometary" object (No. 64), just west of the compressed cluster M 9. It is somewhat cometary in form and has a very black core or head that sharply abuts against the thick stratum of stars; from this it spreads out into a large dark area with much dark detail, filling quite a space close southwest of M 9. It thus resembles a dark comet with a dense and well-defined head and diffused widening tail. The position of the head is $\alpha=17^h9^m57^s$, $\delta=-18^\circ20'.6$.

The upper left picture of this plate shows a chain of several dark spots, two of which (Nos. 127 and 129) are quite small, black and irregular in form. The northern one (No. 129) has a very narrow black streak running from it to the northeast. I have examined these also with the large telescope with a result similar to that of the spots previously mentioned. Their position is $\alpha=18^h55^m$, $\delta=-5^\circ40'$.

In the fourth, or lower left, picture is a small, round, well-defined, black spot (No. 98) less than 3′ in diameter. This could readily be taken for a black planetary nebula, which, in reality, it may be. Its position is closely $\alpha = 18^{\mathrm{h}}25^{\mathrm{m}}31^{\mathrm{s}}$, $\delta = -26°9'$. It is not seen on a bright part of the sky for it lies in, and near the south edge of, a rather broad dark pathway in the Milky Way. It is some 3′ south and following the star C.D. $-26°13264$ ($10^{\mathrm{M}}_{.}$).

I did not at first believe in these dark obscuring masses. The proof was not conclusive. The increase of evidence, however, from my own photographs convinced me later, especially after investigating some of them visually, that many of these markings were not simply due to an actual want of stars, but were really obscuring bodies nearer to us than the distant stars. In this way it has fallen to my lot to prove this fact. I think there is sufficient proof now to make this certain. For some years I have tried to secure long-exposure photographs of as many of these bodies as possible. This has resulted in the location of a considerable number of them in different parts of the sky. Their apparent preference for the bright regions of the Milky Way is obviously due to the fact that they are more readily shown with a bright background. They are, however, not strictly confined to the Milky Way.

Among the first to look upon these dark places as real matter was Mr. A. C. Ranyard, whose lamentable death occurred December 14, 1894. A short time previous to his death he gave a series of papers on the Milky Way and the nebulae, in *Knowledge*, of which magazine he was editor. In speaking of the dark lane south and east of Theta Ophiuchi on a Lick photograph of mine which he reproduced,[1] he says: "The dark vacant areas or channels running north and south of the bright star [θ Ophiuchi] at the center . . . . seem to me to be undoubtedly dark structures, or obscuring masses in space, which cut out the light from the nebulous or stellar region behind them."

There is a list of starless fields given in Appendix I of Webb's Celestial Objects, taken from the Cape observations of Sir John Herschel. These, however, are quite different from the ones I have been dealing with and are in most cases perhaps only real vacancies among the stars.

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[1] *Knowledge*, 17, 253 (November) 1894.

For some time I have hoped to make a catalogue of the dark markings shown on my photographs of the sky. The exact location of these objects is desirable so that their study with powerful photographic telescopes may be possible. There seems to be no question that some of them are real objects which are either entirely devoid of light or so feebly luminous when seen against the Milky Way as to appear black. As mere curiosities of the sky alone their cataloguing would be desirable, but as real opaque objects between us and the more distant stars their exact location would seem to be important. Their study with the present means of research will be of the highest interest. With this idea in view I have collected a number of these objects shown on my negatives to form the following catalogue.

This catalogue is necessarily incomplete; it is constantly being added to. Later, a more complete list will be printed. The places are closely approximate. An effort was made to measure the positions with exactness, but on account of the nature of the objects it was found to be practical to do this in only a few cases.

All the positions in the catalogue and throughout this paper refer to the epoch 1875.0.

NOTES ON THE CATALOGUE

No.

8 Very much like the dark lanes east of Rho Ophiuchi.

9 This is the middle of a great vacancy extending east and west, the continuous part of which is $2\frac{1}{2}^{\circ}$ long and about $\frac{1}{2}^{\circ}$ wide. It really extends in a more or less broken form for about 6°. There is a wide region extending south of it for a couple of degrees, nearly to the star B.D. $+53^{\circ}$ 750 ($5^{\mathrm{M}}.0$) whose position is $\alpha=4^{\mathrm{h}}6^{\mathrm{m}}58^{\mathrm{s}}$, $\delta=+53^{\circ}18'$. To the east it breaks up into more or less separate spots, somewhat resembling those at the east end of the great lane from Rho Ophiuchi, but unlike that lane it does not originate in a larger vacant space or a nebula. Its borders are not so definite as the Rho Ophiuchi lane. It is approximately bounded by the co-ordinates $\alpha=3^{\mathrm{h}}58^{\mathrm{m}}$, $\delta=+54^{\circ}.7$ and $\alpha=4^{\mathrm{h}}20^{\mathrm{m}}$, $\delta=+54^{\circ}.8$. The individual positions of some of the spots are given in the catalogue.

11 This lane is about $2^{\circ}.6$ long and roughly 22′ across. It is very much like the lanes east of Rho Ophiuchi. See No. 8.

12 South of No. 11. Irregularly round with an extension east from the northeast side.

CATALOGUE OF 182 DARK MARKINGS IN THE SKY

No.	α 1875.0	δ 1875.0	Description
1	$3^{h}25^{m}14^{s}$	+30° 44′	Large, indefinite; diam. $\frac{1}{2}$°
2	3 25 44	+31 54	Indefinite; elongated east and west; diam. 20′
3	3 32 14	+31 34	Irregular; diam. 20′; dark space in nebula
4	3 36 14	+31 24	Very large; indefinite
5	3 39 15	+32 8	Indefinite; elongated northeast and southwest; diam. 1°. B.D. +32°667 ($5^{\mathrm{M}}.8$) near northeast side
6	3 46 31	+55 45	Round; indefinite
7	4 4 14	+28 28	Large, irregular; brighter condensation in southeast part
8	4 7 53	+54 56	The center of No. 11
9	4 9	+54 45	Dark irregular lane
10	4 10 57	+27 58	The bright part of No. 7
11	4 16 37	+54 45	East end of irregular lane
12	4 19 52	+53 58	Isolated dark spot; diam. 24′
13	4 21 17	+54 37	Irregular; diam. 11′
14	4 22 13	+25 31	Very small, bright nebula; diam. 3′
15	4 22 50	+46 21	Elliptical dark marking; diam. 10′ by 15′ slightly northwest and southeast
16	4 23 20	+46 20	Very small; elongated north and south; close to southeast edge of No. 15
17	4 23 25	+46 15	Very small; elongated north and south; close to east edge of No. 15
18	4 23 40	+24 5	Diam. 1°; group of dark spots
19	4 25	+26 0	Large; indefinite; diam. 1°
20	4 27 30	+50 43	In the south part of larger, relatively vacant area nearly 1° in diam.
21	4 27 51	+55 6	Indefinite; irregularly round; diam. 10′
22	4 31	+25 48	Irregular; unequally dark; extended southeast and northwest; diam. 2°
23	4 32 41	+29 38	Diam. 5′; sharply pointed to the southeast
24	4 35 0	+29 30	Diam. 8′; sharply pointed to the south
25	4 42 56	+45 48	Irregularly round; diam. 8′
26	4 46 39	+30 25	Irregular; diam. 5′
27	4 47 11	+30 21	Irregular; diam. 5′
28	4 47 54	+30 26	Irregular; diam. 4′
29	4 58 5	+31 55	Round; indefinite; diam. 10′
30	5 23 8	+12 21	Large dark area with few stars; diam. 67′
31	5 25 8	+12 38	Diam. 30′; extended northeast and southwest; the east part of No. 30
32	5 25 8	+12 20	Dark projection from the south end of No. 31 to the east
33	5 34 36	− 2 32	Dark mass, diam. 4′, on nebulous strip extending south from ζ Orionis
34	5 35 20	+32 21	Round; starless; indefinite; diam. 20′
35	5 39 5	+ 8 51	Round; diam. 13′
36	5 43 5	+ 7 23	Irregular narrow dark lane 2° long, northeast and southwest
37	6 26 14	+10 34	Irregular semi-vacant region 2$\frac{1}{2}$°–3° long
38	6 26 45	+11 10	Irregular vacancy; diam. 1°
39	6 31 8	+10 26	Small, sharply defined.
40	16 7 24	−18 39	Diffused dark spot; diam. 15′; small star in it
41	16 14 24	−19 20	Diffused dark spot; diam. $\frac{3}{4}$°
42	16 18	−23 9	Great nebula of ρ Ophiuchi

CATALOGUE OF 182 DARK MARKINGS IN THE SKY—*Continued*

No.	α 1875.0	δ 1875.0	Description
43.....	$16^{h}23^{m}\ \ ^{s}$	−19° 30′	Large dark region
44.....	16 33	−23 50	Dark lane east from Rho Ophiuchi region
44*a*....	16 38 8	−40 6	Round; diam. 2′. Close south of C.D. −40°10662
45.....	16 39 11	−21 22	Rather definite, 2° long
46.....	16 49 42	−22 32	Irregular; definite
47.....	16 52 12	−22 27	Irregular; definite; diam. 15′
48.....	16 53	−40 30	Fairly well defined; 26′ long
49.....	16 54 30	−33 5	Small; close southeast of and involving C.D. −33°11668 ($9^{\mathrm{M}}3$)
50.....	16 54 42	−34 12	Large, irregular, dark space; 10 mag. star in middle
51.....	16 56 12	−22 12	Definite; diam. 20′
52.....	16 57 14	−22 2	Irregular; diam. 13′
53.....	16 58	−33 25	Large; diffuse, extended north and south
54.....	16 58 30	−34 10	Small, round; diam. 5′; close north of 11 mag. star
55.....	16 59 28	−31 51	Irregular; diam. 16′; best defined southeast
56.....	17 0 43	−31 56	Small; diam. 3′
57.....	17 0 50	−22 43	Diam. 5′; elongated northeast and southwest
58.....	17 2 30	−40 20	Round; diam. 27′; a darker core at northeast side
59.....	17 3 35	−27 19	Sink hole; diam. 1°
60.....	17 4 21	−22 17	Curved; 13′ long; extended northeast and southwest
61.....	17 7 26	−20 18	Small; elongated east and west
62.....	17 8 46	−20 44	Diam. 19′; very black in northwest part
63.....	17 9 19	−21 17	Very large; definite; curved; west end abrupt
64.....	17 9 57	−18 21	Cometary; west of M 9
65.....	17 11 50	−26 34	Definite; elongated east and west
66.....	17 12 2	−26 46	Definite; extended east and west; 8′ long
67.....	17 13 14	−26 44	Definite; extended north and south; joins Nos. 65 and 66
67*a*....	17 14 58	−21 46	Irregular; definite; diam. 13′
68.....	17 15 4	−23 40	Small; irregular; sharply defined; diam. 4′; southwest of No. 72
69.....	17 15 19	−23 48	Very small; irregular; sharply defined; extends north and south 4′
70.....	17 15 30	−23 55	Diam. 4′; sharply defined on west side
71.....	17 15 36	−23 53	Very small; diam. 1¼±
72.....	17 16 17	−23 37	S-shaped
73.....	17 16 46	−24 9	Very small; diam. 1′; extended north and south
74.....	17 17 30	−24 2	Diam. 5′
75.....	17 17 47	−21 55	Curved and scalloped marking
76.....	17 18	−24 18	Irregular; narrow
77.....	17 21	−23 45	Indefinite; diam. 1°
78.....	17 25	−25 30	Very large; diam. 3°
79.....	17 27 44	−19 19	½° long (southeast and northwest); narrow; sharply defined
80.....	17 30 16	−21 11	Small; definite; 3′ long, 1′ wide, southwest and northeast
81.....	17 30 55	−23 51	Very small; diffused
82.....	17 31 0	−23 42	Small; angular; well defined
83.....	17 31 20	−24 6	Irregular; diam. 7′
83*a*....	17 37 57	−19 57	Small, definite; diam. north and south 4′
84.....	17 39	−20 12	Sharply defined; length north and south 22′
84*a*....	17 48 40	−17 9	Round; diam. 16′; extension ¾° south
85.....	17 53 50	−23 1	Trifid nebula. See note

CATALOGUE OF 182 DARK MARKINGS IN THE SKY—*Continued*

No.	α 1875.0	δ 1875.0	Description
86.....	$17^{\mathrm{h}}55^{\mathrm{m}}\ 9^{\mathrm{s}}$	−27° 52′	Diam. 5′; several small stars in it
87.....	17 56 14	−32 6	"Parrot's head"; C.D. −32°13686 ($9^{\mathrm{M}}.8$) central; several smaller stars in it
88.....	17 56 56	−24 7	Length north and south 2′.7, width 0′.5; in M 8
89.....	17 57 20	−24 22	Very small; diam. 0′.5; in M 8
90.....	18 2 20	−28 18	Irregular; diam. 2′.7
91.....	18 2 30	−23 43	Diam. 5′; edge of diffused nebulosity
92.....	18 8 16	−18 16	Black spot, 15′ north and south, 9′ east and west
93.....	18 10 14	−17 52	Cometary
94.....	18 18 37	−10 44	Roundish; diam. 15′; indefinite
95.....	18 18 37	−11 49	Large; indefinite; diam. 30′; elongated
96.....	18 19 7	−10 19	Small; indefinite; elongated
97.....	18 23 7	−10 0	Irregular; not very definite; diam. 1°
98.....	18 25 31	−26 9	Very small; sharply defined
99.....	18 25 48	−21 34	Definite; 11′ long east and west, 3′ wide
100.....	18 26 7	− 9 14	Definite; irregular; curved; 16′ long northwest and southeast
101.....	18 26 7	− 8 54	Definite; irregular; 13′ long northeast and southwest, 4′ wide
102.....	18 30 37	−13 51	Rather definite; 8′ long northeast and southwest, 3′ wide
103.....	18 32 42	− 6 47	Irregular; diam. 4′; one small star in it
104.....	18 40 42	− 4 47	Small, definite, caret-shaped object
105.....	18 41 2	− 7 3	Very small; diam. ½′; close northeast of B.D. −7°4710 ($8^{\mathrm{M}}.9$)
106.....	18 42 11	− 5 13	Diam. 2′; extended north and south; free of stars
107.....	18 42 52	− 5 9	Irregular; diam. 5′; free of stars
108.....	18 42 52	− 6 27	Very small, diam. 3′
109.....	18 42 57	− 7 43	Irregular; diam. 0′.7; close east of B.D. −7°4726 ($8^{\mathrm{M}}.0$)
110.....	18 43 30	− 4 59	Irregular; diam. 11′
111.....	18 44	− 5 6	Region 2° in diam., full of dark structures
112.....	18 44 25	− 6 49	Diffused dark region; diam. 18′±
113.....	18 44 43	− 4 28	Diam. 16′; irregular; small star in west part
114.....	18 46 27	− 7 6	Dark spot with two small stars in it
115.....	18 46 36	− 6 49	Very small and black
116.....	18 46 48	− 7 11	Narrow semi-vacant region running south from B.D. −7°4755 ($9^{\mathrm{M}}.1$)
117.....	18 46 58	− 7 34	Round; very black; diam. 1′; sharply defined
117*a*....	18 47 4	− 5 0	Black; irregular; diam. 7′
118.....	18 47 10	− 7 36	Definite; diam. 2′
119.....	18 47 59	− 4 42	Very small; close northwest of B.D. −4°4623 ($9^{\mathrm{M}}.2$)
119*a*....	18 48 0	− 5 20	Irregular dark region; diam. ½°
120.....	18 48 16	− 4 45	Very small; close southeast of B.D. −4°4623 ($9^{\mathrm{M}}.2$)
121.....	18 48 48	− 4 46	Small dusky spot; 12 mag. star in center
122.....	18 50 11	− 4 55	Small; diam. 4′; narrow extension 4′ north
123.....	18 51 2	− 4 53	Roundish; diam. 1½′
124.....	18 51 5	− 4 31	Small dark spot; diam. 3′
125.....	18 51 45	− 4 33	Dark; lune-shaped; diam. 9′
126.....	18 52 25	− 4 42	Dusky; round; rather definite; diam. 8′
127.....	18 54 52	− 5 37	Irregular; diam. 4½′
128.....	18 55 3	− 4 45	Irregular; dusky; fairly well defined; diam. 10′
129.....	18 55 3	− 5 29	Very black; sharply defined; diam. 5′
130.....	18 55 16	− 5 45	Dusky; not well defined; length 7′

CATALOGUE OF 182 DARK MARKINGS IN THE SKY—*Continued*

No.	α 1875.0	δ 1875.0	Description
131.....	$18^h55^m39^s$	$-4°33'$	Black spot; diam. 0′.7; curve of small stars north
131*a*....	18 56 32	− 4 31	Diam. 3′
132.....	18 57 51	− 4 37	Dark; fairly well defined; diam. 8′
133.....	18 59 29	− 7 5	Cometary; close west of B.D. −7°4852 ($9^{M}.2$)
134.....	19 0 11	− 6 26	Round; diam. 6′
135.....	19 0 57	− 4 7	Large dusky spot; diam. 13′
136.....	19 3 0	− 4 12	Dusky spot; diam. 8′; 10 mag. star near middle
137.....	19 9 32	− 1 33	Projection near south end of No. 138
138.....	19 10	0 0	Great curved semi-vacant lane over 3° in length
139.....	19 11 32	− 1 38	Narrow black spot; 10 mag. star on southeast edge
140.....	19 13 39	+ 4 59	Semi-vacant region; diam. 1°
141.....	19 13 52	+ 1 40	Semi-vacant region north of north end of No. 138; diam. 16′
142.....	19 34 0	+10 18	Large; irregular; close west of B.D. +10°4016 ($8^{M}.7$)
143.....	19 35 30	+10 43	Rather narrow angular marking
144.....	19 54	+34 30	Large semi-vacant region; 6° by 3°
145.....	19 58 16	+37 21	Sharply defined; $\frac{3}{4}$° long east and west
146.....	19 58 49	+35 40	Very small
147.....	20 2 7	+35 1	Very narrow sinuous dark lane 11′ long east and west
148.....	20 45 30	+59 12	Very small; round; indefinite; diam. 3′
149.....	20 46 28	+59 4	Very small; round; indefinite; diam. 2′
150.....	20 47 47	+59 50	Curved dark marking; 1° long
151.....	21 4 41	+55 49	Very small; very dark; diam. 1′
152.....	21 11 30	+61 13	Small; 15′ by 3′ southeast and northwest
153.....	21 17 42	+55 55	Black, irregular; diam. 1′
154.....	21 18 1	+56 5	Narrow; 8′ long northeast and southwest
155.....	21 27 25	+44 25	Round; diam. 13′; indefinite
156.....	21 29 17	+45 2	Diam. 8′; sharp-pointed to north
157.....	21 29 34	+54 1	Round; diam. 5′
158.....	21 32 22	+42 51	Dark spot; diam. 3′
159.....	21 33 30	+42 40	Irregular partially vacant region; diam. 25′
160.....	21 33 40	+55 40	Large, dark, irregular; diam. 31′ northeast and southwest
161.....	21 36 38	+57 5	Small, black spot
162.....	21 37 8	+55 55	Very thin, curved, dark strip north and south
163.....	21 37 38	+56 5	Small; very black; pointed to the south
164.....	21 42 2	+50 31	V-shaped vacancy; diam. 5′–10′
165.....	21 45 10	+59 38	Length east and west 18′, width 1′
166.....	21 47 16	+59 30	Round; diam. 5′; small star on southwest edge
167.....	21 48 8	+59 29	Small; irregularly round; diam. 5′
168.....	21 48 32	+46 41	Small nebula at east end of dark lane
169.....	21 53 38	+58 25	Elliptical black ring; diam. 1°
170.....	21 51 50	+58 22	Irregular black strip 26′ long northeast and southwest
171.....	21 57 43	+58 16	Irregular broken region; diam. 19′
172.....	22 2 43	+58 36	Irregular dark spot
173.....	22 2 53	+59 4	Diam. 4′; rather definite
174.....	22 3 8	+58 28	Narrow; irreg.; 19′ long northeast and southwest
175.....	22 11	+69 19	Large; diam. 1°; bright nebula in north part

No.

13 Irregularly darker inside with a rather sharp, short, narrow extension toward the south.

14 A bright spot in the dark nebula No. 22.

15 See description on page 4.

20 This is the dark object mentioned by Espin in *M.N.*, 58, 334, 1898. It is close north of a small group of faint stars and is $6'$ or $8'$ in diameter. In a somewhat larger vacant space. It is not so definite as No. 15.

22 This is a very large region of obscure nebulosity with several darker streaks in it trending northwest. It is partly separated from another starless region lying about 2° to the west and north. Near the middle of the first of these regions is a small luminous nebula with a faint fan-shaped extension for $10'$ to the northeast. This is undoubtedly a brighter condensation of the great obscuring mass surrounding it. The position of this small nebula is: $\alpha=4^{h}32^{m}13^{s}$, $\delta=+25°30'\!.7$.

25 This is a good example of a dark or more or less starless region.

26 Nos. 26, 27, and 28 are close northwest of B.D. $+30°741$ ($6^{\mathrm{M}}\!.8$). (This star is involved in feeble nebulosity.)

30 There is a nebulous border, $\frac{1}{2}°$ wide, to the semi-vacant region extending 1° east and north from B.D. $+12°803$ ($7^{\mathrm{M}}\!.0$). Apparently this nebulous border extends many degrees east in a very diffused manner. There is a narrow dark lane extending south from the semi-vacant region, running close west of B.D. $+12°803$ to a point nearly 1° due south of B.D. $+12°801$ ($6^{\mathrm{M}}\!.9$). There is a small fan-shaped nebula close north of a small star in the position $\alpha=5^{h}24^{m}30^{s}$, $\delta=+12°3'\!.9$. A small strip of nebulosity extends $5'$ southwest from this small star. These two nebulae are probably the brighter parts of a large obscure nebulosity.

33 See *Astrophysical Journal*, 38, 496, 1913, Plate XX.

35 This has an extension to the southeast. There are three small stars north of it.

37 This region is abruptly terminated on the south side by the nebulosity extending north from 15 Monocerotis. Its west side ends abruptly in $\alpha=6^{h}22^{m}15^{s}$, $\delta=+12°26'$.

39 A small elongated spot in the nebulosity from 15 Monocerotis.

40 This spot and No. 41 are connected with the great nebula about Nu Scorpii and are doubtless denser opaque masses of the nebula. There is a larger dark region belonging to this same system in $\alpha=16^{h}23^{m}$, $\delta=-19°30'$ (No. 43), from which broken dark lanes somewhat like those from Rho Ophiuchi, though less marked, run eastward for some degrees.

42 This object is put in the list because, though not wholly dark, it is partly so, and the evident obscuration produced by it puts it in the class for which the catalogue is made. The semi-vacant region in which this, the great nebula of Rho Ophiuchi, lies is about $3\frac{1}{2}°$ in diameter.

44 This is the middle of the main part of the great vacant lane that runs east from the region of Rho Ophiuchi. It is about $32'$ wide and has its

No.

beginning in a vacant area in which is the star C.D. −24°12695 ($5^{\mathrm{M}}.5$), $\alpha = 16^{\mathrm{h}}22^{\mathrm{m}}38^{\mathrm{s}}$, $\delta = -24^\circ 51'$. It runs slightly northeast to about $\alpha = 16^{\mathrm{h}}40^{\mathrm{m}}$, $\delta = -23^\circ 40'$. From this point it extends in a broken chain of dark spots to a little south of No. 63. Its total length is therefore about 10°.

44*a* This appears to be a real, dark object.

45 Extended northeast and southwest with an irregular projection southwest that connects it with the Rho Ophiuchi region. This appears to be a real, dark object.

46 A string of small stars in the middle.

47 Connected with No. 51 by two sharp lanes.

48 Extended northeast and southwest. Close west and north of C.D. −40°11088 ($8^{\mathrm{M}}.0$). This seems to be a real object.

50 This is an irregular square 15′ in diameter, sharply defined on the east side and diffused on the west, where there are broken extensions for ½°. The star C.D. −34°11418 ($9^{\mathrm{M}}.8$) is near the middle. The stars C.D. −34°11422 ($9^{\mathrm{M}}.8$), −34°11426 ($9^{\mathrm{M}}.8$), and −34°11427 ($9^{\mathrm{M}}.9$) are on the eastern border.

51 Two straggling dark lanes run west to No. 47.

52 One of a straggling group toward the eastern end of the dark lane from Rho Ophiuchi.

55 C.D. −31°13582 ($8^{\mathrm{M}}.9$) near the center.

56 Close east of C.D. −31°13609 ($9^{\mathrm{M}}.0$) and −31°13618 ($9^{\mathrm{M}}.3$). This seems to be a real object.

59 At the west end of a wide broken lane from No. 78.

60 This and No. 57 are the most distinct of a group of dark spots southwest of the square end of No. 63.

62 This seems to be a real object.

63 This is the center of a great curved figure convex to the north 1°37′ long and 19′ wide. See description of Plate III.

64 See description of Plate IV.

72 This is a striking object. It is a thin, curved black marking, the exact form of the letter *s* or the figure 5, as the imagination or point of view may dictate. The southeast branch runs east for some distance passing close south of the star C.D. −23°13370 ($9^{\mathrm{M}}.9$). Its average thickness is about 2′ to 3′. The position in the catalogue is for the southern part of the figure, or the bottom of the *s*. See Plate III.

74 Slightly curved, extending 5′ north from the star C.D. −24°13325 ($7^{\mathrm{M}}.1$). It is sharpest on the west and south sides.

75 B.D. −21°4598 on the north edge. See description of Plate II.

76 This is a broken, black strip, ½° long, extending east and west.

77 Connected by a vacant strip with No. 78.

78 C.D. −26°12152 ($6^{\mathrm{M}}.2$) is south of the middle.

80 Close southeast of B.D. −21°4674 ($8^{\mathrm{M}}.1$).

81 Close southwest of N.G.C. 6401.

No.

82 8′ north of N.G.C. 6401.

83 Narrow extension to the south. Several small stars in it.

84 See description of Plate II.

84*a* B.D. $-19^\circ 9457$ ($9^{\mathrm{M}}.5$) in the center.

85 This is the southern part of the Trifid Nebula, N.G.C. 6514. The dark markings in this nebula are too well known to insert in this catalogue.

86 There is a small cluster close east of this spot. See page 3.

88 There are other well-known markings in this nebula. This one and No. 89 are given as illustrations of dark markings in the sky shown in relief against a nebulous or other luminous background. In this case these may be non-luminous opaque parts of the nebula itself.

90 There are many similar dark spots in this region, but this one is perhaps the most distinct and is fairly characteristic of the others.

91 This is a round, sharply defined, dark bay in the east side of a very diffused nebulosity which extends east from M 8. Photographs on July 26 and 27, 1905, show the planet Uranus in this bay. It was nearly central in the bay on July 27.

92 This black spot is very sharply defined on its east edge but less definite on the west. There is a twelfth magnitude star near the middle with several other small stars. See *Astrophysical Journal*, 38, 496, 1913, Plate XX, for an account of this remarkable object.

93 This object has a sharply defined black head 2′ in diameter with a diffused tail 15′ long running south. See Plate XX, as in last note.

98 See description of Plate IV.

99 The east end is the larger with a faint star in it.

100 Larger at east end; diameter 12′. There is a small star in the southeast edge.

101 Separated from No. 100 by a scattering of small stars.

103 Apparently a real object, the south of several dark markings covering a space 45′ in diameter. They form a notched outline to the northwest side of the large star-cloud in Scutum.

104 This consists of a narrow straight black line 16′ long north and south, with a narrow spur at the south end extending west and north for 5′. The width of these lines is about $0'.7$.

108 This object is not black. There are two narrow diverging lanes extending to the north. Some small stars in it.

110 Free of stars. A small star near the west edge.

111 See page 6 for description.

112 Perhaps due to want of stars.

115 Very narrow, running north and south for 7′; width $1'.4$. It is in a small vacant region.

117 This is on the border of a larger dark region with dark lanes running to the north and northeast. It is separated from No. 118 by several very small stars.

No.

118 More definite and blacker than No. 117.

119*a* This is liberally sprinkled with stars in its northwest half. There are several narrow dark lanes in its southeast portion.

120 The east side is bounded by a curve of very small stars.

122 Round, small, and black. The entire object is fairly well defined and uniformly dark. There is a small star $4'$ to the northeast.

123 This has a narrow spur $4'$–$5'$ long from its northeast edge, and a narrow lane $18'$ long and $1\frac{1}{2}'$ wide from its west side to the northwest.

125 Fairly well defined on the north border. Some faint stars in it.

126 An irregular narrow dark streak extends $9'$ to the northeast; another from its south side runs close to B.D. $-4^\circ 4650$ ($7^{\mathrm{M}}.4$). A small star in the northeast edge.

127 Very black in its southeast portion, diffused on the west side. A short line of faint stars on its west edge.

128 A narrow extension $13'$ long from the northeast side. A small star close west of it. This is in a larger vacant region.

129 Very black, sharply defined, with a considerable small star at the west end. Somewhat rectangular in form with projections to the east. It is the blacker western portion of a dark region $10'$ in diameter. Nos. 127, 129, and 130 are connected in an irregular curve convex to the west.

131*a* This is a very small black spot with a small star on the north border and another on the south, with a fine curve of six or seven very small stars running northwest from its north edge.

132 Tadpole-shaped with B.D. $-4^\circ 4680$ ($9^{\mathrm{M}}.0$) in center of the larger (west) part. Length east and west $16'$.

133 This object begins very close west of B.D. $-7^\circ 4852$ ($9^{\mathrm{M}}.2$). From this it gradually widens and curves north like a plume, with the convex side west, to a distance of about $18'$. The south end is intensely black. The northern part is dark with 4 or 5 considerable (small) stars in it. See description of Plate IV.

134 This is dark, but not sharply defined. There is a small star on the south edge and a similar one on the north edge.

135 The star B.D. $-4^\circ 4698$ ($9^{\mathrm{M}}.2$) is on the north border and B.D. $-4^\circ 4702$ ($9^{\mathrm{M}}.2$) on the southeast border.

138 Perhaps the most striking feature of this great curved "lane" is the definiteness of its eastern outline. To the west it is less definite and in the lower, south, half it consists of dark branching structures which blend into a part of the Milky Way relatively dimmer than that to the east. The stars embraced by the curve to the east are much brighter or coarser than those to the west. If the stars to the west were fewer in number, or fainter, the eastern outline would closely resemble the great head of the star-cloud near M 11 in Scutum. The southern termination is in a small, comet-shaped, very black spot, apparently a real object, sharply defined on its south edge.

No.

139 This spot is the beginning of a great curved lane ending in about $\alpha=19^{\mathrm{h}}14^{\mathrm{m}}$, $\delta=+1\frac{1}{2}^\circ$. This lane is concave to the east, on which side it is more definite.

143 The outline of a square $\frac{1}{2}^\circ$ in diameter, with the west side missing.

144 See page 10.

145 This is a triangular semi-vacant area. It is covered by a sprinkling of considerable small stars with the star B.D. $+37^\circ3736$ ($9^{\mathrm{M}}.5$) in the center.

146 This spot is like a dark border to the southwest side of the star B.D. $+35^\circ3930$ ($7^{\mathrm{M}}.0$).

150 This is the center of the dark marking. Its head is in the position $\alpha=20^{\mathrm{h}}51^{\mathrm{m}}0^{\mathrm{s}}$, $\delta=+59^\circ45'$. The west end or tail is in $\alpha=20^{\mathrm{h}}43^{\mathrm{m}}50^{\mathrm{s}}$, $\delta=+49^\circ20'$. This object is fully described in the text.

151 This appears to be a real object in a slightly larger vacancy.

152 Close north of B.D. $+61^\circ2103$ ($8^{\mathrm{M}}.9$). See *Astrophysical Journal*, 43, 1, 1916. It is there wrongly stated that the object is south of the B.D. star. The west end is larger with a small star in it.

154 The star B.D. $+55^\circ2565$ ($7^{\mathrm{M}}.4$) is at the south end.

155 There are four small stars in a line crossing this object east and west.

156 The star B.D. $+44^\circ3865$ ($4^{\mathrm{M}}.1$) is central.

157 A semicircle of faint stars forms the western border.

158 This spot is similar to those in the region of No. 159. There are some others north and west of this.

159 This is the center of an irregular, partially vacant region 25′ in diameter. It is suggestive of the presence of a dark or faint nebulosity, as there are several small darker spots in it. It lies 40′ northwest of Nova Cygni of 1876, and involves the stars B.D. $+42^\circ4177$ ($5^{\mathrm{M}}.2$), $+42^\circ4172$ ($8^{\mathrm{M}}.4$) and $+42^\circ4164$ ($7^{\mathrm{M}}.7$). It is somewhat extended east and west. A very long exposure on this region would doubtless be interesting.

160 The east side is best defined with two dark projections from it 30′ apart. An irregular, narrow, dark lane runs north from it for 2° to a large nebulous region.

161 This is one of the comet-shaped objects. The head is very sharp and black, 3′ across. This extends north for some 3′, then widens out into a less dense tail about 13′ long.

162 A curve of small stars on the southwest end.

163 There are two dark streams running from this to the north.

165 B.D. $+59^\circ2424$ ($7^{\mathrm{M}}.8$) is on the north edge and B.D. $+59^\circ2427$ ($7^{\mathrm{M}}.7$) near the southeast end.

168 This nebula is 10′ in diameter with over a dozen small stars of different magnitudes in it. There is no central condensation, nor does the nebula condense about any of the stars. There are some dark markings in it. The dark lane is $1^\circ.7$ long and 9′ wide. See *Lick Observatory Publications*, 11, Plate 81.

No.

169 This incloses an island of small stars. The dark ring is broadest on its east side where its thickness is 22′. The northern part is very black and straight.

170 This is the northwest border of the island of small stars. It is sharply defined and darker than the sky.

171 This is at the east end of the island of small stars.

173 There is a small star on the north edge and also one on the southwest edge.

175 This is a large dark spot, extended north and south, 62′ in its largest diameter. In its upper part is the star B.D. +69°1231 ($8^{M}.8$) which is nebulous. This is apparently a large dark nebula, the brighter part of which forms the star +69°1231. In *Monthly Notices*, 69 (December) 1908, Dr. Max Wolf gives a photograph of the nebula, stating that the object was discovered by Dr. Kopff at Heidelberg on October 12, 1908. It is conspicuous on a photograph of mine made with the Willard lens at the Lick Observatory, September 24, 1895, with $5^{h}0^{m}$ exposure. It is also shown on a photograph of mine made with the Bruce telescope, July 20, 1904, with an exposure of $3^{h}1^{m}$. By inadvertence reference to this object was omitted in *Lick Observatory Publications*, 11, where it is cut out by the matting in Plate 83.

I am greatly indebted to Miss Mary R. Calvert for invaluable aid in the preparation of this catalogue.

YERKES OBSERVATORY, WILLIAMS BAY, WIS.
December 10, 1918

STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS[1]

NINTH PAPER: THREE NOTES ON CEPHEID VARIATION

By HARLOW SHAPLEY

At least three classes of stars may be independently used to ascertain the distances of globular clusters through luminosity correlations—Cepheid variables, B-type stars, and the giants of the redder spectral classes. The giant red stars in local parts of the galactic system, as well as in globular clusters, are now known to have a limited dispersion of absolute luminosity; and if we should assume the absolute magnitude in the mean to be the same in both regions, it is readily shown that the resulting parallaxes of the clusters would be closely comparable with the values obtained in the present investigation.[2] The stars of spectral type B (color-class *b*) form a second class of objects that are generally recognized as valuable criteria of distance, not only because of a fairly small dispersion of intrinsic brightness, but also because much is known of the actual absolute magnitudes through studies of community motion and parallactic drift. Apparently, however, there is a tendency for B-type stars to fall into groups that may differ in real brightness by one or two magnitudes,[3] thus introducing a large percentage of uncertainty into the distances depending on magnitude correlations. If, as seems necessary from related investigations, the most luminous *b*-class stars in clusters are equated in luminosity with the galactic B stars that have an average absolute magnitude near zero, the distances of globular clusters (and of all Cepheids) are essentially as now adopted. On the other hand, if the brightest *b*-class stars are equated in luminosity with the most luminous of galactic B's, or if much weight be placed upon a direct application of the fragmentary luminosity-curves of B stars,[1] the distances and all the dependent dimensions would be considerably greater, and all giant red stars and Cepheid variables would be systematically much brighter in clusters than in the general galactic system.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 154.

[2] For example, see Division III of the parallax determinations of Messier 13 listed in Table XXVII of *Mt. Wilson Contr.*, No. 116, 1915. The parallax there derived from red giants is 0″00010; that now adopted is 0″00009. Applying this method we would readily obtain also the absolute magnitudes of all Cepheids, and then derive practically the same individual parallaxes for them as given in the eighth paper.

[3] Kapteyn, *Mt. Wilson Contr.*, No. 82, 1914; Charlier, *Meddelanden från Lunds Astronomiska Observatorium*, Series 2, No. 14, 1916.

The present study has shown that for the third class of bright stars common to clusters and the general system—the Cepheid variables—the luminosity is simply a function of the period of light-variation, and, except for some uncertainty in the zero-point of the absolute scale, may be very accurately known. In consequence the Cepheids become of so much greater weight in the estimation of the parallaxes of globular clusters that other classes of highly luminous stars can best be used only to check the results, or as secondary standards.[2]

Although the distances obtained independently of the Cepheids would, as suggested above, be much the same as we have found, and the same conclusions as to the size and structure of the general system of clusters would result, the computed certainty of the distances and dimensions would be of a much lower order. The adopted numerical values therefore depend mainly upon the Cepheid variables, and the investigation of all phenomena related to the luminosity of these stars becomes appropriate. In particular it is important to see if there is any evidence that the variables of a given period in different systems are unlike in median magnitude. In the present communication three notes are brought together that bear on the question of Cepheid variation.

I. THE RELATION OF PERIOD TO LUMINOSITY

In the second and third sections of the sixth paper of this series we have determined a very definite observational relation between the period of light-variation of a Cepheid variable and its total light emission. The luminosity-period curve, based on observations from seven different stellar systems, is reproduced in the lower part of Fig. 1, where absolute magnitude is plotted against the logarithm of the period in days. The curve is defined by more than 330 stars, of which nearly 40, having periods longer than a day, fall along the curve at points where the variation of magnitude with period is rapid. Considering the uncertainties in the periods and magnitudes for some of the stars, the average deviation from the smooth curve is remarkably small—there is hardly a Cepheid known to be discordant.

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[1] *Mt. Wilson Contr.*, No. 116, p. 81, 1915; cf. also Eddington, *Monthly Notices*, 77, 608, 1917.

[2] Through the method of measuring distance by means of the "25 brightest" stars, the red giants have been used in the rôle of secondary standards to determine the parallaxes of nearly all the clusters of Table V in the seventh paper.

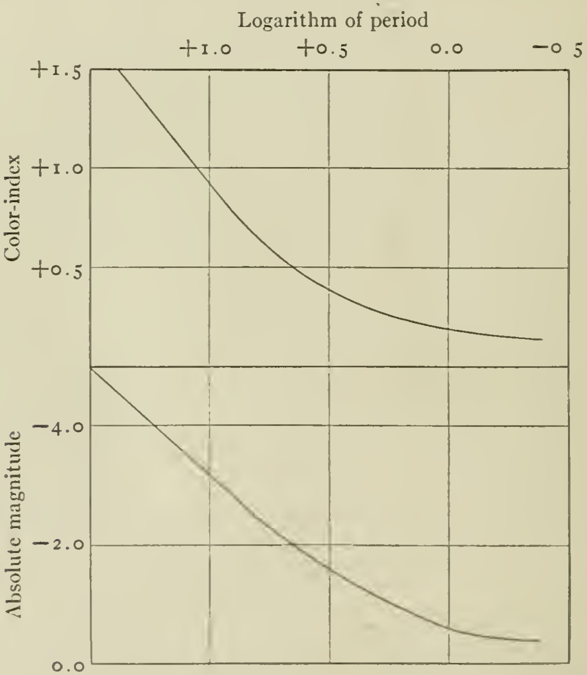

FIG. 1.—*Above:* Relation of period to color-index for Cepheid variables. *Below:* Relation of period to median absolute magnitude for Cepheid variables.

From the curve we see first that the decrease of median brightness with decreasing period is essentially linear until a period of less than three days is reached, and second that the absolute magnitude decreases no farther after the period has become a little shorter than one day. Referring to another curve in the same paper, which is reproduced in the upper part of Fig. 1, we observe that the change of median color-index with period is of much the same general character. A few remarks may be made on the possible interpretation of these curves.

It has been shown in an earlier paper[1] that the relation of period to spectral type and color-index can be provisionally explained (if we adopt a pulsation hypothesis of Cepheid variation) as a result of increasing density with decreasing color-index for giant stars. Thus a general relation, which is known to have a wide application in astronomical and physical problems and which holds without doubt in the present case, gives the period, P, of an oscillation in a gaseous star as a function of the radius and the surface gravity; or, if $\bar{\rho}$ is the mean density,

$$P^2 \propto \frac{1}{\bar{\rho}} \tag{1}$$

For giant stars the dependence of mean density on spectrum is approximately known from eclipsing binaries, and the application of (1) to Cepheid variables of known period gives for each spectral type a mean density of quite the same order as that found from eclipsing stars. Thus, through the intermediary of the mean density, the observed connection of median color-index and period is readily understood.

The reason for the dependence of luminosity on period is not so definite, but the following discussion seems to indicate the physical interpretation. If L be the total luminosity of a giant star, we may write[2]

$$L = \pi r^2 J$$

where r is the radius and J the luminosity per unit of apparent surface. Now $J = f\,(T, \rho_s, i)$; that is, the light intensity per unit area depends, not only upon the effective surface temperature, T, but probably also upon the density at the stellar surface, ρ_s, and the degree of ionization, i—the last involving, it may be, the chemistry of the star as well as its ordinary physical properties. But with good cause we believe that the effective temperature (and hence the color-index, C.I.) is by far the most important factor in surface brightness. Denoting mass by μ, we may write

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[1] *Mt. Wilson Contr.*, No. 92; *Astrophysical Journal*, 40, 448, 1914.

[2] Neglecting darkening at the limb.

$$L \propto r^2 f_1(T) \propto \left(\frac{\mu}{\bar{\rho}}\right)^{\frac{2}{3}} \phi(\text{C.I.}).$$

Introducing (1),

$$L \propto (\mu P^2)^{\frac{2}{3}} \phi(\text{C.I.}) \qquad (2)$$

and we have a physical explanation of the interdependence of the luminosity and period in Cepheid variation.

For the absolute magnitude, M, of stars differing little in spectral type (such as the Cepheid variables for which proper motions are known) we derive from (2)

$$M \propto 2 \log P + \log \mu.$$

It appears from Eddington's theoretical work on giant stars as well as from observational data relative to binaries of various types that the range in the masses of stars is small, so that the approximate theoretical relation

$$M = k \log P \qquad (3)$$

is of the same form as that observed for the Cepheid stars of longer period (Fig. 1).

The inclusion of log μ in (3) would not in the mean disturb the linear form of the relation, for the dispersion in mass is, so far as we know, comparable for giants of different spectral types redder than A. But over a wide range of spectrum ϕ (C.I.) becomes important in maintaining the luminosity in the face of contraction. The increase of surface intensity, with the progression toward bluer spectrum, may wholly counterbalance the tendency to diminish the total light-emission through shortening periods and smaller stars—especially near the blue end of the giant series where critical and asymptotic conditions are more likely to prevail[1]—and in consequence the luminosity-period curve theoretically should flatten out as observed. If, however, the counterbalance is maintained throughout the whole giant series and luminosity depends only on the mass, as some important researches suggest,[1] then (1) the mass apparently determines when in a star's history Cepheid variation may occur, (2) the period never changes materially during the life of the variation except possibly for cluster-type variables, and (3) the luminosity-period law may more appropriately express the mass-period or mass-density relation of Cepheid variation.

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[1] "The concurrent progression toward a definite limit of luminosity, spectrum, and period suggests that, in the evolutionary sequence of stars, Cepheid variation is abruptly limited at or near the blue end of the giant series because of the changing physical conditions in the interiors of the gaseous masses."—*Mt. Wilson Contr.*, No. 151, p. 21, 1917.

It appears from the foregoing consideration that, granting the pulsation hypothesis of Cepheid variation, there is a definite reason for the dependence of total luminosity on period and a fairly plausible interpretation of the almost linear relation between the logarithm of the period and absolute magnitude. Quantitative evaluation of the theoretical equations should be possible eventually, and in fact has already been made tentatively. The computed results roughly conform with observation, but little weight can be attached to this agreement because of the present necessity of rather vague assumptions as to the progression, with spectral type, both of J and of darkening at the limb.

Eliminating the logarithm of the period from the two curves of Fig. 1, we obtain in Fig. 2, in the plot of absolute magnitude as a function of color-index, a curve that may be of considerable importance in Cepheid theories. For comparison with this curve the relation between absolute magnitude and color-index for the giant stars in four globular clusters is plotted along a broken line in the same figure.[2] We note that the change of luminosity with advancing spectral type is about the same for Cepheid variables of the galactic system as for all giant stars of globular clusters. Perhaps our more extensive knowledge of the former can throw light on the general problem of the relationship of successive spectral types.

The constant difference between the curves of Fig. 2, which is certainly larger than any systematic error involved, indicates that for a given color Cepheids are more than twice as bright as other

[1] Various papers by Eddington and Jeans in the *Monthly Notices*, *Nature*, and *The Observatory* during 1916 and 1917. Cf. the tenth paper of this series, p. 8.

[2] Adapted from the mean of the results given in Table II of *Mt. Wilson Communications*, No. 34; *Proceedings of the National Academy of Sciences*, 2, 525, 1916.

stars; or rather, since the largest uncertainty is probably in the colors of the Cepheids, it indicates that for a given magnitude the variables appear to be nearly 0.4 mag. bluer than other giant stars. If minimum color and magnitude of the Cepheids be substituted for the median values, the two curves become nearly coincident.

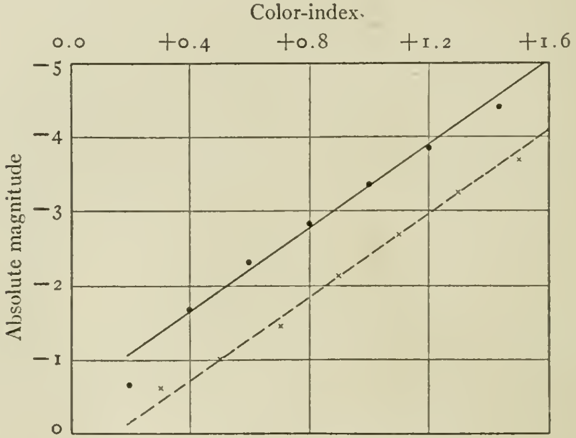

FIG. 2.—Relation of color-index to absolute magnitude for Cepheid variables. The broken line represents the corresponding relation for the giant stars of Messier 3, 5, 13, and 15.

II. THE COMPOSITE COLOR-CURVE OF 103 CLUSTER-TYPE VARIABLES

The determination, from plates made with the 60-inch reflector, of the magnitudes of a sequence of reference stars in Messier 3, permits the transformation to the Mount Wilson system of the light-curves derived by Bailey.[1] Since details of the derivation of magnitudes for the comparison stars will appear elsewhere, it will suffice for the present note to give only the final photographic and photo-visual results. The first two columns of Table I contain the designation of the comparison star by Bailey and the corresponding number in von Zeipel's catalogue.[2] The Harvard and Mount Wilson photographic magnitudes, in the following columns, differ in part because of the well-known systematic difference between

[1] *Harvard Annals*, 78, Part 1, 1913.

[2] *Annales de l'Observatoire de Paris*, 25, 1, 1908.

the two systems, but mainly because of the somewhat provisional nature of the published Harvard magnitudes for this sequence of stars.[1] By plotting against each other the numbers of the third and fourth columns a fairly smooth curve is obtained which can be used to change the observed photographic magnitudes from one system to the other.

TABLE I

COMPARISON STARS IN MESSIER 3

Designation		Harvard Photographic Magnitude	Mt. Wilson Photographic Magnitude	Mt. Wilson Photo-Visual Magnitude	Color-Index
Harvard	von Zeipel				
a...........	740	13.50	14.10	13.40	$+0\overset{\mathrm{M}}{.}70$
b*...........	238	13.58	14.27	12.48	+1.79
c...........	640	13.98	14.40	13.22	+1.18
d...........	263	14.22	14.53	13.41	+1.12
e...........	250	14.50	14.81	13.91	+0.90
f...........	218	14.98	15.04	14.51	+0.53
g...........	227	15.28	15.25	14.60	+0.65
h...........	258	15.70	15.52	15.54	−0.02
k...........	609	16.00	15.66	15.70	−0.04
l...........	1131	16.23	15.81	15.48	+0.33
m...........	1055	16.49	15.95	15.26	+0.69
n...........	1327	16.82	16.17	15.49	+0.68
o...........	....	17.15	16.43	15.78	+0.65
p...........	....	17.63	16.79	16.38	+0.41

* Variability suspected by Bailey.

The close similarity of the photographic light-curves of the typical variables in Messier 3 makes possible the general representation of them all by the single mean curve[2] which is tabulated in the first two columns of Table III and drawn as a full line in Fig. 3. Whatever irregularities may exist in the individual variations are smoothed out, of course, in this mean; and in particular the range may be slightly too small because the rapid variation at maximum is concealed to some extent by the length of exposures necessary on the Harvard plates. Also it is not likely, except in the mean curve, that the light actually remains constant for three hours at minimum. It is to be noted in this mean result, however, that both the form of the curve and the photographic amplitude of 1.2 magnitudes are much the same as those usually observed for isolated cluster-type variables.

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[1] Cf. *Mt. Wilson Contr.*, No. 115, p. 13, n. 1. In a recent letter Professor Bailey writes that the magnitudes are being revised on the basis of the Harvard Polar Sequence.

[2] Derived from Bailey's Table XIX, *op. cit.*, p. 89.

The derivation of visual or photo-visual curves for each of these variables would entail much work, but a mean photo-visual curve can be derived with considerable certainty and without difficulty through the determination of a composite color-curve. Making use of the remarkable similarity of the variables and noting that the slightly differing periods are not commensurable, we may, if we choose, interpret the hundred magnitudes which are derived for variable stars from a single photograph of the cluster as a hundred different observations distributed at random along the light-curve of one variable—that is, along the mean light-curve. Similarly, the determination of the color at a given instant for the hundred variables gives immediately a hundred observations for the determination of a mean color-curve. Some of the periods, however, are not known accurately enough to allow the computation of phases for the time of observation, and the curve, therefore, gives color as a function of photographic magnitude rather than of time. To distinguish this property it will be called a composite color-curve.

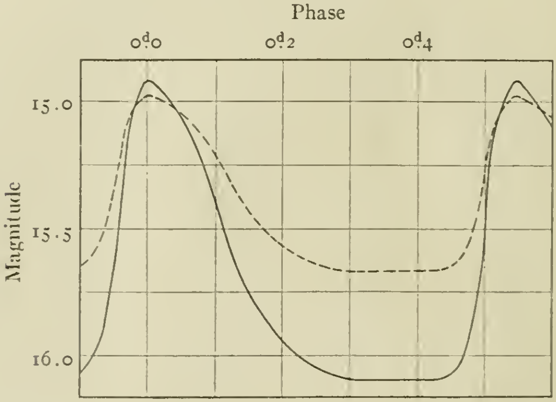

FIG. 3.—Mean photographic and photo-visual light-curves of 103 cluster-type variables in Messier 3.

For the measures of color, two pairs of photographs of Messier 3 were made on December 20, 1917, each pair consisting of a

TABLE II

Variable	Plates 4309 and 4310		Plates 4311 and 4312	
	Photographic Magnitude	Color-Index	Photographic Magnitude	Color-Index
173	15.06	$-0\overset{\mathrm{M}}{.}15$	14.68	$-0\overset{\mathrm{M}}{.}37$
181	16.33	....	16.30	+0.18
214	15.54	−0.01	15.68	+0.08
221	16.28	+0.36	16.34	+0.47
222	15.91	+0.19	15.68	+0.37
223	15.41	+0.29	15.54	+0.21
226	15.70	+0.06	15.92	+0.19
234	15.70	+0.35	15.87	+0.47
249	15.06	+0.03	15.64	+0.39
252	15.70	+0.12	15.95	+0.27
253	15.77	+0.44	15.51	+0.17
264	16.23	+0.44	16.24	+0.23
278	15.20	+0.08	15.39	+0.12
279	15.77	+0.22	15.95	+0.52
296	15.33	+0.17	15.61	+0.39
305	15.70	+0.28	15.95	+0.52
318	13.95	+1.61	13.88	+1.52
321	16.20	+0.43	16.05	+0.24
328	15.91	+0.22	15.99	+0.52
338	15.20	−0.08	15.39	+0.05
351	15.87	+0.29	15.97	+0.36
353	15.65	+0.21	15.61	+0.21
355	15.49	+0.32	15.47	+0.34
361	16.20	+0.38	16.12	+0.38
362	15.83	+0.39	15.86	+0.39
366	15.91	+0.33	15.51	+0.36
374	15.14	+0.21	15.34	+0.12
378	16.24	+0.42	16.05	+0.15
379	15.61	+0.32	15.68	+0.25
384	14.47	−0.29	14.83	−0.09
391	16.20	....	16.18	+0.38
396	15.91	+0.22	16.09	+0.29
397	15.94	+0.31	16.09	+0.75
404	16.46	+0.64	16.03	+0.53
411	16.20	+0.46	16.09	+0.35
425	15.98	+0.34	16.09	+0.59
451	15.41	+0.20	15.64	+0.30
455	15.77	+0.01	15.84	+0.17
479	15.36	+0.20	15.61	+0.30
537	15.93	+0.43	16.12	+0.50
541	14.88	+0.14	15.08	−0.05
616	14.62	−0.16	15.01	+0.09
623	14.97	−0.03	15.34	+0.18
644	16.35	+0.53	16.05	+0.55
650	15.91	+0.06	16.03	+0.08
656	14.97	−0.16	15.61	+0.34
679	15.41	+0.25	15.68	+0.22
700	15.91	+0.24	15.92	+0.42
710	16.46	+0.72	16.34	+0.49
713	16.20	+0.67	15.87	+0.13
759	16.28	+0.59	16.13	+0.63
765	15.14	+0.21	15.61	+0.40

TABLE II—*Continued*

Variable	Plates 4309 and 4310		Plates 4311 and 4312	
	Photographic Magnitude	Color-Index	Photographic Magnitude	Color-Index
775	16.33	$+0^{\mathrm{M}}.66$	16.23	$+0^{\mathrm{M}}.37$
776	15.40	+0.22	15.68	+0.25
818	15.21	+0.27	15.25	+0.22
840	15.91	+0.09	16.10	+0.44
873	15.14	+0.14	15.44	+0.26
922	15.77	+0.27	15.83	+0.37
948	15.91	+0.56	15.08	+0.16
965	15.91	+0.33	16.05	+0.50
999	15.70	+0.33	15.92	+0.34
1003	15.55	+0.38	15.54	+0.32
1041	16.26	+0.62	16.06	+0.21
1057	15.91	+0.28	16.05	+0.40
1090	15.91	+0.63	15.94	+0.48
1105	15.87	+0.37	15.97	+0.49
1122	15.94	+0.48	16.09	+0.59
1126	15.33	+0.41	15.28	+0.12
1164	16.38	+0.66	16.12	+0.36
1181	16.28	+0.44	16.24	+0.41
1184	15.06	+0.02	15.34	+0.04
1187	15.66	+0.38	15.68	+0.35
1201	16.09	+0.46	16.23	+0.49
1202	15.26	+0.15	15.61	+0.27
1207	15.05	+0.12	15.39	+0.26
1222	15.77	+0.56	15.54	+0.27
1231	15.91	+0.22	16.09	+0.55
1234	15.55	+0.22	15.73	+0.39
1250	14.90	+0.16	15.08	+0.12
1253	15.91	+0.63	15.92	+0.44
1259	15.11	−0.07	15.73	+0.27
1265	15.11	+0.07	15.28	+0.15
1268	15.70	+0.27	15.01	−0.15
1277	15.87	+0.11	16.05	+0.20
1291	15.83	+0.20	15.92	+0.44
1292	15.91	+0.47	15.50	+0.56
1298	15.02	+0.10	15.21	+0.08
1307	15.83	+0.62	16.03	+0.57
1308	15.91	+0.28	16.05	+0.19
1317	15.33	+0.20	15.24	−0.10
1320	15.87	+0.20	15.76	+0.30
1321	15.70	+0.13	15.87	+0.24
1330	15.66	+0.34	15.76	+0.33
1353	15.87	+0.24	16.06	+0.30
1357	15.83	+0.39	16.05	+0.32
1373	15.14	+0.07	15.30	+0.23
1380	15.98	+0.31	15.73	+0.13
1384	15.41	+0.09	15.61	+0.34
1386	15.91	+0.26	15.97	+0.12
1388	15.40	+0.08	15.76	+0.42
1400	15.60	+0.18	15.92	+0.52
1406	15.41	+0.08	15.64	+0.37
1409	16.26	+0.74	16.23	+0.17
1422	15.10	−0.06	15.47	+0.25

photographic and a photo-visual plate. The photo-visual plates were interrupted when half exposed to obtain the photographic exposures, thus insuring the essential coincidence of the mean times of the two observations. Very rarely will an observation occur at such a phase of the light-variation that appreciable error in the color can arise from this method of observing. The results of the measures are given separately for the two pairs of plates in Table II. The first column contains the number of each variable in von Zeipel's general catalogue, the second and fourth contain its observed photographic magnitude on the Mount Wilson system, and the third and fifth the difference between this magnitude and the corresponding photo-visual result. No. 318, a bright variable to which no type or period has been assigned, is not included in the discussion.

A noticeable diversity of color for a given magnitude is evident in the plots of the individual results of Table II. This may be attributed to accidental error, or to real differences in the variables, or perhaps to actual differences in the color for a given magnitude on the ascending and descending branches. But taking the mean of the color-indices, grouped in order of photographic magnitude, a very definite composite color-curve is obtained for each pair of plates. As no material difference appears in the results from the two pairs, the normals for each are combined into the following means, which are graphically shown in Fig. 4:

Mean Photographic Magnitude	Number of Variables	Mean Color-Index
14.78	7	−0.14
15.11	23	+0.05
15.38	26	+0.19
15.64	39	+0.27
15.89	58	+0.34
16.12	38	+0.41
16.33	13	+0.55

From the mean photographic curve and this composite color-curve we obtain immediately for the variables of Messier 3 a mean photo-visual curve. The co-ordinates appear in the first and fourth columns of Table III, and are plotted as a broken line in Fig. 3. Again the form of the curve, its amplitude, and the color variation

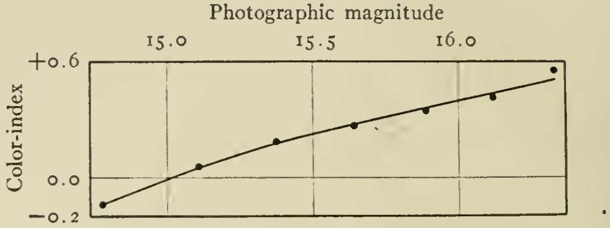

FIG. 4.—Composite color-curve of 103 cluster-type variables in Messier 3

TABLE III

MEAN PHOTOGRAPHIC AND PHOTO-VISUAL LIGHT-CURVES OF CLUSTER-TYPE VARIABLES IN MESSIER 3

Phase	Mt. Wilson Photographic Magnitude	Color-Index	Photo-Visual Magnitude	Phase	Mt. Wilson Photographic Magnitude	Color-Index	Photo-Visual Magnitude
0.000...	14.90	$-0^{\mathrm{M}}.07$	14.97	0.271...	16.07	$+0^{\mathrm{M}}.42$	15.65
0.011...	14.93	−0.06	14.99	0.283...	16.08	+0.42	15.66
0.023...	14.96	−0.04	15.00	0.294...	16.09	+0.43	15.66
0.034...	15.00	−0.01	15.01	0.305...	16.09	+0.43	15.66
0.045...	15.04	+0.01	15.03	0.317...	16.10	+0.43	15.67
0.057...	15.11	+0.05	15.06	0.328...	16.10	+0.43	15.67
0.068...	15.17	+0.07	15.10	0.339...	16.10	+0.43	15.67
0.079...	15.23	+0.11	15.12	0.350...	16.10	+0.43	15.67
0.090...	15.32	+0.16	15.16	0.362...	16.10	+0.43	15.67
0.102...	15.39	+0.18	15.21	0.373...	16.10	+0.43	15.67
0.113...	15.48	+0.22	15.26	0.384...	16.10	+0.43	15.67
0.124...	15.58	+0.26	15.32	0.396...	16.10	+0.43	15.67
0.136...	15.67	+0.29	15.38	0.407...	16.10	+0.43	15.67
0.147...	15.73	+0.31	15.42	0.418...	16.09	+0.43	15.66
0.158...	15.79	+0.33	15.46	0.430...	16.09	+0.43	15.66
0.170...	15.84	+0.34	15.50	0.441...	16.08	+0.42	15.66
0.181...	15.87	+0.36	15.51	0.452...	16.06	+0.42	15.64
0.192...	15.91	+0.37	15.54	0.464...	16.01	+0.40	15.61
0.204...	15.94	+0.38	15.56	0.475...	15.93	+0.38	15.55
0.215...	15.98	+0.39	15.59	0.486...	15.77	+0.33	15.44
0.226...	16.01	+0.40	15.61	0.497...	15.53	+0.24	15.29
0.237...	16.03	+0.41	15.62	0.509...	15.22	+0.11	15.11
0.249...	16.04	+0.41	15.63	0.520...	15.06	+0.02	15.04
0.260...	16.06	+0.42	15.64	0.531...	14.96	−0.04	15.00

are all sensibly the same as for the isolated cluster-type variables of the galactic system. The quantitative agreement of these distant stars with the local variables in phenomena of color is also important and favorable evidence of the accuracy of the photovisual and photographic magnitude scales relative to each other, and of the absence of light-scattering in space.

III. ON THE RELATION OF COLOR TO PERIOD FOR CLUSTER-TYPE VARIABLES

In the third and fourth sections of the sixth paper of this series[1] attention has been called to certain properties of the different kinds of cluster-type variables. In ω Centauri, for instance, the three subclasses defined by Bailey differ from each other in the form and amplitude of the mean light-curve, and most clearly in length of period. In Messier 3 and Messier 5, where nearly all variables are of the most typical kind, a second subclass is only sparingly represented, but in Messier 15 the variables fall into two nearly equal groups. It appears definitely that the median intrinsic luminosity in all subclasses is photographically the same—an important phenomenon that a full interpretation of these variables must explain.

The differences in period of the subclasses naturally suggest corresponding differences in the effective density; or it may be that the character of the underlying pulsation differs somewhat in the various groups. On the basis of the first alternative, if the mean densities increase with shortening periods[2] while the photographic magnitudes do not change, we must expect the change in the surface area to be compensated by greater intensity of radiation per unit of surface, and investigations of the color phenomena for the different subclasses should afford a partial test.[3]

In Messier 15 the faintness of the variables makes a detailed study of individual color-curves inadvisable at present, and accordingly the method used in the preceding note is employed to obtain composite color-curves for the two groups of variables. Professor Bailey has kindly communicated, especially for the present investigation, the preliminary results of his study of the periods and light-curves of the 53 variables in this cluster.[4] One or two have periods longer than a day, for several the periods have not yet been found, and for others accurate photometric measures are not possible.

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[1] *Mt. Wilson Contr.*, No. 151, 1917.

[2] See sec. I of this *Contribution*.

[3] ω Centauri is too far south for detailed color investigations at Mount Wilson. Color-curves are being derived for the variables of Messier 5.

[4] A preliminary report on his work is abstracted in *Popular Astronomy*, 25, 520, 1917.

TABLE IV

COMPOSITE COLOR-CURVES OF VARIABLES IN MESSIER 15

VARIABLE	APPROXIMATE PERIOD	PLATES 4284–4285		PLATES 3986–3987	
		Photographic Magnitude	Color-Index	Photographic Magnitude	Violet-Index
2	0^{d}66	16.13	+0^{M}51	16.21	−0^{M}10
3	0.39	16.18	+0.44	15.90	+0.03
4	0.31	16.27	+0.44	15.85	−0.12
5	0.38	15.90	+0.16	16.43	−0.29
6	0.67	16.13	+0.39	16.11	+0.05
7	0.35	16.27	+0.50	15.99	−0.16
8	0.65	15.42	+0.06	16.03	−0.01
9	0.72	15.49	+0.08	16.36	−0.10
10	0.40	16.27	+0.42	16.32	−0.12
11	0.34	16.18	+0.23	16.03	−0.07
12	0.59	16.22	+0.28	16.34	−0.14
13	0.57	16.38	+0.36	16.42	−0.02
14	0.38	16.10	+0.35	16.08	−0.20
15	0.58	16.13	+0.19	16.26	−0.24
16	0.67	15.53	−0.20	15.85	−0.10
18	0.37	15.49	−0.16	15.90	−0.07
19	0.57	16.47	+0.65	16.47	....
20	0.69?	15.98	+0.33	16.03	−0.19
22	0.72	15.98	+0.26	15.52	+0.13
23	0.67	16.13	+0.36	16.31	....
24	0.37	16.52	+0.68	15.78	+0.02
25	0.67	16.27	+0.33	16.42	−0.16
26	0.40	16.18	+0.26	16.37	....
29	0.57	16.10	+0.30	16.37	−0.11
30	0.41	15.56	−0.09	16.03	−0.02
31	0.44	15.90	+0.19	16.58	....
32	0.61	16.13	+0.31	15.75	−0.08
35	0.38	15.90	+0.19	16.44	−0.28
38	0.38	15.53	−0.23	16.44	−0.20
39	0.39	16.06	+0.32	15.85	−0.10
40	0.38	16.10	+0.33	15.90	+0.02
41	0.42?	16.27	+0.56	16.03	−0.01
42	0.36	15.49	−0.18	16.32	−0.27
43	0.29??	15.66	−0.05	15.96	−0.03
44	0.60	15.82	+0.08	16.35	−0.19
45	0.66	16.30	+0.54	15.96	−0.08
48	0.38	15.56	−0.16	15.82	0.00
50	0.30	16.22	+0.28	16.43	−0.17
51	0.42	16.30	+0.36	16.22	−0.06

The 39 variables with periods less than a day, for which photovisual and photographic magnitudes have been obtained from a pair of plates made on December 19, 1917, are listed in Table IV.

As a preliminary to the derivation of the magnitudes and colors of the third and fourth columns, it was necessary to study the comparison stars by means of a series of polar comparisons. At the same time colors were determined for a considerable number of other stars in the cluster. Since the discussion of magnitudes follows the customary lines, all details may be omitted from this note.

The color-index of each variable is plotted against photographic magnitude in the upper part of Fig. 5, the dots indicating variables with periods between $0^d.57$ and $0^d.71$ and the crosses indicating those with periods between $0^d.29$ and $0^d.44$. As in the preceding note, all dots may be interpreted as representing different observations on a single mean variable of period about $0^d.64$, and the crosses as representing different observations on a typical variable in the second group, with mean period of about $0^d.36$. The usual absolute values of the color and the usual increase of redness with decreasing brightness are shown by the observations; but no material difference appears in color for the two groups, except, perhaps, near maximum, where there is a suggestion that the stars with longer period are redder.[1]

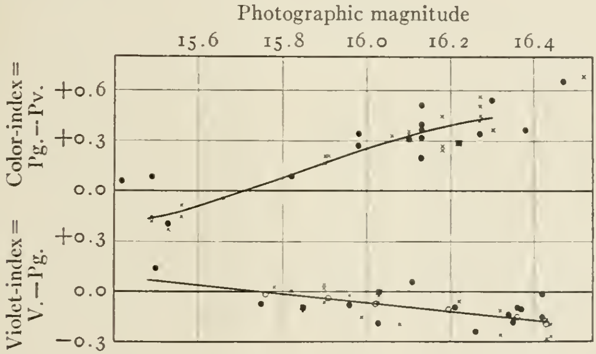

FIG. 5.—Composite color-curves of cluster-type variable stars in Messier 15; the dots refer to the longer-period subclass, crosses to the shorter-period subclass, and open circles to means of six values.

The test for difference in color has been carried into the region of shorter wave-lengths by a pair of plates made September 11, 1917, in which, as a companion to an ordinary photographic plate, a photograph was made on a Seed 27 plate through a violet-color screen that transmits very little to the red of λ 4000 and has a maximum transmission at about λ 3700. The results are tabulated in the last two columns of Table IV and plotted in the lower part of Fig. 5, dots and crosses being used as before and open circles indicating means of six values taken in order of photographic magnitude.

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[1] This would be in keeping with results for Cepheids with periods longer than a day. Cf. Fig. 1.

TABLE V

Relation of Violet-Index to Color-Index for Giant Stars in Messier 15

Star	Photographic Magnitude	Color-Index (Pg.−Pv.)	Violet-Index (V.−Pg.)
a..........	14.14	$+1^{\mathrm{M}}.32$	$+0^{\mathrm{M}}.70$
b..........	14.51	+0.79	+0.15
c..........	14.58	+1.18	+0.42
d..........	14.81	+1.18	+0.33
e..........	14.76	+1.06	+0.46
f..........	14.98	+1.04	+0.14
g..........	15.01	+0.75	+0.13
h..........	15.23	+0.93	+0.28
k..........	15.49	+0.70	+0.16
l..........	15.68	+0.68	−0.03
m..........	15.97	+0.29	+0.01
n..........	16.02	+0.21	0.00
t..........	16.10	+0.18	−0.05
u..........	16.60	+0.66	−0.06
x..........	16.05	+0.64	0.00
y..........	14.51	+1.23	+0.55
z..........	15.07	+0.08	−0.09
100..........	14.17	+1.40	+0.78
103..........	13.09	+0.45	+0.36
104..........	14.11	+0.61	+0.36
105..........	12.81	+0.61	+0.34
106..........	14.17	+1.15	+0.63
107..........	14.51	+1.14	+0.46
109..........	14.69	+1.28	+0.36
110..........	14.18	+1.40	+0.65
108..........	14.15	+1.41	+0.97
112..........	14.23	+1.52	+0.82
113..........	14.15	+0.60	+0.01
115..........	14.10	+1.29	+0.87
155..........	14.42	+1.66	+0.65
173..........	14.55	−0.37	−0.43
201..........	16.08	+0.46	+0.04
222..........	16.00	+0.27	−0.02

A full consideration of the decrease of violet-index, V.−Pg., with decreasing photographic brightness, which indicates a smaller range in violet magnitude than in photographic, must be deferred until more definite information is at hand relative to the scale of violet magnitudes. As it now stands the violet-index for these cluster-type variables decreases as color-index increases—a result quite the contrary of the relation of the two indices shown in Table V for a number of typical invariable giant stars in this same cluster.

SUMMARY

1. The redder a Cepheid variable the greater is its intrinsic luminosity; a nearly linear relation connects absolute magnitude and color-index.

2. The general characteristics of the luminosity-period curve of Cepheid variation (Fig. 1) appear to have a reasonable explanation on the pulsation hypothesis.

3. For giant stars in at least four globular clusters the decrease of brightness with decreasing color-index parallels the same phenomenon for Cepheids. When median brightness and color are considered, the variables of a given absolute brightness are nearly four-tenths of a magnitude bluer than other giant stars (Fig. 2).

4. The composite color-curve of more than 100 cluster-type variables has been determined from two pairs of photographs of Messier 3 (Fig. 4).

5. In range of variation, form of light-curve, and phenomena of color the variables of a globular cluster apparently are exactly comparable with the average isolated cluster-type variable (Fig. 3). No evidence has been found that in different stellar systems Cepheids differ systematically either in median luminosity or in nature of variation.

6. In Messier 15 two subclasses of cluster-type variables which have distinctly dissimilar mean periods show neither a difference in median photographic magnitude nor a difference in color at the median magnitude. At maximum, stars with shortest periods may be slightly bluer.

7. Preliminary violet magnitudes of cluster-type variables indicate a smaller amplitude than in photographic light; for the giant invariable stars of clusters the violet-index, V. − Pg., increases with color-index and hence with increasing photo-visual brightness.

Mount Wilson Solar Observatory
December 1917

THE INFRA-RED ARC SPECTRUM OF IRON

By H. M. RANDALL and E. F. BARKER

Captain W. de W. Abney,[1] in 1881, in mentioning his failure to photograph the infra-red arc spectrum of iron, concluded that only those metals which volatilize at low temperature possess an infra-red spectrum. In 1912 H. Lehmann[2] published the infra-red emission spectra of numerous elements obtained by the phosphorescent method, the extreme wave-lengths being about 15,000 A. Here the infra-red spectra were most successfully photographed for the easily volatilized substances, while no lines were found for iron. In the photographic region Geiger[3] has obtained results extending nearly to 1 μ. The four wave-lengths which he gives beyond 0.9 μ have, however, not been verified by the present investigators. It would appear that he had attempted to extend his measurements beyond the proper limits of his observational method. More recently K. Burns[4] has worked in the photographic infra-red region, his published values extending to 0 9 μ. In this same region Meggers and Kiess[5] have recently obtained excellent results beyond 0.9 μ, and some of their values in that region have been included in our tables. The region covered in the present investigation extends from 9000 to 30,000 A and has been found to contain some fifty measurable lines. Iron was found to be a difficult material to work with, the first attempts being very unpromising, but gradually the conditions necessary for the production of an infra-red spectrum sufficiently intense and steady for bolometric measurement were learned, so that in the region covered the infra-red iron spectrum is at present the richest in the number of its measured lines. As a large majority of these lines are weak, it is evident that the iron spectrum preserves in this region the characteristics of its photographic region. The extent and distribution of this long-wave spectrum, together with the exactness of the measurements now possible, suggest that an attempt to extend the work of Gale and Adams[1] on the pressure-shift of iron lines into the infra-red might meet with success. Should any of the lines in the neighborhood of 25,000 A belong to their *a* group and the law hold that the shift with pressure varies as the third power of the wave-length, there should be a change in the wave-length of over 4 A for an increase in pressure of 10 atmospheres, while lines of the *d* group would show a shift of more than 17 A. As it is probable that shifts of 1 A would be readily observed under conditions easily obtainable, it is evident that it is now possible to extend investigations of pressure-shifts of arc lines into the infra-red region.

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[1] *Proceedings of the Royal Society*, **32**, 343, 1881.

[2] *Annalen der Physik*, **39**, 53, 1912.

[3] *Ibid.*, **39**, 782, 1912.

[4] *Lick Observatory Bulletin*, **9**, 27, 1913.

[5] *Scientific Papers of the Bureau of Standards*, No. 324, June 1918.

As the experimental arrangement of the mirror spectrometer by which the line spectrum of the iron, produced by a grating, is passed over the slit of a linear thermopile has been described in an earlier paper,[2] it will not be discussed here. The methods of observation are also essentially the same. The grating was that used in previous work and was kindly lent us by Professor Hussey, of the Detroit Observatory. It is a flat six-inch grating with 15,000 lines per inch, ruled by Dr. Anderson, and has given very good service. The slits of collimator and thermopile were at first approximately 0.2 mm wide and covered a spectral region in the neighborhood of 1 μ of about 6 A, and at 3 μ of 3.6 A. Later the entire spectrum was examined with a slit 0.5 mm wide, and a very appreciable number of weaker lines were found and measured. No systematic differences between the values of wave-lengths measured with the slits of different width were noticed.

Through the kindness of Professor Campbell, of the department of chemistry, we obtained the American ingot iron with which the major part of the work was done. This iron is over 99.8 per cent pure, the largest single impurity being copper, which is present in the proportion of approximately 0.04 per cent. We also obtained through the courtesy of Professor Terry, of Wisconsin, a sufficient quantity of Burgess electrolytic iron to check up the results obtained with the other iron. None of the impurities, found in small traces only in both irons, were found to give lines; in fact, as later experience has shown, iron less pure would probably have sufficed, as it is only when there are large quantities of volatilized metal present in the arc that the lines appear with sufficient intensity to be measured.

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[1] *Astrophysical Journal*, 35, 10, 1912.

[2] H. M. Randall, *Astrophysical Journal*, 34, 1, 1911.

The iron was placed in a boring in the positive carbon, the diameter of the hole being as large as the three-quarter-inch carbon would permit. To secure greater localization of heat in the metal the carbon was occasionally nearly sawed off immediately below the boring. The carbons used showed the several infra-red K lines and also two others, whose approximate values are 11,439 A and 10,692 A. As these values do not correspond to any infra-red lines previously measured their origin is at present unknown.

The currents necessary to volatilize the metal sufficiently to produce measurable lines from the point of view of intensity and steadiness were very large, in general in excess of 60 amperes. Lines which are broadened unsymmetrically may very well have their measured values appreciably altered by the use of such currents, as was shown in earlier work, particularly with caesium,[1] although more recently[2] several barium lines capable of measurement with relatively small currents gave substantially the same values for all currents. At present there seems to be no way of avoiding large currents, and any estimation of the accuracy of determinations of wave-length must be qualified through this possibility of unsymmetrical broadening which cannot be allowed for in bolometric measurements on arc lines. Recently the inclusion of a very large inductance in the circuit has very effectively prevented the frequent extinction of the arc during observations.

The wave-lengths given in the table are in angstrom units as measured in air according to the Rowland scale, the spectrometer being calibrated with the Hg line 10,140.15 A as measured by Volks.[3] The intensities of the lines are indicated by the mean of the galvanometer deflections in millimeters of the separate determinations, two or more in number, when the 0.5 mm slit was used.

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[1] H. M. Randall, *Astrophysical Journal*, **34**, 12, 1911.

[2] *Ibid.*, **42**, 200, 1915.

[3] Inaugural Dissertation, Tübingen, 1914.

These numbers given in the first column of the table are at best but a very rough indication of the relative intensities of the lines, as the deflections for any one line vary between wide limits, depending upon the state of the arc, which is always very unsteady. The column headed "Maximum Variation from Mean" will give a good idea of the reproducibility of results by the methods here used, as the values are those obtained independently by two observers working at different times, the apparatus having been dismounted and otherwise used in the meantime. In general, unless otherwise stated, the values given are considered correct to within 1 or 2 A. This estimate has been verified whenever more accurate methods have furnished standards for comparison. In the present work six lines between 0.8 μ and 1 μ, measured two or more times, show a mean variation from values obtained photographically by Meggers and Kiess of 0.9 A, the maximum single difference being −1.9 A. This is the region of the spectrum where the bolometric method is least accurate. The smaller dispersion of the grating here, together with the intensity of the general radiation in the neighborhood of 1 μ, produces a strong background of rapidly varying intensity, so that the location of lines superimposed upon it becomes very difficult. For the region between 0.8 μ and 1 μ the photographic method is the more sensitive, yielding more lines; the lines so measured should be more accurate than can be obtained by our method. Beyond 1 μ, however, the bolometric method apparently becomes increasingly the more sensitive, and accordingly our values in and beyond this region become increasingly the more reliable.

A number of the lines marked with asterisks have been measured but once, two years ago this summer. They were not found a year later when an attempt was made to measure them. While the measurements, when made, were so normal in all respects that we have great confidence in the lines, still the fact that they were not found again places them somewhat in doubt. The two strongest of the lines in this region, 25,987 A and 26,229 A, were found and remeasured last year, which strengthens the view that conditions at the first measurement were particularly favorable to the formation of a spectrum, and that for some reason it was not possible to reproduce those conditions.

In a spectrum so rich in lines as that of iron it will naturally occur that a number of the infra-red lines will coincide, within the limits of accuracy obtainable in these measurements, with the higher orders of shorter lines. When the shorter lines are weak

Int.	Wave-Length, Rowland Scale	No. of Determinations	Maximum Variation from Mean	Remarks	Int.	Wave-Length, Rowland Scale	No. of Determinations	Maximum Variation from Mean	Remarks
50...	9001.1 A	3	1.1 A	(*a*)	15...	14710.7 A	2	0.2 A	
40...	9090.3	2	0.8	(*b*)	10...	14744.2	2	0.5	
20...	9210.4	3	2.3	(*c*)	20...	14828.2	3	0.9	
20...	9257.1	3	2.0	(*d*)	20...	15054.2	3	0.8	
20...	9407.8	2	0.6	(*e*)	35...	15212.5	2	0.5	
15...	9740.0	3	1.4		70...	15295.7	4	0.9	
15...	10063.3	2	1.7	(*f*)	30...	15396.4	3	0.6	(*j*)
30...	10144.3	4	2.7		25...	15624.8	2	0.5	
20...	10216.0	3	2.1		40...	15771.2	2	0.6	
20...	11608.0	3	1.0		25...	15815.1	3	0.6	
25...	11640.8	2	0.1		25...	15820.8	2	0.5	
50...	11884.2	4	0.8		15...	16165.9	2	0.1	
75...	11974.7	3	0.9		15...	16317.2	2	0.5	
25...	12033.5	2	0.8		10...	16397.2*	1	....	
45...	13564.1	2	0.9		15...	16494.6*	1	....	
50...	13899.4	2	1.4	(*g*)	10...	16608.0*	1	....	
10...	13978.5	2	0.1		10...	18856.2*	1	....	
15...	14124.4	2	0.6		5...	25341.5*	1	....	(*k*)
40...	14237.1	4	2.1	(*h*)	15...	25661.5*	1	....	
40...	14287.9	4	1.6		25...	25987.4	2	0.3	
100...	14401.8	3	0.4		20...	26229.4	2	1.1	
20...	14440.4	3	2.7	(*i*)	5...	26487.3*	1	....	
75...	14513.3	3	0.5		5...	26727.4*	1	....	
35...	14558.3	4	1.8						

EXPLANATION OF REMARKS

(*a*) Meggers and Kiess value reduced to Rowland scale 8999.8 A.

(*b*) Meggers and Kiess 9089.1 A, mean of two near lines.

(*c*) Possible error ±3 A, Meggers and Kiess 9210.41.

(*d*) Possible error ±3 A, Meggers and Kiess 9259.0, mean of two near lines.

(*e*) Meggers and Kiess 9407.9, mean of two near lines. Meggers and Kiess 9739.0.

(*f*) (The foregoing means are weighted means, each line being weighted according to its estimated intensity.)

(*g*) Possible error ±3 A.

(*h*) Possible error ±3 A.

(*i*) Possible error ±3 A.

(*j*) Absorption by water too complete to be 2d order of K7699.4 (10).

(*k*) Possibly 4th order 6336.

their higher orders do not produce effects which are measurable radiometrically. With the strong lines absorbing screens have been used when possible to eliminate effects of higher order. In other instances the complete absorption of the observed lines by varying thicknesses of water and glass have shown them to be first-order lines of long wave-length. While some of the results in the foregoing table appear to be second orders of strong lines, it is thought, nevertheless, that in each case the effect measured is that of a superposed first-order line.

We have been very materially assisted in part of this work by Mr. Jean Cooley, whose aid was made possible through a grant by the American Academy of Science, to which we wish to acknowledge our indebtedness.

Physical Laboratory, University of Michigan
November 19, 1918

DISCUSSION OF SOME EVIDENCE ON THE ORIGIN OF RADIATION IN THE TUBE-RESISTANCE FURNACE[1]

By ARTHUR S. KING

In two recent papers[2] which give his very interesting observations with the tube-resistance furnace, Dr. Hemsalech has reached some conclusions based on evidence which is not in agreement with experimental results obtained by the writer. In the interest of a proper correlation of all available data, so that we may the sooner see what conclusions may be safely drawn, a brief account of these points of difference will be given.

Dr. Hemsalech, in observing the iron spectrum produced in a carbon-resistance tube at atmospheric pressure, did not find certain lines, notably the group near λ 4900 which is strong in the arc, until his furnace reached a temperature of about 2500° C. At this temperature he considers that the strong ionization known to take place within the tube, together with the potential difference of a few volts at its extremities, give a conduction of electricity through the vapor equivalent to a low-tension arc, and that lines of this type require such an electrical condition for their production in the furnace.

The effect of the ionized state within the tube on the radiation of the inclosed vapor is a matter which needs full investigation; but sufficient data on the electrical side are not yet available to justify the conclusion that the inclosed vapor carries a considerable part of the current, as compared with the highly conducting graphite tube, under the conditions which I have found to give spectra favorable for examination. In a former contribution[3] I reported some observations in which I repeated the experiments of Harker and Kaye[4] on the ionization current between exploring electrodes and then measured the resistance between insulated electrodes placed within the tube as compared with that of the tube itself, titanium vapor at 2600° C. being present. The results showed the resistance of the vapor to be relatively high.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 162.

[2] *Philosophical Magazine* (6), **36**, 209, 281, 1918.

[3] *Mt. Wilson Contr.*, No. 73; *Astrophysical Journal*, **38**, 315, 1913.

[4] *Proceedings of the Royal Society*, **86A**, 379, 1912.

Aside from the question as to how nearly the furnace at high temperature approaches a low-tension arc, the high-temperature iron lines referred to by Dr. Hemsalech have been obtained by me at temperatures much below those at which he considers that the electrical conductivity of the vapor becomes important. In my paper[1] on the furnace spectrum of iron all of the lines of the λ 4900 group are recorded as appearing with fair intensity at temperatures between 2100° and 2300°, while the two strongest were perceptible at a temperature between 1800° and 1900°. Since the publication of that paper I have occasionally photographed the iron spectrum under the more favorable conditions of improved furnace construction and the use of the 15-ft. concave grating spectrograph, the first order of which gives very bright spectra with a dispersion of 1 mm = 3.7 A. The group at λ 4900 proved more sensitive than the former results indicated, the two strongest lines, λ 4921 and λ 4958, being visible at 1650° with an exposure of 110 min., while the entire group, including the weaker components of the doublets, was well developed at 2000° with an exposure of 15 min. In fact the later experiments have shown that the classification which I have adopted does not require temperatures which Dr. Hemsalech regards as giving a low-tension arc. Iron lines of Classes I, II, and III may be classified according to their behavior at 1700° and 2000°, while 2350°, in conjunction with the arc intensities, serves to place the lines of Classes IV and V. This latter temperature was used as the high-temperature stage in a study of the ultra-violet portion of the iron spectrum now being completed. Higher temperatures increase the width of reversals and may bring out a very few extra lines, but they soon give difficulty on account of the stronger continuous ground, the emission of which finally balances the emission of most of the metallic lines, while the more reversible ones appear in absorption.

As the furnace of Dr. Hemsalech was operated at atmospheric pressure, which might affect the vaporization point of iron, I have made a series of tests with air present, but the differences between Dr. Hemsalech's observations and mine are not to be ascribed to this feature. I obtained all of the lines of the λ 4900 group at atmospheric pressure, the temperature not rising above 1850°. An interesting effect, however, was a much increased chemical action with air present. Iron, when fused in the furnace, forms a carbide in vacuum, which sticks to the tube but does not penetrate deeply. With air present this action was much stronger. The iron ate into the tube and, when the latter burned thin through long use, dropped through the wall in several places. The tube was used in air without protection other than being in the steel chamber, the window-holder of which was removed. When operated first in vacuum and then with air present the change to atmospheric pressure caused a temperature increase of from 50° to 100° with the same impressed voltage, though the thinning of the tube gave a lower average current. The iron spectrum was very bright, but the most notable difference relative to the vacuum spectrum was a high intensity of the more reversible lines. The cyanogen (nitrogen?) bands were strong at 1850°.

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[1] *Mt. Wilson Contr.*, No. 66; *Astrophysical Journal*, 37, 239, 1913.

Obviously it must be assumed that there is no large discrepancy between the temperature measurements of Dr. Hemsalech and the writer. The Wanner type of pyrometer was used in both cases, and with ordinary care this instrument, when properly adjusted with the standard lamp, and the current through the pyrometer lamp kept constant by means of a milammeter, gives very consistent readings. The extreme upper and lower temperatures measured by us were nearly the same, my photographs showing, like his, that a few iron lines are emitted at as low a temperature as 1500° C., the formation of the carbide evidently lowering the melting-point of iron.

Dr. Hemsalech in his latest paper[1] describes an experiment in which he avoids exposing the vapor to a potential difference by constructing a chamber resting on a graphite plate, the latter carrying the heating current. Judging from the drawing, the maximum temperature given by the hot plate was applied along only about one-seventh of the periphery of the four-sided chamber. The fusion of both iron and copper in this inclosure failed to show a spectrum for either, and Dr. Hemsalech concludes from this that their line spectra are not to be excited by purely thermal means. Granting, however, that a potential difference was thus avoided, it would seem that if the temperature were high enough (pyrometer readings are not given) and the means of observation sufficient a large group of iron lines which are subject to thermo-chemical excitation should have appeared. This is the stimulus which Dr. Hemsalech believes produces the spectrum shown by the tube furnace up to 2500°, and there was full opportunity for the formation of iron carbide and for chemical action resulting from the presence of oxygen.

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[1] *Op. cit.*, p. 290.

I must further call attention to two investigations in which furnaces, which seem to differ in no essential respect from this "hot plate" furnace, produced line spectra. The first case was when the writer[1] vaporized substances within a bored-out carbon placed horizontally and heated at its middle by an arc formed between the side of this carbon and a vertical electrode beneath. With this crude furnace, some twenty-eight lines of iron and three of copper were photographed. I may say in passing that the lines from the various elements studied in this furnace have corresponded fully with their low-temperature spectra when produced by the tube-resistance furnace at present in use.

The second furnace of this type was constructed by Hale, Adams, and Gale[2] in the course of their investigation of sun-spot spectra. A powerful arc passed between two horizontal carbons and heated the upper side of a carbon tube containing the substance to be vaporized. The temperature inside the tube was probably below 2000°, as titanium could not be melted, but a fairly rich iron spectrum consisting of the low-temperature lines was photographed in the green-yellow region. In both of these arc-heated furnaces a potential difference in the iron vapor is avoided.

A third point in the last paper[3] by Dr. Hemsalech on which I wish to present additional data is the failure of his tube-resistance furnace to show the spectra of copper, silver, and zinc. He was unable to obtain any lines of these elements even at high temperature and concludes that in addition to showing no spectroscopic reaction at moderate temperatures their vapors are so poorly conducting that they do not respond to the potential gradient when more highly heated.

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[1] *Astrophysical Journal*, 21, 236, 1905.

[2] *Mt. Wilson Contr.*, No. 11; *Astrophysical Journal*, 24, 185, 1906.

[3] *Op. cit.*, p. 288.

As to the appearance of copper lines in the furnace, I must again refer to my observations with the tube heated by an external arc. To obtain further data on the question raised by Dr. Hemsalech I have used copper and silver in the tube-resistance furnace and have photographed some of the stronger lines of each element, both in vacuum and at atmospheric pressure. The temperatures ranged from 2000° to 2150° and the spectrum was examined from λ 3600 to λ 5800. With silver, λλ 3682, 4055, 5209, 5466 appeared distinctly and with copper, not only λλ 5106, 5700, 5782, observed previously with the external-arc furnace, but also λ 5153 and λ 5218 were now photographed. Some strong arc lines in this region failed to appear in the furnace at the moderate temperature used, but these are mainly of diffuse structure, a type which in general requires higher temperature. The period of strong emission is brief, the line λ 5106 of copper narrowing during a ten-minute exposure from a width of over an angstrom to a sharp maximum. This is readily explained by the behavior of the copper and silver, which, instead of sticking to the carbon and remaining in the hot portion, condense near the cool ends of the tube. This could be avoided by a spraying apparatus, giving a continuous supply. Zinc was not tried with this furnace, but in previous experiments I have found it to give the same trouble, forming filmy slag across the opening of the tube.

Summarizing these experiments, it is seen that for the high-temperature lines of iron, for the production of the iron spectrum without a potential gradient, and for the spectra of copper and silver in the tube furnace, some feature of Dr. Hemsalech's experiments has prevented the observation of spectrum lines which can nevertheless be produced under conditions which I believe invalidate the conclusions which he draws from his results. I make no plea for the existence of a purely thermal radiation, which I have long maintained is not to be proved by means of the tube furnace; still a clear disproof of the possibility of producing spectra by this means is yet to be presented. Dr. Hemsalech's work has done much to show the importance of possible chemical actions, and I believe this feature of the furnace radiation deserves to be taken into account at every point. But whatever processes of this nature may take place within the tube in any given experiment, they are controlled by the temperature, and according to present evidence it appears legitimate to employ the electric current to produce this temperature.

Mount Wilson Solar Observatory
November 19, 1918

THE INFRA-RED ARC SPECTRA OF COBALT, NICKEL, MANGANESE, AND CHROMIUM

By H. M. RANDALL and E. F. BARKER

Measurements in the infra-red region of the spectrum have yielded such fruitful results in developing the series relations of numerous elements that the major part of the work in this field has been done for this express purpose. There remain, however, the same general reasons for mapping out the spectra of all elements beyond the photographic region on the side of long wave-lengths as exist for the numerous investigations within that region. As it seemed probable that the technique developed during the measurement of the infra-red spectrum of iron[1] would serve equally well for the allied metals, cobalt, nickel, manganese, and chromium, it was decided to continue the work with these elements. The experimental arrangement remained the same, as did also the experimental methods. With the exception of manganese, which volatilized readily and gave a comparatively steady arc which yielded the strongest lines of any of the metals of this group, the metals were very similar to iron in their action in the arc. Nickel and cobalt produced possibly an even more uncertain arc than iron. While it would not be easy to say wherein the differences lay, it was nevertheless the experience of both observers that a certain amount of facility in using each substance in the arc was necessary before an arc of an intensity and steadiness sufficient for observation would result.

The materials used were the ordinary c. p. metals of commerce. This degree of purity seems at present sufficient, as no lines due to any of the impurities likely to be found in the samples used were actually found. In fact a large majority of the lines appeared sufficiently intense for measurement only when a fairly large quantity of material was in the arc and there was obviously excessive volatilization, the arc burning very noisily. The currents employed were again large, being in general between 60 and 80 amperes.

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[1] Randall and Barker, "Infra-Red Arc Spectrum of Iron," *Astrophysical Journal*, **49**, 42, 1919.

The spectrum of each material was in general twice searched, once with narrow slits corresponding to a spectral region before the thermopile of from 6 A at 1 μ to 3.6 A at 3 μ, and once with double slit-widths. With cobalt the region covered extended from 0.9 μ to 2 μ; with nickel from 0.9 μ to 3 μ; the region between 2 μ and 3 μ, however, was gone over but once. With manganese the region from 0.9 μ to 2 μ was covered twice and from 2 μ to 3 μ once; with chromium from 0.9 μ to 3 μ twice.

NICKEL

Wave-Lengths Rowland Scale	Maximum Variation from Mean	Intensity	Number of Determinations	Remarks
8967.5......	0.0	100	2[a]	
9522.1......	0.0	40	2	Meggers and Kiess 9520.3
10195.1......	1.5	50	3	
10301.4......	0.0	25	2	
10330.0......	0.7	25	2	
10378.1......	1.0	40	3	
10980.4......	0.3	50	2	
11198.1......	0.0	35	2	
11591.1......	2.0	30	3	Possible error ±3 A°
13553.7......	0.2	20	2	
13722.6......	0.5	50	2	
13829.6......	1.5	30	3	
13969.0......	0.2	20	2	
14102.1......	0.3	20	2	
14874.7......	0.5	30	2	Shaded to Violet
16313.0......	1.0	15	2	
16363.0......	0.2	100	2	
16409.4......	....	50	1	Very doubtful; could not be verified
16495.5......	....	120	1[b]	
16868.5......	0.7	15	2	
16999.6......	1.4	60	5	
17986.8......	0.5	20	2	
18040.6......	0.8	15	2	

[a] Meggers and Kiess 8967.3 mean of two near lines on Rowland Scale. A number of lines shorter than 8967.5 show a mean difference of +0.6A from Meggers and Kiess values, maximum difference being 1.2 A.

[b] Doubtful; subsequent search showed more indications of a line. Line of equal value found once in Fe.

In a number of cases the values obtained agree within the limits of experimental error with previously measured lines of other elements. In each case, however, the strongest lines in the spectra of these other elements have not appeared, so that the lines in question have been considered new lines properly belonging to the substance under examination. In the case of manganese measurements on a number of the lines of greatest intensity were made by our colleague, Dr. Sleator, the results being practically identical with our individual determinations.

COBALT

Wave-Lengths Rowland Scale	Maximum Variation from Mean	Intensity	Number of Determinations	Remarks
9096.5......	1.2	10	2	M. and K. 9095.8
9357.2......	0.4	40	2	M. and K. 9357.4
9545.8......	0.9	100	3	M. and K. 9544.9
9598.4......	1.4	30	2	M. and K. 9598.3
9661.3......	....	20	1	Found but once, doubtful
10046.6......	3.3	30	3	Possible error ± 5 A°
10128.7......	0.1	15	2	
10170.7......	1.9	20	3	
10359.1......	3.9	15	3	Possible error ± 5 A°
10689.7......	0.2	40	2	
11633.5......	0.3	20	2	
11894.5......	0.5	10	2	
14062.0......	1.7	40	....	
14559.0......	0.6	15	2	
14610.9......	0.0	35	2	
14680.9......	0.6	20	3	
14958.0......	1.0	30	2	
15209.6......	1.2	15	2	
16132.6......	0.3	50	3	
16256.9......	2.1	50	3	Possible error ± 3 A°
16387.5......	1.5	30	3	
16447.2......	1.4	20	2	
16573.8......	0.2	30	2	
17004.9......	0.3	50	2	
17080.4......	2.6	30	3	Possible error ± 5 A°
18175.5......	0.8	30	2	
18273.8......	0.4	20	2	
19778.7......	0.0	30	2	

In the case of chromium there are between 0.9 μ and 1.3 μ twenty-five measurable lines, while in the corresponding region of manganese there are but six lines, the region between 0.9 μ and 1 μ being entirely free, a very remarkable fact in view of the richness of this region for nearly all materials so far examined. In manganese in the interval from 1.3 μ to 1.7 μ are found fifteen lines, the majority of its entire number, while the corresponding region for chromium is quite empty, there being but two or three lines. From 1.7 μ onward chromium has sixteen lines, while manganese has shown no lines of measurable intensity. This alternate arrangement of the spectra of these nearly related elements is rather striking, though probably without special significance. It is noticeable that there is in chromium, as in the case of iron, a group of lines found in the region of 2.5 μ upon which but a single measurement has been made. These measurements were in every way normal and all the lines showed complete absorption by a water screen 1 cm thick. We have therefore considerable confidence in their reality but can give them here only provisionally, as we have been unable to find them a second time. It might be noticed that this group, as well as the one in iron whose members could not be measured at will, lies in the absorption band of water vapor at 2.5 μ, and that during the summer months the basement-room in which the work was carried on was comparatively damp. As by our optical arrangement the radiation from the arc passes through about 3 m of air it is quite possible that considerable energy is absorbed by the intervening vapor.

MANGANESE

Wave-Lengths Rowland Scale	Maximum Variation from Mean	Intensity	Number of Determinations	Remarks
8672.3......	1.1	80	3	
8703.1......	0.6	100	2	
8739.9......	0.4	80	2	
11378.4......	0.0	15	2	
11614.4......	2.0	40	5	
11782.8......	2.7	60	4[a]	Possible error ±3 A?
12307.9......	....	....	....	Very doubtful; not found a second time
12900.3......	1.4	80	4	
12976.5......	0.9	40	5	
13294.7......	1.1	50	3	
13318.5......	0.6	30	5	
13416.5......	0.5	80	2	
13500.7......	1.3	100	3	
13626.3......	1.8	200	6	
13685.2......	0.2	80	2	
13864.4......	1.6	100	5	
13997.6......	1.6	120	5	
14970.5......	3.8	30	4	Possible error ±3 A
15218.6......	0.5	80	3	
15263.8......	0.8	200	3	
15965.6......	1.5	200	3	
17336.0......	0.6	80	2	
17608.3......	1.0	20	3	

[a] Value possibly influenced by 11771.7 K, carbon impurity.

In the tables the mean of the observed values appears in the first column measured in angstrom units. The values when not otherwise stated are supposed to be accurate within one or two angstrom units. The intensities are determined by a very rough

CHROMIUM

Wave-Lengths Rowland Scale	Maximum Variation from Mean	Intensity	Number of Determinations	Remarks
8947.3	0.3	160	2	Meggers value on Rowland scale 8947.5
8977.1	0.4	30	2	Meggers value on Rowland scale 8977.1
9015.4	2.8	300	6[a]	
9140.6	0.7	20	2	
9206.3	2.0	30	3	
9290.6	1.6	150	4	Meggers value 9290.8
9447.9	0.6	150	3	
9574.6	1.2	150	3	
9670.9	1.1	50	4	
9732.2	1.5	120	5	
9948.4	1.1	20	4	
10082.0	0.9	20	2	
10486.3	1.3	35	2	
10673.4	0.6	30	2	
10819.9	0.8	40	2	
10906.2	1.3	60	4	
11016.0	2.9	80	5	Possible error ±4 A
11157.6	1.4	90	6	
11311.9	0.7	20	2	
11337.1	0.6	40	2	
11392.3	2.6	50	5	Possible error ±3 A
11483.3	1.0	40	2	
11611.4	0.8	100	5	
13462.1		20	1	
15680.0	3.7	30	4	Possible error ±3 A
15860.5	0.6	30	3	
18479.1	4.1	30	3	Possible error 5 A
18583.5	1.6	30	4	
18654.2	1.1	30	2	
18717.0	0.5	20	2	
25459.6*		15	1	
25489.7		10	1	
25560.4		10	1	
25583.6		20	1	
25665.4		10	1	
25708.8		10	1	
25784.6		5	1	
25819.6		10	1	
25849.7		20	1	
25902.2		10	1	
26232.0		20	1	

[a] Broad line. Possible error 4 A. Meggers value 9016.5 (Mean).

* The following lines are to be considered doubtful until verified.

mean of the galvanometer deflections during measurement. They have some significance as rating the intensities of the lines of any one element, but have very little as a means of comparison of intensities between lines of various elements.

A number of the lines in the tables are multiples of strong lines of short wave-length. It is thought, however, that in each case it has been established that the higher-order line of short wave-length is superposed by a line of long wave-length of the first order. In some cases, where it has not been possible to screen out the line of higher order, it is quite probable that its presence has somewhat affected the value ascribed to the line under observation.

As an infra-red region up to 1 μ was regarded as properly within the field of photographic methods, only a superficial examination of this region was made in this work. This largely accounts for the fact that we have not measured all the strong lines which the recent work of Meggers and Kiess shows to be here. Such lines as we have measured, as the comparisons in the tables show, have a somewhat higher accuracy than we have claimed for them, taking the photographic values as standards. The mean departure of all our measured lines below 1 μ, 19 in number, including quite a number of lines shorter than 0.9 μ not contained in the tables, is 0.6 A, while the maximum variation is 1.9 A. The superiority of the photographic method in registering faint lines is made very evident by these investigations, and it may be safely concluded that, except possibly in special cases, the infra-red spectrum up to 1 μ is properly a photographic region.

Surprisingly enough, the same cannot yet be said of the region immediately beyond 1 μ, in which photographic observations have been taken with apparently as great success, in the case of Ni to 10,843 A, Co to 11,623 A, and Fe to 10,689 A. A comparison of the results obtained photographically and radiometrically shows no cases which are unquestionably in agreement, though there are in the foregoing regions twelve radiometrically determined lines and many more photographic lines.

Confidence in radiometric measurements in this region is based upon the following general considerations: The radiometric methods here employed, as well as that earlier used by Lewis,[1] which is similar, have given values between 0.7 μ and 1 μ, which have been substantiated by subsequent photographic results. The results under comparison are in fact an instance of such a confirmation. The same lines beyond 1 μ have been measured by different observers using different gratings and making determinations in higher orders for the strong lines with closely agreeing results, and finally with those substances for which there are series relations there have been numerous instances in which the radiometric values beyond 1 μ have so perfectly met the requirements that the confidence in their accuracy has become very great.

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[1] *Astrophysical Journal*, 2, 1, 106, 1895.

Since the particular lines in question have been measured under usual conditions of observation and have shown no peculiarities during observations, but have, on the other hand, been quite normal in all respects, there appears to be no reason for questioning their accuracy. In this work also we wish to express our indebtedness to the American Academy of Arts and Sciences for a grant from the Rumford fund which enabled us to obtain the services of an assistant.

Physical Laboratory
University of Michigan

Minor Contributions and Notes

A STAR WITH DISAPPEARING BRIGHT LINES

In the *Journal of the Royal Astronomical Society of Canada* for November 1918 (**12**, 462), Director J. S. Plaskett, of the Dominion Astrophysical Observatory at Victoria, shows from measures of four spectrograms that the radial velocity is variable of the star Boss 5918, which is B.D. +47°3985 ($\alpha = 22^h 53^m$, $\delta = +48° 9'$, mag. = 5.20, spectrum = B3). An examination of our plates and measures of this star brings out a further point of interest, which is worthy of mention, namely, that the spectrum now does not show bright hydrogen lines (or bright fringes beside the dark lines), although they were conspicuous in 1913.

Our available plates are as follows:

No.	Date	G.M.T.	Quality	Taken By
I 3446	1913 July 28	$18^h 52^m$	Good	B, S
3457	Aug. 4	17 28	Very good	F, S
3520	Oct. 9	14 31	Ti weak	F, S
5345	1918 Aug. 5	18 22	Good	W, S
5363	Aug. 30	21 34	Good	B, S

B = Barrett; F = Frost; S = Sullivan; W = Miss Wickham.

The first two plates were measured in 1913 by myself and yielded values of −11 and −15 km per sec. as the radial velocity from the four dark hydrogen lines, ϵ, δ, γ, and β. It was noted in the measures of the first plate that there were "bright fringes on each side of Hβ." On the second plate, settings were made on the edges of the fringes. Expressed in kilometers of velocity, and in angstroms, the widths were:

Bright fringe toward violet	−239 to −111 km = 2.06 A
Dark	−111 to + 16 km = 2.05 A
Bright fringe toward green	+ 16 to +178 km = 2.61 A

In other words dark β was of the same width as the more refrangible bright fringe and somewhat narrower than the fringe toward the green. If we should consider the dark line as superposed on a bright band 6.7 A wide, it was displaced toward the violet from the center by about 0.3 A, or about 17 km.

On the third plate the fringes at β are strong, and at γ the one on the side toward green is visible. This plate was not measured, because the comparison lines were too weak to yield a good result.

This kind of appearance at Hβ is not at all unusual for A spectra with bright lines, but it is not often that we can establish a variation in the intensity of the emission lines. Our two plates taken in August 1918, however, do not display a trace of the bright emission lines, and the entry was made on the observing card at the time by Professor Barrett, after examining the plates: "Bright lines variable?"

Mr. Plaskett makes no mention of bright lines in the notes describing this spectrum, and in response to my inquiry he states that no bright fringes are visible upon a re-examination of the Victoria spectrograms.

Professor E. C. Pickering's letter with regard to the matter reads as follows:

> An examination of our photographs of the spectrum of H.R. 8731, α 22^{h}8^m, δ +48°9′, by Miss Cannon, shows that on a photograph taken with the 8-inch Bache Telescope on November 18, 1887, and on photographs taken with the 11-inch Draper Telescope on November 5, 1904, October 27, November 14, and November 19, 1905, all the lines are dark. Hβ is a well-marked dark line with no appearance of bright edges. On a photograph taken November 3, 1913, with the 8-inch Draper Telescope, the line Hβ is not clearly seen and certainly is not present as a strong dark line.

A list of stars having bright hydrogen lines was published by Mrs. Fleming in *Harvard Annals*, **56**, Part VI, pp. 181–84. It describes the variations of the lines so far as known at the date of that publication. Miss Cannon's recent important paper on the same subject is No. 3 of Vol. **76** of the *Harvard Annals*.

EDWIN B. FROST

YERKES OBSERVATORY
January 3, 1919

Reviews

Science and Learning in France, with a Survey of Opportunities for American Students in French Universities: an Appreciation by American Scholars. By JOHN H. WIGMORE, Editor-in-Chief. Chicago: A. C. McClurg & Co., 1917. Pp. 454. Illustrations 58. Price, $1.50, cloth; $1.00, paper.

"To the Scholars of France, worthy custodians of their country's intellectual greatness, this volume, prepared in a time when France has reached the heights of moral greatness, is offered with heartfelt admiration and sympathy in the name of the Scholars of America."

There are two introductions: the first, entitled "The Mind of France," by former President Charles W. Eliot, which occupies three pages and is adorned by a photograph of Rodin's Le Penseur; the second, a charming essay of a dozen pages entitled "The Intellectual Inspiration of Paris," written in that city in September 1916 by George E. Hale.

The different branches of learning, from anthropology to zoölogy, are dealt with in successive chapters, each under the editorial care of a drafting committee of well-known American teachers. The chapters are embellished with excellent portraits of distinguished French scholars and with views of famous educational buildings of Paris and of the interiors of some of the laboratories. The aim in each section is to show the splendid intellectual past in France in that department and to indicate the present opportunities for American students who may desire to add a residence in Europe to their preparation for a specialty. Of course Paris receives the major part of the attention, but practical information is given in two appendices, of which the first is entitled "Educational Advantages for American Students in France: with a History of the Recent Changes in its University System," by Professor James Geddes, Jr., of Boston University; the second discusses "Institutions of Higher Learning, Their Organization, Degrees, Requirements, Fees, etc.," by Professor C. Vibbert, of the University of Michigan.

A third brief appendix of twelve pages on practical suggestions to the intending graduate student is also by Professor Vibbert.

The book makes a valuable beginning in introducing the learning of France to the American student. It would, doubtless, be improved in a second edition. Its errors in the departmental chapters are probably more those of omission than of commission, although the reviewer notices both in the section on astronomy. The general approval of Americans of the purpose of the book is evidenced by the list of about 1000 "sponsors," of whom it is said, "These American scholars have expressed a cordial desire to join with the authors in making this book a national homage, offered from the Universities of America to the Universities of France."

We bespeak for the book a wide circulation and use. It should be in every college library.

F.

THE

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THEORY OF IMPERFECT GRATINGS

By C. M. SPARROW

A complete bibliography of the theory of imperfect gratings would make a long list of papers by many writers. Of these writers, four perhaps stand out from the others. The classical researches of Rowland[1] and of Rayleigh,[2] employing what may be called the conventional methods of analysis, form the basis of most of the work that has been done. Besides these, Cornu[3] has developed a method of peculiar elegance for investigating the focal peculiarities of gratings, and Michelson[4] a method which is specially adapted to the study of periodic errors of spacing. In the present paper an attempt is made to present the whole subject afresh from the standpoint of the so-called vector method. This method, used so brilliantly by Cornu in treating elementary diffraction phenomena, was first applied to the perfect grating by A. L. Kimball.[5] It is a method of great power for some problems, enabling us to avoid the use of much complicated analysis, particularly in those problems in which qualitative insight is sought rather than a numerical solution. Where the numerical result is desired, the graphic solution obtained from the geometric representation is often the shortest road; in other cases we can pass at once from the vector diagram to its analytic equivalent, the geometric intuition serving as a guide in keeping the analysis in its simplest form.

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[1] *Phil. Mag.* (5), **35**, 397, 1893; *Astronomy and Astrophysics*, **12**, 129, 1893; *Physical Papers*, p. 525.

[2] *Coll. Papers*, I, 415; III, 47.

[3] *Comptes Rendus*, **116**, 1215, 1893.

[4] *Astrophysical Journal*, **18**, 278, 1903.

[5] *Phil. Mag.* (6), **6**, 30, 1903; see also Wood, *Physical Optics*, p. 203, for a simple exposition.

The first part of the paper considers systematically the various kinds of defects which are likely to occur in practice. Many well-known results will appear in the new garb; many others, not so well known, are nevertheless contained by implication in the already published work of others, but have hitherto failed to receive the emphasis which their importance demanded. In order to increase this emphasis, curves showing the forms of diffraction patterns have been included, and numerical data which have been obtained by ordinary processes of calculation. The second part of the paper applies some of the results obtained in the first part to an examination of the limitations of the method of coincidences as a means of measuring wave-length. It contains, in addition, a theory of the defective Fabry and Perot interferometer with consequences relating to this method of measuring wave-lengths.

In the discussion that follows we shall consider the grating to have $2N$ grooves, and shall in general denote by n the number of a particular groove, counted from the center of the grating, n being positive or negative according as the groove is on one side or the other of the center. It will be necessary to recall briefly the main features of the vector treatment of the perfect plane grating. Let Δs_n and ϕ_n be the amplitude and phase (with respect to the central groove) of the disturbance due to the nth groove. If we represent this disturbance by a vector of length Δs_n and polar angle ϕ_n the vector sum of all such vectors will represent the amplitude and phase of the resultant disturbance. Intensities are obtained by squaring the amplitudes. For the perfect grating, Δs_n is independent of n, and

$$\phi_n - \phi_{n-1} \equiv \Delta\phi_n$$

is constant. The order in which the vectors may be added is completely arbitrary, but if we add them in the order of increasing n, the vector polygon obtained is equilateral and equiangular and is therefore inscriptible in a circular arc. Let us denote by θ the angle of diffraction, and by θ_0, θ_1, ... the angles corresponding to the central image, first order spectrum, etc. For $\theta = \theta_0$, $\Delta\phi = 0$, and the vector polygon is a straight line of length $2N\Delta s$. As we pass away from this direction, $\Delta\phi$ increases, and the vector polygon curls up, the length of the broken line remaining constant, until, when $\Delta\phi = \pi/N$, the polygon closes completely: this is the direction of the first minimum. As we go still farther, the polygon laps over itself on the circumscribing circle, giving the first subsidiary maximum when it has gone about one and a half times round, the second minimum when it has gone twice round, and so on. (The well-known formula for the intensity,

$$J = \frac{J_o \sin^2 N\Delta\phi}{\sin^2 \Delta\frac{\phi}{2}}$$

is obtainable at once from the vector diagram without the use of trigonometric identities.) The brightness of the maxima steadily decreases until $\Delta\phi = \pi$. From this point on, an increasing $\Delta\phi$ is equivalent to a negative phase-difference of decreasing absolute value (i.e., $\Delta\phi$ is replaced by $\Delta\phi - 2\pi$). The vector polygon gradually straightens out, going through the previous series of changes in the reverse order, until in the direction θ_1 it becomes perfectly straight again. Between this and the second order the same changes are repeated, and so on.

At the pth minimum on either side of a principal maximum,

$$\Delta\phi = p\pi/N.$$

So long as this is small, the polygon will approach very closely to the circumscribing circle, and can be replaced by it. As N is in practice quite large and the values of p which interest us are usually small, we are led to consider an infinite number N of infinitesimal vectors ds, of phase-difference $d\phi$. Instead of the vector polygon we have a vector curve. But

$$\frac{ds}{d\phi} = \rho,$$

the radius of curvature of the curve. In the case of the perfect grating just considered ds and $d\phi$ are independent of n, so that ρ is a constant. In general ds and $d\phi$ are functions of n, so that

$$\rho = f(n).$$

Thus f is expressible as a function of s or ϕ, or both, giving us the intrinsic equation of the vector curve. The form of f is determined by the nature of the defects of the grating, and the properties of such gratings are completely expressed by the properties of the family of vector curves; just as the properties of the perfect grating are expressed by those of a family of circular arcs of constant length.

The defects of gratings fall into two fundamentally distinct classes according as they involve a variation of $d\phi$ or ds. We shall call them phase errors and amplitude errors, respectively. The phase errors are practically the most important; they have also received the fullest treatment by analytic methods. The amplitude errors which actually occur are usually such as to produce only minor defects in the performance of the grating, but we should include under this head certain "defects" which may be artificially introduced in order to accomplish some particular end. We will begin with a discussion of phase errors, and will thus assume that ds is independent of n.

In the direction θ_k of a principal maximum of the kth order, the phase-difference between successive disturbances should be constant and equal to $2k\pi$, provided the grating is perfect and the instrument is in adjustment. That this is not the case is due in practice to one or more of several causes: (1) the spacing of the lines is not uniform; (2) the surface is not accurately plane; (3) the telescope and collimator are not in adjustment.[1] The possibility arises of several of these causes being present at once and producing errors which more or less compensate; this is in fact actually the case with the concave grating. The essential results of the theory are all included in a treatment of errors due to non-uniform spacing.[1] We shall assume, therefore, unless otherwise stated, that the surface of the grating is perfectly plane, and that telescope and collimator are in adjustment, but that the distance between the $(n-1)$th and nth grooves is a function of n, $f(n)$, the nature of which will be further specified.

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[1] Crookedness of the lines is a fourth defect, which does not readily lend itself to treatment by the vector method. See the papers of Rayleigh referred to above for a discussion of the consequences of this class of error.

The defects of spacing of actual ruled gratings may be divided into four classes: (1) Errors of "run," a small progressive change of spacing caused by variation of temperature during the ruling, etc. (2) A sudden "jump" or discontinuity in the ruling, the grating interval being constant except at a single point. Michelson has pointed out the possibility of an error of this kind; the actual frequency of its occurrence is a matter of speculation. (3) A periodic variation in the spacing, usually of the pitch of the screw, producing "ghosts." (4) Accidental variations of spacing, inherent in the imperfections of the ruling engine.

Each of these kinds of error, except the last, has its appropriate form of $f(n)$. Instead of $f(n)$ it will be convenient to represent the grating interval by $a+f(n)$, where a is the interval at the center. The phase-difference between successive disturbances is then

$$\frac{2\pi \sin\theta}{\lambda}[a+f(n)].$$

From this we must subtract $2k\pi$ for the spectrum of the kth order, in order to get $d\phi$. We are concerned only with the phenomena near a principal maximum. We may therefore put

$$\frac{2\pi a \sin\theta}{\lambda}-2k\pi=d\phi_0,$$

where $d\phi_0$ is thus the excess of phase-difference over $2k\pi$ for two grooves at the center of the grating. Thus

$$d\phi=d\phi_0+\frac{2\pi \sin\theta}{\lambda}f(n).$$

But we have also

$$\frac{2\pi a \sin\theta k}{\lambda}=2k\pi,$$

---

[1] Conversely, any distribution of the grooves has its parallel in the aberrations of an optical instrument, the treatment given here being equally applicable.

and since we are only concerned with values of θ near θ_k, and the variation of spacing is also small, we may replace θ by θ_k, giving

$$d\phi = d\phi_0 + \frac{2k\pi}{a} f(n).$$

The length ds of the elementary vectors being constant, n is proportional to s, the length of arc of the vector curve, measured from the center (say $n = Qs$). Hence dividing by ds, we have finally

$$c = c_0 + kF(s) \tag{1}$$

$$F(s) \equiv Af(Qs)$$

where c is the curvature of the vector curve at the point s and c_0 the curvature at the center of the curve. c_0 is that parameter of the family of curves which varies with θ, the diffraction angle. The effect of the error is thus to superpose upon the curvature c_0 a curvature which varies from point to point of the curve, but which is independent of c_0 and hence of θ. The fact that errors of spacing produce no effect on the form of the central image is an immediate corollary of this.

Certain general properties of these curves may now be noted. A given length of arc of the curve corresponds to a definite portion of the ruled surface. The value of c_0 determines the point of the diffraction pattern, being proportional to the phase of this point. Draw the osculating circle (curvature c_0) at the center of the vector curve, and take an arc $2s_0$ equal in length to the vector curve. This circular arc is the vector curve for a perfect grating of spacing a; it corresponds to the same diffraction angle θ as the actual vector curve. Now the angle between the tangents at the ends of this arc is $2c_0s_0$; this then is the extreme variation in phase between the end grooves of the grating of spacing a. If we choose s_0 equal to unity we see that c_0 is half the phase of the corresponding point of the diffraction pattern, the zero of phase being taken as that corresponding to the *direction of the principal maximum of an ideal grating of spacing a.* While this direction furnishes a convenient zero it is not the proper standard. We should rather take as our zero the phase corresponding to the direction of a principal maximum of an ideal grating with the *same mean spacing as the actual grating.* A perfect grating with the same mean spacing would have the same total distance between the first and last grooves as the actual grating. In the direction of a principal maximum the phase of these two grooves is the same, hence in the actual grating the vector curve which corresponds to the proper zero of phase has parallel tangents at its two ends. It has seemed necessary to go into this point in some detail, because it has proved a source of confusion in the work of some other investigators. It is characteristic of the vector method that it fixes attention on the law of spacing, by means of the variation of curvature, rather than on the total "aberration," which is indicated by the variation of the slope of the curve to the polar axis.

Errors of run.—As we may assume these to be small, we may write

$$c=c_0+\beta s+\gamma s^2+\delta s^3+ \ . \ . \ . \tag{2}$$

We shall speak of the successive terms as representing linear, quadratic, etc., errors, these corresponding to the aberrations of the second, third, etc., degree of ordinary optics. There is a fundamental difference between the terms of odd degree and those of even degree. For the errors of odd degree, the middle of the vector curve $c_0=0$ is a center of symmetry, the curvatures on each side of this point being equal and opposite. The tangents at the two ends of this curve being parallel, the mean spacing is the same as the spacing at the center. If we superpose now a uniform curvature c_0, the shape of the curve will be independent of the sign of c_0; the diffraction pattern is therefore symmetrical with respect to the true direction of the principal maximum. Thus for errors of odd degree, *the spectral images are symmetrical and in their true position.* As we pass away from the center of the pattern, the point of inflexion of the curve moves toward one end; the curvature of one half is thus always greater than that of the other half, and the curve will not close completely. The minima of intensity will thus not be "clean" but will always have some illumination. For small errors, the amplitude of this "error of closure" of the vector curve will be proportional to the error, and hence to the order of the spectrum. The intensity of illumination at the minimum will thus be proportional to the square of the order.

The simplest case of an error of odd degree is represented by

$$c = c_0 + \beta s$$

or

$$\phi = c_0 s + \tfrac{1}{2}\beta s^2.$$

The vector curves are Cornu spirals, as may be seen by putting

$$s = s' - \frac{c_0}{\beta}.$$

This furnishes a simple proof of the well-known fact that an error of this type simply changes the focus of the grating. Just as the form of the spectral images of a perfect grating in exact focus is identical, for large N, with that of an aperture of the width of the ruled surface, so the diffraction pattern of such a grating out of focus can be found in the usual way by the Cornu spiral. From the linearity of the curvature formula (2) it follows that small errors may be considered as producing their effects independently, their curvatures being simply superposed. Therefore by a suitable change of focus the effect of a linear error may be removed, the curvature of the vector curve becoming uniform, and the diffraction pattern that of a perfect grating.

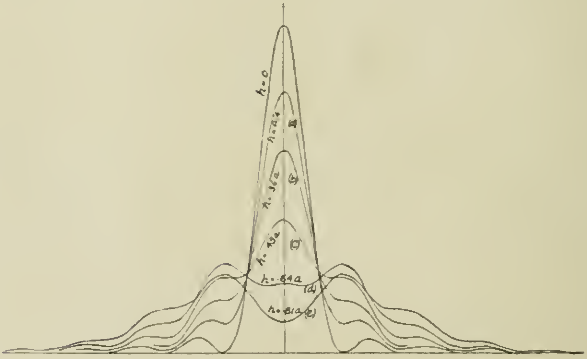

FIG. 1

A few curves are given (Fig. 1) showing the form of these diffraction images. It will be noted that they embody the general features of the effect of the errors of odd degree mentioned above. They may be interpreted either as due to a linear error of spacing or as the extra-focal images of a perfect grating. The quantity h denotes the maximum displacement of a groove from its true position.

Cubic error.—The next case in order of simplicity is where

$$c=c_0+\delta s^3$$

or

$$\phi=c_0 s+\tfrac{1}{4}\delta s^4.$$

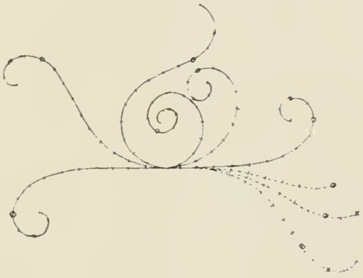

FIG. 2

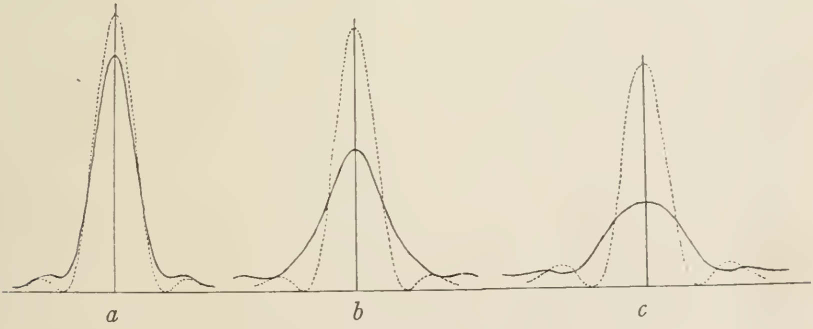

FIG. 3

The general form of the vector curve resembles the Cornu spiral, but the convolutions converge more rapidly to the limit points. We cannot obtain all our intensities from a single curve, as with the Cornu spiral, but must use the entire family of curves of which c_0 is the variable parameter. The length of spiral used depends on the magnitude of the error. A few of the spirals are shown in Fig. 2 (solid lines). They are so drawn that for $c_0=0$ the spiral turns through one right angle in the length 10 (marked by small circles). The diffraction pattern corresponding to this error is shown in the solid curve (a) of Fig. 3. If this curve is compared with (a) of Fig. 1, which corresponds to the same value of h, it will be seen that the distortion of the spectral image is less for the cubic error. This is to be expected, as the central portions of the spirals of Fig. 2 are straighter than those of the Cornu spiral. The length 12 of the spirals, indicated by crosses, corresponds to

$$h=1\cdot 2^4\cdot a/4=0.5184a.$$

The corresponding diffraction pattern is shown in the solid curve (*b*) of Fig. 3. The curve (*c*) of Fig. 1 corresponds to a somewhat smaller value of h, but the distortion is greater. The whole length of spiral corresponds to

$$h=1\cdot 4^4\cdot a/4=0.9604a;$$

the diffraction pattern is shown in (*c*) Fig. 3. This error amounts to a maximum shift of groove of practically a whole grating interval, but there is still some definition left. A linear error of the same size destroys it almost completely.

The question now arises: How far can the effect of this cubic error be eliminated by focusing? The effect of focusing is to superpose a linear error of opposite sign, so that the vector spiral is given by

$$c=c_0-\beta s+\delta s^3\,,\qquad \phi=c_0 s-\tfrac{1}{2}\beta s^2+\tfrac{1}{4}\delta s^4. \tag{3}$$

In order to determine the proper value of β we can visualize the effect of focusing as follows: in the right-hand half of the curve the cubic error shifts all grooves to the right, the linear error shifts them to the left, so that near the center a groove is to the left of its true position, while farther out it is still shifted to the right. A very close approximation to the best focus will be when the greatest shifts to the right and left are equal, for then the absolute value of the shifts will be a minimum. The phase is a maximum when $c=0$, or $s^2=\dfrac{\beta}{\delta}$. Substituting this value of s in (3), we have

$$\phi_{\max}=-\tfrac{1}{4}\frac{\beta^2}{\delta}.$$

At the end of the curve

$$\phi=-\tfrac{1}{2}\beta s_0^2+\tfrac{1}{4}\delta s_0^4\,;$$

equating this to $-\phi_{\max}$, we get a quadratic from which we derive

$$\beta=\delta(\sqrt{2}-1)s_0^2=0.414\delta s_0^2$$

and for this value

$$\phi_{\max}=\frac{3-2\sqrt{2}}{4}\delta s_0^4=0.172\frac{\delta s_0^4}{4}.$$

Taking this value therefore is equivalent to reducing the greatest displacement of a groove to 0.172 of its former value. The dotted curves in Fig. 2 show the corresponding modifications of the vector spirals for $c_0=0$ (only half of each spiral is shown). The corresponding diffraction patterns are shown in the dotted curves in Fig. 3. The compensation in (*a*) is almost perfect, the maximum intensity being 0.984; in (*b*) it is still good, the maximum being 0.933; while in (*c*) it is 0.796, or somewhat better than for (*a*) without focusing. We thus see that even when the groove shift is as much as a whole interval, the grating will behave tolerably well when focused.

Odd errors of higher degree.—From the foregoing it is easy to see how we should expect the terms of the 5th, 7th, . . . degree to affect the definition. For the same value of the maximum shift h, the effect will diminish as the degree increases; on the other hand, the compensation by focusing is less perfect. We can form some estimate of the extent of this compensation in a manner similar to that used above for the cubic error. We thus find that for an error of the fifth and seventh degree, h can be reduced respectively to 0.25 and 0.297 of its original value.

Errors of even degree.—The effect of errors which are symmetrical with respect to the center of the grating is quite different from those we have just been considering. In the first place, the vector curve is symmetrical with respect to the normal at its middle point; the curve thus closes completely for some value of c_0, and the minima are in general "clean." (The only exceptions to this arise when, for large errors, two minima and a maximum coalesce to form a single minimum, which may then have some illumination.) In the second place the diffraction pattern is unsymmetrical, because as c_0 increases or decreases, the curve on the one hand is curling up, and on the other is straightening out. It follows from this that in the third place the maximum illumination is not at $c_0=0$, but at some value of c_0 opposite in sign to the curvature produced by the error. To fix ideas, let the error be positive, that is, let the grating interval increase toward the ends of the grating. Then the maximum occurs for some negative value of c_0, that is, nearer the central image. It might be inferred that the maxima are nearer the central image than their true position, but this is not the case, for the mean spacing of the lines is not the same as the spacing at the center. The true position of the maximum is given, as we have said, by that vector curve the tangents at whose ends are parallel. An inspection of the curves will readily show that this curve corresponds to a larger negative value of c_0 than that for the maximum, so that the displacement of the maximum is really away from the central image for a positive error and hence toward it for a negative error.

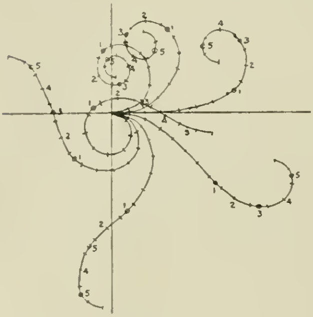

FIG. 4

The most important error of this class, and the only one that will be considered in detail, is the quadratic error. The vector spiral is given by

$$c = c_0 + \gamma s^2$$

or

$$\phi = c_0 s + \tfrac{1}{3}\gamma s^3.$$

Some of these curves are shown in Fig. 4. The full length 15 of this spiral corresponds to

$$\tfrac{1}{3}\gamma s_0^3 = 2\pi.$$

That is, compared with a grating of spacing a, the end grooves are displaced a whole interval. Compared with a grating of the *same mean spacing*, however, the end grooves are in their true position, and the greatest displacement of a groove from its true position is shown by a simple calculation to be

$$\tfrac{1}{3}\gamma s_0^3 \cdot \frac{2}{9}\sqrt{3} = 0.385 \cdot \tfrac{1}{3}\gamma s_0^3,$$

or about three-eighths of the value found by the first process.

The diffraction patterns corresponding to these spirals are given in Fig. 5. The lengths 7.5, 9, 10.5, 12, 13.5, 15 correspond to the curves 1, 2, 3, 4, 5, 6 respectively; the first five of these lengths are indicated by the small numbers in Fig. 4, the full length of the spiral corresponding to curve 6. Similar curves are given by Rayleigh in a discussion of the aberrations of optical instruments, but as we have drawn them here they are shifted in the opposite sense, owing to the different choice of the zero of phase mentioned above. As all of the foregoing treatment applies equally well to the aberrations of an optical instrument, it is worth while showing from that point of view just how this choice of zero differs. A wave-front *ABC* possessing third-order aberrations has different curvatures in the two halves *AB* and *BC* (*B* being the apex of the wave). Rayleigh's choice amounts to taking the normal to the wave-front at *B* as the axis of the instrument, whereas we would take the line from *B* to a point equidistant from *A*, *B*, and *C*.

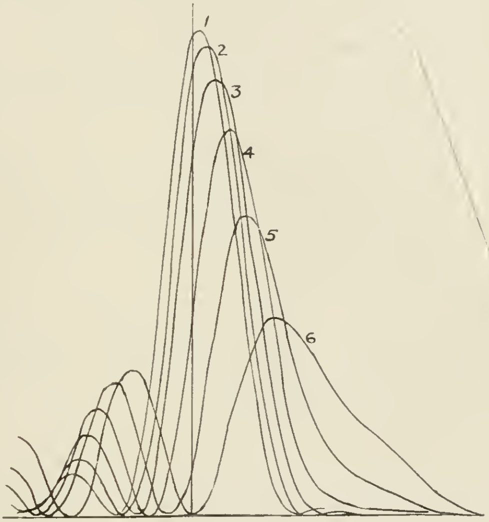

Fig. 5

The series of curves in Fig. 5 show how the quadratic error affects the accuracy of the grating in the absolute measurement of wave-lengths, and shows, if demonstration were necessary, the unreliability of the instrument for this purpose. The displacement of the maximum is very closely proportional to the error of spacing, and amounts to about $\frac{1}{6}\gamma s^3$. Thus the largest error considered here would introduce errors of about $1/R$, where R is the resolving power of the instrument. The error introduced in the method of coincidences comes from another source, and is, as we shall see, of a much higher order of small quantities.

Just as in an optical instrument the aberrations of the third order are those that give most trouble, so in gratings it is these errors of even degree in spacing that constitute the greatest defect. If, therefore, there were some convenient way of introducing such errors artificially, we could rid ourselves completely of the terms of the second degree, and to a considerable extent of those of higher degree. Rayleigh has suggested a slight rotation in azimuth of the telescope or collimator lens. This might prove practicable, but a possible source of trouble arises due to the dispersion of the lens, making it impossible to compensate simultaneously for two coincident spectra.

The presence of the quadratic error may be detected by examining the two halves of the grating separately. The two halves will have different foci, and also the spectra will be less displaced. If in the expression

$$c = c_0 + \gamma s^2$$

we consider each half separately, we have a quadratic error of one-fourth the original amount, combined with equal and opposite linear errors for which

$$\beta = \tfrac{1}{2}\gamma.$$

If we assume with Rayleigh that the change in focus is just perceptible when it involves a change of phase of a half-period, we would just be able to detect this change for

$$\gamma s_0^3 = 4\pi,$$

an error represented in our diagram approximately by curve 4, Fig. 5. The shift of the maximum would be reduced to one-fourth its original value, measured in terms of the width of the spectral image, but as the new images would be twice as wide, the shift in terms of angular measure would only be cut in half. Taking Wadsworth's figure for the possible accuracy of measurement, we could just detect this shift when the error amounts to one-sixth of a grating interval. Apparently, then, the observation of the shift should be more sensitive than that of the change of focus. Taking therefore

$$\tfrac{1}{3}\gamma s_0^3 = \frac{\pi}{2}$$

as the smallest value we could detect, and remembering that the greatest shift of a groove from its true position is only three-eighths of this, we see that it should be possible to detect a quadratic error involving a maximum groove-shift of $0.1a$ by observations on the spectrum of the first order.

Discontinuous errors.—Michelson has called attention to the possibility of a sudden "jump" in the ruling such that the spacing on each side is uniform, but that one interval has the value $a+qa$, where q may be positive or negative. The possible cause of such a discontinuity is given as the release of strains in the speculum metal by the ruling. One may be permitted some skepticism as to the fact of this kind of error having actually occurred. Speculum plates which are figured and polished in one operation do show evidence of strain by a gradual change of figure in the course of a year or so, so that it is necessary to season the plates before they are given their final figure if permanence is desired. The release of strain is gradual, however, and it is difficult to believe that such a change would take place in the ruling without a corresponding change in the figure of the surface which would render the grating unusable. Other possible causes of such a jump in the ruling might be adduced, however, so that it is well to bear it in mind. The theory of this type of error is given by Michelson, as well as the expression for the intensity when the jump is in the middle of the ruling. These results are obtained so readily by the vector method that it is worth while to give the theory from this point of view.

On each side of the break the spacing is uniform; the corresponding portions of the vector curve are therefore circular arcs of the same radius. For the spectrum of the kth order the circular arcs make the constant angle

$$2\pi kq=2\theta$$

with one another. Fig. 6 shows a set of the vector curves for the case in which the discontinuity is in the middle of the ruling. For all curves of the family the centers of the arcs lie on the lines OP, OQ making an angle of $2\pi kq$ with one another. The phase is the angle OPB. If ON is the bisector of POQ, the perpendicular BH on ON represents half the amplitude. We have

$$\text{angle } BON = \tfrac{1}{2}\left(\frac{\pi}{2} - \phi - \theta\right)$$

$$BH = OB \cos \tfrac{1}{2}(\phi+\theta) = 2OP \sin\frac{\phi}{2} \cos \tfrac{1}{2}(\phi+\theta) = s_0 \frac{\sin\frac{\phi}{2}}{\frac{\phi}{2}} \cos \tfrac{1}{2}(\phi+\theta),$$

agreeing with Michelson's result except for a change in notation. It is easy to calculate, by simple trigonometry, the resulting

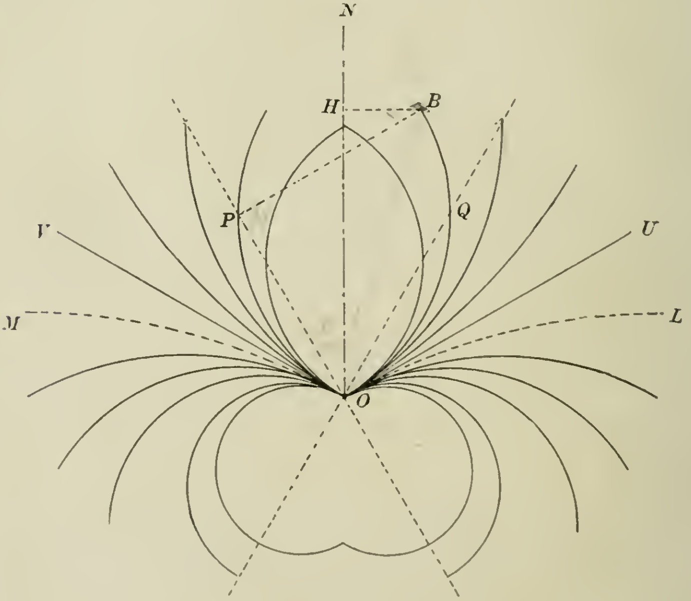

Fig. 6

intensity for any number of breaks at any points of the ruling by this method; the use of graphic methods is probably a simpler way of obtaining numerical results.

There are two sets of minima. The more important of these, and the only one given by Michelson, corresponds to the zeros of the factor $\cos \frac{1}{2}(\phi+\theta)$. These occur when the halves of the vector curve meet on the y-axis. The other set is given by the zeros of $\sin\frac{\phi}{2}$. These occur when each half of the vector curve closes independently. The first minimum obviously belongs to the first set. The values of ϕ corresponding to the maxima are given by Michelson as approximately $3(\theta-2\pi n)/4$. These values, however, are with respect to the true direction for a grating with interval a; that is, the zero of phase is taken to correspond to the vector curve consisting of the lines OU, OV. The proper zero of phase corresponds to the dotted arcs OL, OM, so that the maxima are given by subtracting θ from the above. The diffraction patterns are given by Michelson. They present all the characteristic peculiarities of the errors of even degree, though the error cannot be represented by an algebraic expansion, being of infinite degree. If the break occurs at some point other than the middle of the ruling, the error is partly odd and partly even, so that the asymmetry is diminished.

Periodic errors.—The theory of the "ghosts" produced by periodic errors in the spacing has been so often investigated that little can be added to the quantitative aspects of the theory. Michelson's method in particular, employing Fourier expansions for the law of spacing, is peculiarly adapted to the treatment of this kind of error. The vector method, however, gives us immediate insight into the qualitative aspects of the problem, enabling us at once to deduce all the important properties of this class of errors.

Let the $2N$ grooves of the grating consist of a group of m grooves repeated p times. Consider the diffraction pattern of a single group. The error of spacing of this group may be any kind whatever, provided it is not periodic.

To fix ideas, let the error be of even degree. We shall then have some such series of vector curves as is shown in Fig. 7. The curves, the tangents at whose ends are oppositely directed, correspond to the true direction of the first minima for a grating of the same mean spacing; the curves (a) and (b) correspond similarly to the true direction of the second minima, and so for minima of higher order. It is easy to see that these curves cannot close completely, provided the m grooves cannot be divided into two or more smaller identical groups. If we now take the p groups, we have a grating of the same mean spacing, and the vector curve for the directions just considered will consist of a repetition of loops similar to the curve for a single group (see Fig. 8). That is, the vector "errors of closure" for the single groups are all in phase. But now the principal maxima are narrowed down to $1/p$ of their former width, and these directions become those of a series of secondary maxima. The amplitude of the ghosts is thus proportional to the amplitude of the error of closure of the single group, and we have seen that this is proportional, within the first order of small quantities, to k, the order of the spectrum. Hence the intensity of the ghosts is proportional to k^2.

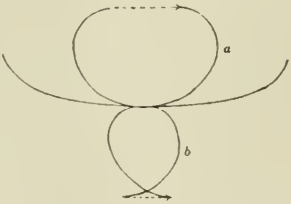

FIG. 7

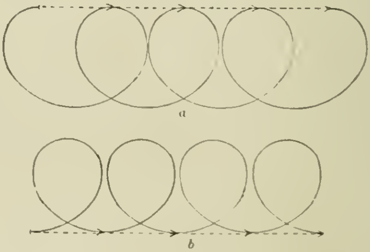

FIG. 8

Accidental errors.—Let us now suppose that the grating interval differs from the mean by a random small quantity $\frac{a\epsilon}{2\pi}$, the values of ϵ being distributed according to the law of error. We can no longer use the continuous vector curve, but must return to the vector polygon. In the direction of the central image the vector polygon is a straight line, in the direction of a principal maximum of the kth order the polygon has become a zigzag line. Consider a direction in which the order of interference is q, where q is in general not integral. The successive sides of the vector polygon have now inclinations $2\pi q+q\epsilon_1$, $4\pi q+q\epsilon_2$, . . . We can resolve each elementary vector along and at right angles to its "true direction" (i.e., the direction it would have had if the spacing were perfect). The first part gives us a set of vectors of magnitudes $\cos q\epsilon_1$, $\cos q\epsilon_2$, equally inclined to one another. As $\cos q\epsilon$ differs from unity by small quantities of the second order, the resultant of this group of vectors gives us the amplitude and phase which we would have had if the spacing had been perfect. The second group gives a set of vectors of magnitude sin $q\epsilon_1$, sin $q\epsilon_2$, . . ., or $q\epsilon_1$, $q\epsilon_2$, . . ., neglecting second-order quantities, also equally inclined to one another. If now q is not integral, the distribution of these vectors in phase becomes, on the average, symmetrical. The problem becomes that of finding the "expectation" of amplitude of $2N$ vectors of random phase with amplitudes distributed according to the "law of error." The theory of this problem has been given by Rayleigh, and we thus find that the "expectation" of amplitude is

$$\sqrt{2N}q\epsilon_0,$$

where

$$\epsilon_0=\sqrt{\Sigma\epsilon^2}$$

is the measure of precision of the ruling. The effect of the accidental errors will thus be to produce scattered light between the spectra of intensity proportional to $2Nq^2\epsilon_0^2$. The intensity of the scattered light is thus proportional to the mean square error of position of the grooves and to the square of the order of the spectrum. The term "order" should here be taken to mean the order of interference; thus for a point midway between the first and second orders we take $q=1.5$. The intensity of a principal maximum varies as N^2, so that the relative intensity of the scattered light varies as $1/N$. The total energy in a spectrum is, however, proportional to N, so that the proportion of energy in the scattered light is independent of N, as we should expect.

Amplitude errors.—We now pass to the consideration of a fundamentally different class of errors. These arise when the amplitude of the disturbance from the nth groove is a function of n, while the phase-difference between successive grooves is constant. For such errors it is convenient to take ϕ as our independent variable and write

$$\frac{ds}{d\phi}\equiv\rho=f(\phi).$$

The vector polygon is equiangular but not equilateral. In the direction of a principal maximum, therefore, it always straightens out completely, so that the form of the diffraction pattern is independent of the order. It follows also that the spectral images are always symmetrical and in their true position. The question as to whether the minima are clean or not depends, as with the phase errors, on whether $f(n)$ is an even function of n or not. In the classification of errors of this type we may repeat exactly the same list as was given for phase errors, namely, (1) f is a polynomial in ϕ; (2) f is subject to discontinuous variations; (3) f is periodic; (4) f is subject to accidental variations. Errors of the first class will naturally arise from the gradual wear of the ruling point; other types of this defect belong to other forms of spectroscopic apparatus. Errors of the second type may arise from a sudden breakage of the diamond point during the ruling; there are also certain artificially produced modifications of the grating coming under this head. Properly speaking there is no amplitude error of the periodic type to be looked for in the process of ruling the grating, but we shall consider one case in which such a variation of amplitude may be made to serve a useful purpose. The fourth type of error is always present.

If we screen off a portion of the ends of the grooves of one part of the grating the amplitude of the disturbances coming from these grooves is cut down proportionally. This process of artificially producing amplitude errors not only serves to give a concrete picture of any particular problem, but affords a means of practically compensating all except the accidental errors.

Generally speaking, the progressive change in form of groove due to wear of the ruling point is a trivial defect. The calculation of the intensity when $f(\phi)$ is represented by an algebraic expansion is a matter of some difficulty, but if we write

$$ds = ds_0 e^{-a\phi}$$

we have

$$\rho = \rho_0 e^{a\phi},$$

$$s = \frac{\rho_0}{a}(e^{-a\phi} - 1),$$

the vector curve is readily seen to be a logarithmic spiral, and the intensity is given by

$$\rho_0^2 \cdot \frac{e^{-2a\Phi} - 2e^{-a\Phi} \cos \Phi + 1}{1 + a^2},$$

where Φ is the total range of phase. The form of the diffraction pattern is that of a prism with finite absorption, a case already treated by H. M. Reese,[1] Wadsworth,[2] and the writer.[3] It is there found that the change in resolving power for a total variation of amplitude of $1/e$ is almost negligible, and even for a variation of $1/e^2$ is only 4 per cent. While the wear of the ruling point would not follow any such law in general, the above figures serve as a fair index of the order of magnitude of the effect to be expected. The breakage of the point, producing a sudden change in amplitude, is easily treated by the vector diagram. Thus suppose we have half of the grating giving an amplitude s and the other half giving an amplitude μs. The appropriate vector curve consists of two circular arcs of radii $\frac{s}{\phi}$ and $\frac{\mu s}{\phi}$, and of lengths s and μs, having a common tangent at their point of junction. The expression for the intensity is readily seen to be

$$\frac{s^2}{\phi^2}(1-2\mu\cos\phi+\mu^2),$$

or in general, for lengths x_1, x_2 of ruled surface, with amplitudes A_1, A_2,

$$I=A_1^2+A_2^2-2A_1A_2\cos\Phi.$$

The use of screens to cut off a portion of the ruled surface has a practical importance to which Rayleigh has called attention, but which has not been utilized to the fullest extent. We will consider some of the general aspects of the problem, limiting the discussion to symmetrical screens, for which the minima are free from illumination. Suppose first that the amplitude is reduced at the ends of the ruled surface. The circular arcs which constitute the vector curve of the complete grating now have a smaller radius of curvature at their extremities. Thus in the true direction of the first minimum the vector curve has not closed completely, the closure taking place for a larger value of the phase. The central portion of the image is thus widened out, with a consequent loss of resolving power. On the other hand, the linear dimensions of the closed loop are smaller than those of the corresponding circle; hence when the first subsidiary maximum has been reached it has a smaller relative amplitude than the complete grating. The general effect of screening the ends of the grating is thus to widen the image and at the same time to diminish the relative prominence of the secondary maxima. Screening the central portions of the image produces exactly the opposite effect. It is unnecessary to repeat the complement of the preceding argument. Diffraction patterns for discontinuous screening of the center are given by Rayleigh.[1] A typical example of the effect of screening the ends arises when the aperture of the optical train is less than the aperture of the grating. This case has in effect been worked out by Airy and by Rayleigh, the intensity being expressed by means of the Bessel function of the first order. Finally, if we screen off portions of the grating intermediate between the middle and the ends, we get a diffraction pattern which is in a sense a compromise between the two extreme cases given above, in which we obtain an increased resolving power without unduly enhancing the prominence of the subsidiary maxima. This case is discussed by Rayleigh,[2] who finds that if at the points of trisection of the grating three-eighths of the ruled surface is blotted out we get a fairly satisfactory compromise. The vector curves are readily constructed for this case, and the analytic expression for the intensity is readily obtained from such diagrams. The problem of finding the best compromise deserves wider investigation, but unfortunately the methods available for its solution seem to be almost entirely empirical. It may be noted in this connection that the grooves most affected by the phase errors of even degree are intermediate between the center and the ends of the ruled surface, so that the use of screens would in general improve the symmetry of the image.

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[1] *Astrophysical Journal*, 13, 199, 1901.

[2] *Phil. Mag.* (6), 5, 355, 1903.

[3] *Astrophysical Journal*, 44, 76, 1916.

Periodic errors of amplitude.—Consider a grating in which every pth groove is missing, what would be the effect on the diffraction pattern? To solve this problem, consider first the vector diagram of the complete grating. In the direction of the qth minimum the sum of the disturbances from all the grooves is zero. If we draw all the vectors from a common origin we will have $2N$ vectors symmetrically distributed about this point. If now we obliterate every pth groove, the corresponding vector diagram will be obtained by taking away $2N/p$ vectors, which will be in general symmetrically distributed about the origin, and which will therefore have zero for their sum. The sum of the remaining vectors will thus be zero, so that in general the minima of the defective grating will coincide with those of the complete grating. An exception arises when $q=2N/p$, $4N/p$, etc., for in this case the vectors subtracted will all be in phase, and will have a resultant $2N/p$; the resultant of the remaining vectors will have the same magnitude. The effect of omitting the grooves will therefore be to produce a series of ghosts of equal brightness $1/p^2$ in the directions of the principal maxima of a grating having an interval pa.

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[1] *Coll. Papers*, I, 415.

[2] *Ibid.*, III, 88.

This device has been suggested to the writer by Dr. J. A. Anderson,[1] of Mount Wilson, as a means of obtaining secondary standards for wave-length measurement with a grating, using the method of coincidences. It would make possible the use of a smaller number of selected spectral lines of known homogeneity, for purposes of comparison.

Accidental errors of amplitude.—These are due largely to the crystalline structure of the speculum combined with its brittleness. The range of the variation of amplitude is usually greater for the smaller wave-lengths. Let the amplitude of the successive disturbances differ from a certain mean value A by random quantities ϵA. Consider any direction in which the illumination would be zero for a perfect grating. For the actual grating the vectors are distributed symmetrically in phase, but are of unequal magnitude. We can break up this sum into a sum of $2N$ vectors of amplitude A plus a sum of $2N$ vectors of random amplitudes distributed symmetrically in phase. The sum of the first set is zero, that of the second, by the results of Rayleigh, referred to above, is on the average $A\sqrt{\Sigma\epsilon^2}$. Hence the expectation of intensity is $A^2\epsilon_0^2 N$. The effect of the accidental errors of amplitude is to produce scattered light of an intensity independent of the order. Practically all gratings show this scattered light, which can be distinguished from that due to errors of spacing by examining the neighborhood of the central image, where the errors of spacing produce no effect. It seems to be perfectly feasible to determine, by measuring the intensity of this scattered light, the magnitude of the accidental errors, both of spacing and groove form.

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[1] The writer is indebted to Dr. J. A. Anderson for his permission to refer to this. Dr. Anderson has actually ruled a small grating of this type, but the results obtained have not yet been published.

The method of coincidences.—The discovery of inadmissible errors in Rowland's table of wave-lengths threw a great deal of suspicion on the method of coincidences as a means of constructing standard tables. Michelson[1] then showed that a particular kind of error of spacing would be sufficient to account for the errors actually found. Unless it can be shown, however, that such defects as are unavoidable in the best gratings will produce systematic errors of measurement, a complete case can hardly be said to be made out against the method. Miss Howell's[2] very careful work has reopened the question (1) by pointing out other sources of error which were present in Rowland's work; (2) by showing that with some gratings, at least, good measurements can be made. Our task here is to investigate just what defects impair the usefulness of gratings for accurate measurement, together with the size of the possible errors introduced.

To begin with we may notice that coincident spectra of different orders will be in focus simultaneously. Consider the wave-fronts of two coincident orders. The effective errors of spacing are proportional to the order, and hence vary inversely as the wave-lengths, so that the phase variations of the wave-front are in this ratio. The linear magnitude of the distortions of the wave-front are thus the same, or the two wave-fronts coincide.

In the second place, variations of the form of groove or other amplitude errors cannot affect the measurements, as they affect neither the symmetry nor the positions of the spectral images. The same is true of the phase errors of odd degree. There remain the phase errors of even degree as the only defects which can affect the measurements with any systematic error.

We have seen that such errors displace the maxima from their true positions. We shall limit the following discussion to quadratic errors. If we plot the position of the maximum against the magnitude of $\frac{1}{3}\gamma s_0^3$, the coefficient of the quadratic error, we get the curve (*a*) of Fig. 9. Except for the larger values of $\frac{1}{3}\gamma s_0^3$ this curve is sensibly a straight line. Thus the displacement of the maximum, measured in phase (that is, in terms of the width of the image), is proportional to the magnitude of the error. The error is proportional to the order of the spectrum, while the width of the image is proportional to the wave-length; the angular displacement of the maximum is therefore proportional to $k\lambda$. It follows that if all measurements are made on the maxima, no errors can be introduced in the ratio of two wave-lengths. The same is true if we

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[1] *Astrophysical Journal*, 18, 298, 1903.

[2] *Ibid.*, 29, 230, 1914.

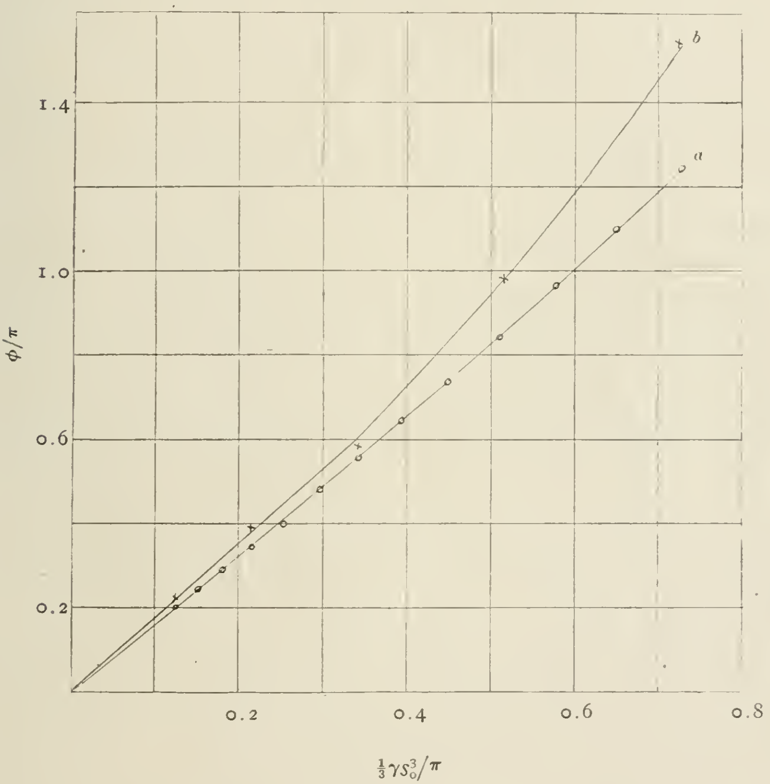

FIG. 9

make all settings on the center of gravity of the central maximum, for if we plot the shift of this we get the curve (*b*), which is approximately straight. The error committed arises entirely from the fact that the images are unsymmetrical, and dissimilar for the different orders, and hence when we set on an image, the particular portion of the image set on depends on the form of the image, and on the intensity of the line in a way that cannot be predicted. The distance between the two curves gives us an approximate upper limit to the magnitude of the error committed. The greatest value of this within the range of errors considered gives about $\pi/6$. This is about one-sixth of the angular separation of the principal maximum and the first minimum. For a grating of 100,000 lines the error would be about 2 in 1,000,000, say 0.01 A in the neighborhood of the D lines. This would suppose, however, a grating so bad that the spectral images were nearly symmetrical doublets. If the asymmetry of the images is no greater than 1 of Fig. 5 the limit of error would be about 0.001 A. Whether such gratings are hard to find or not an asymmetry of this amount should be easily noticeable, so that it should be possible, by simple examination of the spectral images, to decide whether the results of the measurements should have this accuracy. There are other tests for the presence of quadratic error, such as covering half of the grating and measuring the change of focus or observing the shift of the image, but they are not nearly so sensitive as the asymmetry test. It may be considered that in view of the multiplication of secondary standards obtained by the Fabry and Perot interferometer there is no need to use gratings for this purpose. We shall see in what follows that the interferometer measurements are subject to an error of the same kind as the grating, and further, the grating remains the only instrument for measurements on absorption lines. The writer hopes at some future date, when he can get access to a number of gratings, to pursue the subject farther.

The defective Fabry and Perot interferometer.—Consider a narrow beam of parallel light falling on the interferometer plates. Let Δs_0 be the amplitude of the first transmitted beam. Then the amplitude of the *n*th beam may be written

$$\Delta s = \Delta s_0 e^{-kn}$$

The vector polygon consists of a series of vectors whose phases are in arithmetical progression and whose amplitudes are in geometric progression. The summation leads to the well-known Airy formula for the intensity. If the logarithmic decrement of the successive interfering beams is not too great, the writer has shown that we can replace the polygon by a logarithmic spiral. Hence we may write

$$ds = ds_0 \epsilon^{-kn} = \rho_0 e^{-\frac{k\phi}{d\phi}} d\phi = \rho_0 e^{-\frac{k\phi}{c_0 ds}} {}_0 d\phi \,.$$

Hence

$$\int_0^\infty \cos\phi ds = \frac{k}{c_0^2 ds_0^2 + k^2} \qquad \int_0^\infty \sin\phi ds = \frac{c_0 ds_0}{c_0^2 ds_0^2} + k^2 \,.$$

Here c_0, the curvature of the spiral, is the change in phase for unit length of the spiral. Hence $c_0 ds_0$ is the phase-difference between successive interfering beams ($=\phi_1$, say), while k is the logarithmic decrement for two successive beams. Squaring and adding, we get the expression for the intensity previously obtained.[1] If the beam covers an area dS of the plates, it contributes to the total intensity the two vector components

$$\frac{kdS}{\phi_1^2 + k^2}, \quad \frac{\phi_1 dS}{\phi_1^2 + k^2} \,.$$

Let the distance z between the plates be given by

$$z = z_0 + f(x, y),$$

where z is the distance at any assumed origin on the plates, and x, y are the co-ordinates of dS. We have

$$\frac{2\pi}{\lambda} z = \phi_1 = \Phi + \frac{2\pi f}{\lambda},$$

where Φ is the phase-difference at the origin. We thus get for the final intensity the sum of the squares of the two integrals

$$\int \frac{kdS}{\left(\Phi + \frac{2\pi f}{\lambda}\right)^2 + k^2}, \qquad \int \frac{\left(\Phi + \frac{2\pi f}{\lambda}\right) dS}{\left(\Phi + \frac{2\pi f}{\lambda}\right)^2 + k^2} \,.$$

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[1] Sparrow, *op. cit.*

The images will not in general be symmetrical. Before examining the general case we will assume a few special forms of f. Taking circular plates, with the origin at the center, let

$$z=z_0+\frac{\lambda b r^2}{2\pi R^2},$$

where R is the radius of the plates, we can readily integrate, and get for the intensity

$$J=\frac{k^2}{b^2}\left\{\tan^{-1}\frac{bk}{k^2+\phi_1^2-\frac{b^2}{4}}\right\}^2+\frac{k^2}{2b^2}\left\{\log\frac{\left(\phi_1+\frac{b}{2}\right)^2+k^2}{\left(\phi_1-\frac{b}{2}\right)^2+k^2}\right\}^2$$

The images are symmetrical. The curve is shown in (*b*) Fig. 10. Perhaps the simplest assumption we can make as to f is that f is constant and equal to 0 for the fraction h of the total area, and equal to the constant $\gamma\frac{\lambda}{2\pi}$ for the remainder. The expression for the intensity is

$$J=\left[\frac{hk}{\phi_1^2+k^2}+\frac{(1-h)k}{(\phi_1+\gamma)^2+k^2}\right]^2+\left[\frac{h\phi_1}{\phi_1^2+k^2}+\frac{(1-h)(\phi_1+\gamma)}{(\phi_1+\gamma)^2+k^2}\right]^2$$

For $h=\frac{1}{2}$ we get a symmetrical curve. For $h=\frac{1}{3}$ we get the curves (*c*), (*d*), (*e*), for which $\gamma=0.18$, 0.27, 0.36 respectively.

Before examining the consequences of this asymmetry, let us look a moment at the general question. In the first place, it is evident that the number of surfaces which will give the same diffraction pattern is infinite, for we can rearrange the elements dS in any way so long as the distances are preserved. If we change the sign of ϕ_1 and f simultaneously, the pattern is unaltered. But changing the sign of f replaces the surface by its reflection in the xy plane, and since this is equivalent to a change in the sign of ϕ_1, the diffraction pattern is turned end for end. If the pattern is symmetrical therefore, f and $-f$ give the same pattern. Thus a concave and a convex spherical curvature give the same pattern. Choose z as independent variable, and pass planes z, $z+dz$, intercepting between them the area dS. Let z be the plane through the center of gravity of the surface. Then if dS/dz is an even function of $z-\bar{z}$, the elements are *balanced* against one another, so that when the surface is reflected in the plane $z=z$, there results simply an interchange of equal elements, and we have symmetry. It is not quite clear whether this condition is necessary for symmetry, as the difficulties of a general solution of the problem are considerable. We can see now, however, why the spherical curvature should give symmetrical images for this condition is exactly fulfilled in this case, dS/dz being in fact constant. With the notation adopted, the quantity b represents the extreme deviation in phase over the plates. By examining the expression for the intensity it can be seen that the shape of the diffraction curve depends on the ratio of γ to k. The value of k corresponding to the investigations of Fabry and Perot is log 0.75 or about 0.3, which is therefore the value assumed in these calculations. The better methods of polishing the films used by Pfund and others probably reduce this value of k, and hence make higher demands on the accuracy of the plates. Thus the errors of path difference corresponding to the curves given are (a) 0; (b) 0.05λ; (c) 0.032λ; (d) 0.043λ; (e) 0.064λ.

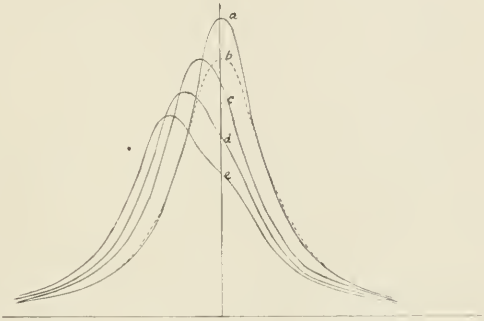

FIG. 10

The importance of this for us lies in the fact that the method of measuring wave-lengths with this instrument is essentially the *method of coincidences*. The ratio of two wave-lengths is found by measuring the separations of the plates in terms of the two. If the images are unsymmetrical, an error arises, just as with the grating, from the uncertainty as to which part of the fringe is set on. The distance between the center of gravity and the maximum for the curves drawn is (a) 0; (b) 0; (c) 0.05; (d) 0.10; (e) 0.14.

The separation of the fringes is 2π in terms of ϕ. Thus if the variation of path difference is 0.04λ the error of setting may amount to one-sixtieth the distance between the fringes, or about double the limit of measurable accuracy. But this requires that the distance between the plates be constant to within $\lambda/40$, or that each plate be correct to within half this amount if the plates are tested separately. We thus see what a high precision of figure is necessary to approach the accuracy which the instrument is supposed to give. Even if the plates are figured accurately a slight variation in the condition or thickness of the silver film may introduce the errors. Somewhat larger errors are caused when $h=\frac{1}{4}$. This is probably an extreme case, but the value $\frac{1}{3}$ does not give the upper limit of possible asymmetry.

It may be argued, however, that the method of using different separations of the plates, which is used in order to eliminate the phase-change in the film, also eliminates any systematic error in setting on the fringes. In the first place, in order that this method should be valid, the two plates when used with different etalons must be always placed in the same relative position. In the second place, the wave-length must be calculated from the *difference* of the separations, just as is done in absolute measurements. Actually, two different methods have been used. Pfund[1] calculates the phase-change from the difference, which thus gives the average value of the phase-change over the plates, and then uses the larger separation alone to calculate the wave-length, thus reintroducing any error arising from asymmetry. Fabry and Buisson[2] used for the smaller separation the Newton ring system localized *in the plates*. In this system of fringes each fringe is produced by that small portion of the plates in its neighborhood, and the fringes are necessarily symmetrical. Thus in determining the phase change by this method, measurements on symmetrical fringes are compared with those on (possibly) asymmetrical fringes.

There is another cause of asymmetry which exists even with perfect plates. This is due to the fact that the diameters of successive rings vary as the square roots of the natural numbers. The dispersion of the instrument thus varies, and the asymmetry of the rings increases as we approach the center. Thus if we measure the central rings, as is usually done, we have to deal with an asymmetry the amount of which depends on the fractional part of the order. Errors arising from this cause may affect even absolute measurements, unless care is taken always to use such distances that the fractional order is the same for both separations of the plates. All of the foregoing discussion of the shape of the plates has assumed that the rings were not too near the center. To find the exact shape of a particular fringe, we should have to replace ϕ_1 by μ_2 where r is the radius of the fringe corresponding to the phase ϕ_1 and r_0 corresponds to the zero of phase.

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[1] *Astrophysical Journal*, 28, 197, 1908.

[1] *Ibid.*, 169, 1908.

We thus see that the errors of the method of coincidences are not confined to the grating, but are to be looked for in the interferometer as well. That the measurements with the latter instrument have been more satisfactory is due perhaps to the fact that good interferometers have been compared with poor gratings, but also, and in large measure, we think, to the fact that the users of the former instrument have applied the knowledge acquired in other fields regarding the standardization of light sources, atmospheric dispersion, etc. The theoretical advantages of the two instruments may be compared as follows: The defects of the gratings are fixed. If we have a good instrument we can make measurements as good as the best; if it is defective a definite limit is set to the accuracy of all our measurements. With the interferometer a good pair of plates may become defective through rotating one of them, or through tarnish or non-uniformity of the silver film; on the other hand, we may more or less average out the errors of a defective pair by this process, since each silvering or change of relative position makes virtually a new pair of plates. We think the tests for the accuracy of the grating can be made more sensitive than those for the interferometer, because the disappearance of the subsidiary maximum on one side in an intensity curve like (1), Fig. 5, is more certainly detected than a slight general asymmetry of a smooth curve like (*c*), Fig. 10. The writer hopes at some future date to examine this question experimentally.

Rouss Physical Laboratory
University of Virginia
July 1, 1918

STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS[1]

TENTH PAPER: A CRITICAL MAGNITUDE IN THE SEQUENCE OF STELLAR LUMINOSITIES

By HARLOW SHAPLEY

The investigations reported in the preceding *Contributions* point to the close equality of the median apparent magnitudes of cluster-type variables as a significant phenomenon in Cepheid variation. The absolute brightness corresponding to this median value marks the limit fainter than which variability of the Cepheid type evidently does not occur. The discussion in the present paper tends to show that the same absolute magnitude, or one nearly the same, also marks a critical point in the dynamical history of stars in general. The result might have been anticipated, for the concentration of the median luminosities of cluster-type variables to a very narrow interval would require a similar concentration for other stars of like mass and color if Cepheid variation is regarded as accidental rather than inevitable in a star's development. We must keep in mind, however, the possibility that the variation itself might operate to hold a star indefinitely long at the critical magnitude.

From an unpublished study of Messier 3 (N.G.C. 5272) the frequency of photo-visual magnitudes for various classes of colors is given in Table I. The catalogue from which this table was constructed is based upon about ten thousand measures of magnitude. Much care has been taken to avoid errors, and the results are believed to be dependable, both as to actual colors and as to relative frequencies. Only stars between 2′ and 11′ from the center of the cluster and brighter than the seventeenth photo-visual magnitude are included in Table I. All known variables are omitted, as well as stars with doubtful colors or magnitudes. The general discussion of the table we reserve for the paper on Messier 3, using for the present investigation only the relation of star numbers to magnitude.

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[1] *Contributions from the Mount Wilson Observatory*, No. 155.

TABLE I

FREQUENCY OF PHOTO-VISUAL MAGNITUDES IN MESSIER 3 FOR EACH COLOR-CLASS (DISTANCE FROM CENTER 2′.0 TO 11′.3)*

Limits of Photo-Visual Magnitude	<*b*5	*b*5 to *a*0	*a*0 to *a*5	*a*5 to *f*0	*f*0 to *f*5	*f*5 to *g*0	*g*0 to *g*5	*g*5 to *k*0	*k*0 to *k*5	>*k*5	<*f*5	>*f*5	All Colors
12.20–.39	......	......	......	......	......	......	......	......	......	1	......	1	1
.40–.59	......	......	......	......	......	......	......	......	1	1	......	2	2
.60–.79	......	......	......	......	......	1	......	1	......	2	......	4	4
.80–.99	......	......	......	......	......	1	......	......	4	......	......	5	5
13.00–.19	......	......	......	......	......	......	......	5	2	......	......	7	7
.20–.39	......	......	......	......	......	......	......	1	2	......	......	3	3
.40–.59	......	......	......	......	......	1	......	4	1	......	......	6	6
.60–.79	......	......	......	......	......	......	4	3	......	......	......	7	7
.80–.99	......	......	......	......	1	......	7	6	......	......	1	13	14
14.00–.19	......	......	1	......	......	2	2	5	1	......	1	10	11
.20–.39	......	......	......	......	......	1	3	3	......	......	......	7	7
.40–.59	......	1	......	1	3	16	6	......	......	......	5	22	27
.60–.79	1	......	......	1	1	11	11	2	......	......	3	24	27
.80–.99	......	......	......	1	1	3	7	4	......	......	2	14	16
15.00–.19	......	......	1	2	......	7	17	3	......	......	3	27	30
.20–.39	......	1	4	4	15	19	13	1	......	......	24	33	57
.40–.59	......	9	23	12	15	26	1	......	......	......	59	27	86
.60–.79	......	8	6	4	2	28	8	2	......	......	20	38	58
.80–.99	......	2	6	......	7	17	5	......	......	......	15	22	37
16.00–.19	2	2	2	......	12	14	6	......	......	......	18	20	38
.20–.39	1	1	......	4	16	13	1	1	......	......	22	15	37
.40–.59	......	1	......	9	21	19	......	......	......	......	31	19	50
.60–.79	......	......	2	20	26	11	1	......	......	......	48	12	60
.80–.99	......	2	4	24	38	1	......	......	......	......	68	1	69
All magnitudes	4	27	49	82	158	191	92	41	11	4	320	339	659

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*To save space in the table two tenth-magnitude red stars and one of eleventh magnitude with color-class *a* are omitted. Probably they are not physical members of the cluster.

Plotting the data of the last column, we obtain, as in the upper part of Fig. 1, the luminosity-curve for all colors, which shows a conspicuous, sharp maximum at photo-visual magnitude 15.50. This combines the luminosity-curve of the reddish giants with an abnormal or at least irregular luminosity-curve of the bluer stars. In the lower half of Fig. 1 the two are given separately. The curve for the redder giants is as smooth and complete as could be expected for a small group of giant stars.[1] The other, though otherwise a regular curve for which the fainter limit is set by the

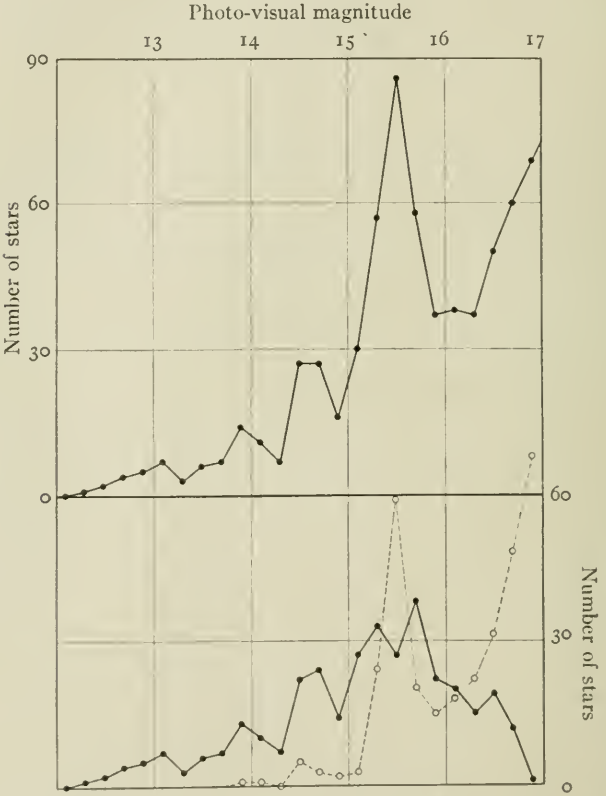

FIG. 1.—Frequency of photo-visual magnitudes in Messier 3. *Above:* All colors. *Below:* Full line, color-index $\geq +0.60$; broken line, color-index $< +0.60$.

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[1] Actually this curve is also a composite of several distinct luminosity-curves, as may be inferred from the columns of Table I.

limit of the catalogue, shows a remarkable gathering of stars between magnitudes 15.20 and 15.60. A smooth curve would allow less then 15 stars to this interval; the actual number is 83, exclusive of known variable stars.

The median photo-visual magnitude of the variables has not been directly determined, but it is known[1] to lie between 15.25 and 15.35. Therefore, if all the known variables within the measured area were included, with median values of their magnitudes and colors, the excess of stars between 15.20 and 15.60 would be about 1200 per cent.

The observed luminosity-curve for the blue stars is not of the regular form originally expected and most readily explained. If, with the variables excluded, a smoothly increasing curve had resulted, we would have assumed that the variables are peculiar stars intermingled with a normal aggregation. On the other hand, if a smoothly increasing curve resulted only when all the variables were included at their median, maximum, or minimum brightness, we would have assumed that the variables are normal stars undergoing a stage in stellar evolution that is common to nearly all stars with sufficient mass to attain this critical luminosity. The actual condition, however, is accounted for less easily.

A question naturally arises concerning the constancy of the light of the stars that form the excess over the number required for a smooth luminosity-curve. In the area covered by the manuscript catalogue of stars in Messier 3 there are about 100 typical cluster-type variables—all nearly identical in every known characteristic;[2] and there are about 70 of these additional stars, which, though closely resembling the variables in magnitude and color, do not in a single case appear to undergo a definite variation of light. Are they incipient variables, or stars in which a variation once existed but is now disappearing or has completely vanished? Or, through the fortunes of position and motion, have they avoided the cause of variation or, by relatively slight differences in physical structure, have they been able to withstand it?

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[1] See Figs. 3 and 4 in the second part of the preceding *Contribution.*

[2] Except, of course, in distance from the center, and possibly, therefore, in space velocity.

The stars in question may be readily selected from the catalogue, and a future detailed study of their magnitudes may help to solve a problem which is of much significance in the full interpretation of Cepheid variation, and which may be of some importance in getting at the internal structure of a star. Meanwhile we have an indication of the possible result through the following tabulation of the means of the photographic residuals:[1]

Photo-visual magnitude 15.20 to 15.60:
  Color-index 0.00 to +0.20, Mean Pg. residual ±0.103 (33 stars)
  Color-index +0.20 to +0.40, Mean Pg. residual ±0.083 (19 stars)
  Color-index +0.40 to +0.60, Mean Pg. residual ±0.071 (37 stars)
All magnitudes and colors, Mean Pg. residual ±0.082 (800 stars)

The interval of brightness near median magnitude is the most comfortable of all for measurement, being neither too bright nor too faint, and the accidental errors should be less than for the average. The mean residual for all stars includes a considerable number of values referring to objects which later were rejected because of uncertainty; therefore this mean is slightly greater than it should be for a fair comparison.

It is evident then that the 33 stars most nearly like the variables in median magnitude and color are subject to uncommonly large magnitude deviations. Moreover, one or two uncertain results are not responsible for the large average residual, as one-half of the deviations exceed ±0.10. While awaiting further study our provisional conclusions are that Cepheid variation affects some of these stars to some extent, and that it may be possible to witness the beginning or end of this type of phenomenon. If so, the initial and final stages must be short compared with the whole duration of mature variability, for of all the variables in this cluster none is yet known to have an intermediate amplitude or period.[2]

Since the mean photographic median magnitude is definitely known from Bailey's extensive studies, we shall be able to examine more closely the relationship of the variables to these 33 stars if we put the frequency analysis on a photographic basis. Rearranging the data of Table I and plotting the numbers for color-class *a*0–*a*5, we obtain the photographic luminosity-curve given in Fig. 2*a*. The maximum frequency corresponds to a brightness fainter than the median, 15.50, by less than a tenth of a magnitude.

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[1] The residuals will appear in the forthcoming catalogue of Messier 3. The magnitude of each star is the mean of from two to six determinations.

[2] Cf. sec. IV of the sixth paper. Variable No. 37 belongs to a distinctly different subtype.

A similar investigation of the photographic luminosity-curve for Messier 13—the only other globular cluster for which extensive color data are available[1]—results in the curve (Table II) illustrated in Fig. 2*b*, showing a similarly close coincidence of maximum frequency and median magnitude (15.03).

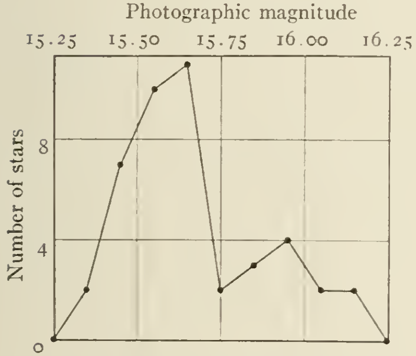

FIG. 2*a*.—Photographic luminosity-curve of blue stars in Messier 3.

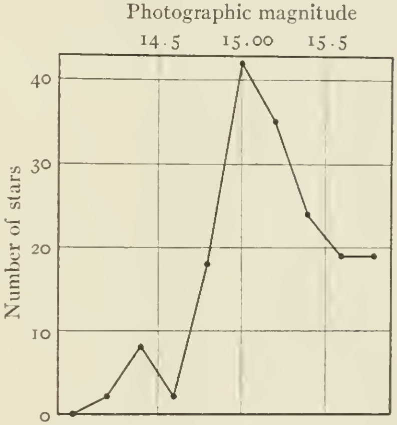

FIG. 2*b*.—Photographic luminosity-curve of blue stars in Messier 13.

In Messier 13 the few faint variable stars have not been thoroughly studied, and it is necessary to obtain a hypothetical median magnitude by adding 1.28 to the mean magnitude of the 25 brightest stars.[2] And also, since the number of stars of color-class *a* is not large, the interval of color is extended in both directions, thus including numerous stars with negative color-indices. In fact, the maximum is chiefly due to stars with color-class *b*7 to *a*0—a comparatively infrequent type[1] in Messier 3.

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[1] *Mt. Wilson Contr.*, No. 116, p. 51, 1915. Instead of photographic, as erroneously printed, the limits of magnitude in this table should read photo-visual. The change to the photographic system for the present curve is made with sufficient accuracy by assuming mean colors and magnitudes for each tabulated interval. For Messier 3, however, the transformation was carried out rigorously.

[2] See sec. V of the sixth paper and Table II of the seventh paper.

TABLE II

FREQUENCY IN MESSIER 13 OF THE PHOTOGRAPHIC MAGNITUDES OF HIGH-LUMINOSITY BLUE STARS

Interval of Photographic Magnitude	*b*0 to *b*5	*b*5 to *a*0	*a*0 to *a*5	*a*5 to *f*0	*f*0 to *f*5	Color-Index < +0.60
12.70–12.89	1	....	....	....	....	1
.90–13.09	....	....	....	1	....	1
13.10– .29	....	....	....	....	1	1
.30– .49	....	....	....	....	....	0
.50– .69	....	....	....	....	....	0
.70– .89	....	3	....	....	....	3
.90–14.09	....	....	....	....	....	0
14.10– .29	....	....	....	2	....	2
.30– .49	....	1	1	....	6	8
.50– .69	....	....	1	....	1	2
.70– .89	....	5	6	7	....	18
.90–15.09	10	24	3	2	3	42
15.10– .29	4	21	9	....	1	35
.30– .49	1	11	7	3	2	24
.50– .69	....	5	6	5	3	19
.70– .89	....	....	3	4	12	19

The luminosity-curves of Figs. 2*a*, *b* render probable the hypothesis that the median magnitude is an epochal point in the luminosity of a cluster star whether or not its light is periodically variable. As a partial and provisional interpretation of the congestion of blue stars at this magnitude, we may refer to the luminosity-period curve of Cepheid variation and in particular to the constancy of absolute magnitude for all periods less than a day. If we adopt the theory of pulsations for these variables, it is clear that a closely analogous curve would connect luminosity with mean density, and accordingly a progression through the whole range of densities possessed by cluster-type variables would demand no change in median brightness. Now, if a more or less uniform progression of density with time is a property of all stellar evolution (a natural and almost necessary assumption, if stars are either contracting or expanding), there may come an era in the history of every massive star when no change of magnitude will occur throughout a long interval of time, although the variation of the mean density is making conspicuous progress. Such a condition will result in a maximum in the frequency-curve of luminosity, while it would produce no irregularity in a graph of the frequency of density.[1] Therefore, the perfect counterbalance, at the median magnitude of cluster-type variables, of decreasing surface area with increasing intensity of light-emission, which can account for the flattening out of the luminosity-period curve, may explain at the same time the accumulation of invariable stars about this critical point.[2]

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[1] The small difference in the mean color at the maxima of the two curves is probably larger than the combined zero-point errors in the two color scales.

Assuming, however, on the basis of the theoretical studies of Eddington and Jeans, that the brightness of a giant star is independent of the density and is essentially a simple function of the

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[1] In the third note of the preceding paper we find in Messier 15 neither a change of color nor an appreciable difference in magnitude for the two groups of variable stars which differ decidedly in period and presumably in mean density. The mean periods are about three and five-eighths of a day, respectively, and the mean densities must differ in the ratio of three to one unless two different classes of vibration are involved. How such a change in the effective density could occur (with its supposedly necessary change in surface area) without a distinct alteration either in the total light-emission or in the median color is not obvious. A detailed study of the two groups of variables in red or violet light may supply the answer.

The solution of this problem may lie in a further consideration of the theoretical discussion and observations of the preceding contribution (*Astrophysical Journal*, **49**, 24, 1919). From equation (2) on page 28 the absolute magnitude may be written

$$M \propto \frac{2}{3}(2 \log P + \log \mu) + \log \phi \text{ (C. I.)}$$

where P is the period of pulsation, μ is the mass, and ϕ (C. I.) determines the surface brightness. The observations referred to in the foregoing paragraph suggest that M and ϕ (C. I.) do not change for cluster-type variables. Therefore $P^2\mu$ remains essentially constant, and the variables of the shorter-period subtype are three times as massive, in the mean, as those of the longer-period subtype. Further, within the uncertainty of the observations of color and magnitude, the period of an individual cluster-type variable does not change during the life of the variation, for it is hardly conceivable that the mass alters appreciably. Every period is associated with a definite mass, and only one period is possible for a given star. (Cf. Russell, *Science*, N. S., **49**, 136, 1919.) It appears, therefore, that the period-luminosity curve is best interpreted on the basis of Eddington's theory of a giant star. In any case the critical luminosity is to be associated with the flattening of the period-luminosity curve.

[2] There are large numbers of fainter blue stars in both Messier 3 and Messier 13, as is shown by Table I of this paper and by Tables XXI, XXIV, and XXV of the second paper, *Mt. Wilson Contr.*, No. 116. One is reminded of the differences in mean absolute magnitude found by Kapteyn and Charlier for various groups of galactic B-type stars.

mass, then the congestion of stars at the median magnitude would indicate a sharp maximum in the frequency of stellar masses. Such an assumption suggests interesting and unexpected consequences in the interpretation of the luminosity-period curve[1] and in the study of the relative ages of stellar systems.

Of the two clusters discussed above, one is unusually rich in typical cluster-type variables but poor in *b*-class stars of the median magnitude; the other is poor in variables but remarkably rich in the number of stars of the median magnitude with negative color-indices. There may be much significance in this compensatory difference, but our material is still too scanty to warrant further hypothesis. Future work must also decide to what extent the cluster phenomena may be modified in their translation to the general stellar system.

Meanwhile, through investigations of composite luminosity-curves for other clusters, we may look into the generality of the maximum at the median magnitude. Lacking color-indices, we have not the power to determine quantitatively the influence of the giant red stars on the composite curves, but we shall be able to extend the work to fainter magnitudes and to learn something of the relative importance of this maximum frequency near absolute magnitude zero.[2]

A summary of the photographic magnitudes in four clusters appears in Table III. Messier 13, which affords a good example of a cluster with few variables, has been remeasured for this table because the data of Table II are too incomplete for the fainter stars. For none of the four systems do the measures extend over the whole photographic plate, the condensed central regions, for instance, being necessarily avoided. Over large representative areas, however, all stars are included so that correct relative frequencies are obtained. Care has been taken that peculiarities which might be associated with galactic planes should not unduly prejudice the results. After correcting for distance from the center and irregularities of the measuring scale, the measures are retained in the original scale readings, the exact evaluation for fainter stars being impossible from the present material. Except for a possible divergence near the ends of the scale, the average value of an interval is one-third of a magnitude.

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[1] See the first note of the preceding *Contribution*.

[2] The adopted absolute value of the photographic median magnitude is −0.23; cf. sec. IV of the sixth paper.

The relative luminosity-curves are plotted in Fig. 3, each adjusted so that the median magnitude is on the same vertical line. For both Messier 2 and 13 the median photographic brightness is the hypothetical value based on the brighter stars. The sharpness of all the maxima has probably been somewhat lessened through the process of smoothing out scale irregularities.

TABLE III

FREQUENCY OF THE MAGNITUDES OF 4015 STARS IN FOUR CLUSTERS

Corrected Scale Reading	Messier 2	Messier 5	Messier 13	Messier 15
<10	2	1	1	17
10.5	0	0	2	12
11.5	1	0	6	17
12.5	3	2	13	39
13.5	6	10	21	62
14.5	19	26	41	100
15.5	46	45	51	151
16.5	76	79	59	148
17.5	95	109	53	161
18.5	94	126	62	148
19.5	84	110	69	161
20.5	79	89	97	186
21.5	92	79	147	212
22.5	122	90	228	266
Total	719	766	850	1680
Median Pg. Scale	17.2	17.7	14.1	15.4
Median Pg. Mag	15.90	15.26	15.03	15.63
Number of variables included	5	70	3	50

In comparing these maxima with the median magnitude it should be remembered that the latter is subject to some uncertainty of measurement and the former to a displacement of unknown amount and direction through the inclusion of the giant red stars. The possible effect of the superposition of the luminosity-curves of the red giants is illustrated in the curve of Fig. 4, where, for Messier 3, the total numbers of Table I are plotted, after transforming them to the photographic system. This curve should be compared with that of Fig. *2a*.

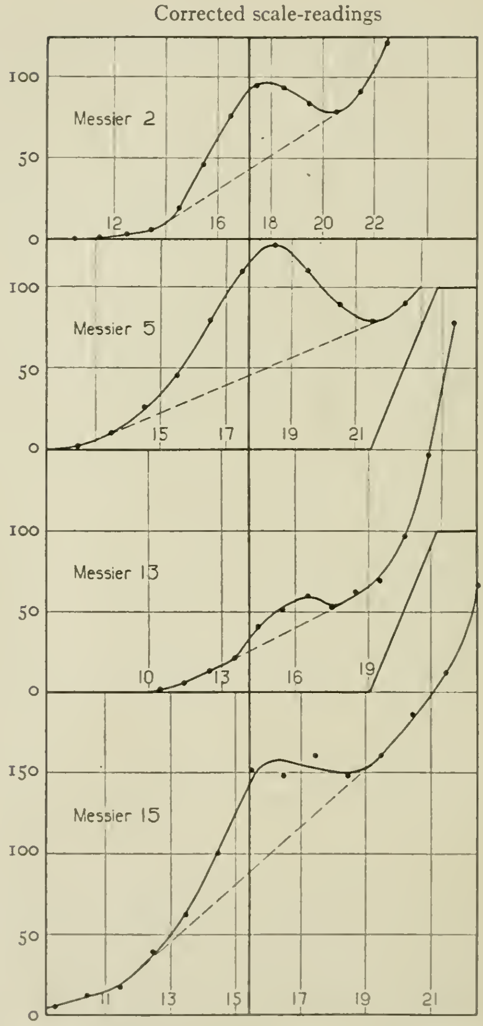

FIG. 3.—Photographic luminosity-curves of stars of all colors in four globular clusters, illustrating the maximum frequency near zero absolute magnitude. The median magnitude for each cluster is indicated by the heavy vertical line. The number of stars measured in different clusters (ordinates) is not proportional to the total number in the cluster.

From the evidence of this series of diagrams there can be little doubt that in all these clusters we have a concentration of the magnitudes of high-luminosity blue stars about the median magnitude of cluster-type variables.

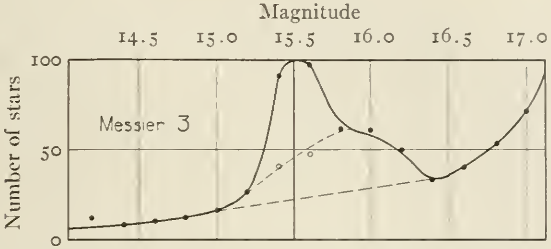

FIG. 4.—Photographic luminosity-curve of stars of all colors in Messier 3. Excluding variable stars we have the broken line with circles.

SUMMARY

The absolute photographic magnitude −0.2 defines a limit of stellar luminosity at which the liability to Cepheid variation suddenly stops. All known variables of the class appear to surpass this limiting brightness, which is probably coincident with a turning-point in the development of the internal structure of a star. Whether or not analogous dynamical changes occur, regardless of variability, at the same epoch in the evolution of luminosity for all giant stars, has been discussed for several globular clusters with the aid of luminosity-curves derived from the magnitudes of 5000 stars. For a graphical representation of the results reference may be made to the figures on the foregoing pages.

Table I affords the complete luminosity-curves for all giant stars in Messier 3 redder than color-class *g*0. Confirming earlier work, the final results for this cluster show that the highly luminous stars are red, that the fainter ones are blue. The infrequence of cluster-type variables in the Hercules cluster (Messier 13) is compensated for by the richness of extremely blue stars at the median magnitude.

MOUNT WILSON SOLAR OBSERVATORY
December 1917

AERIAL PHOTOMETRY

By M. LUCKIESH

Owing to the nature of the material presented in this paper it is impractical to provide the usual summary; therefore for the convenience of the reader the following outline is given of the chief details in their order of appearance in the text.

INTRODUCTION

- Source of daylight illumination
- Variation of sunlight on a clear day
- Variation of skylight on a clear day
- Relations of sunlight and skylight for a clear day
- Relations of sunlight and skylight on four typical days
- Maximum total illumination at noon on clear days as function of season and of cloudiness
- Skylight illumination in terms of total illumination
- Russell's conclusion of 135,000 meter-candles as maximum illumination outside earth's atmosphere
- Another method of determining maximum solar illumination which yields 126,000 meter-candles
- Conclusions

APPARENT REFLECTION FACTORS OF EARTH AREAS

- Terminology—reflection factor and relative brightness
- Table I—reflection factors of fields, barren land, woods, inland water
- General summary of reflection factors of earth areas in summer; in mid-autumn
- Statements of Abbot on earth's albedo

LANDSCAPE

- General characteristics
- Attempt at averaging reflection factors
- Explanation of low reflection factor
- Range of measurements of reflection factor
- Attempt to measure average relative brightness of landscapes

CLOUDS

- Reflection factors of upper surface of different types of clouds
- Explanation of high value

- Haze
  - Two types of haze
  - Rough determination of earth haze
  - Effect of absorption and scattering of haze on earth's relative brightness from high altitudes
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  - False horizon due to earth haze

- Sky
  - General sky brightness
  - Variation of sky brightness with altitude
  - Table II—brightness of earth areas in terms of sky brightness
  - Checks between Tables I and II
  - Check on mean reflection factor of earth
  - Determination of relative brightness of upper and lower sides of horizontal opaque white sheet
  - Case of an opaque white surface disappearing against the sky
  - Data of Kimball and Thiessen for clear and smoky atmosphere

- Water
  - Variation of brightness expected from reflection at the surface
  - Variations due to objects mirrored, to beds, to suspended matter, etc.
  - Variation of trip over shallow inland, deep inland, and ocean water
  - Hue considerations
  - Table III—brightness of inland waters in terms of sky brightness
  - Same for deep ocean water
  - Evidence that brightness of water is generally due chiefly to diffusion of light in the water
  - Numerical considerations
  - Brightness at forty-five degrees incidence

INTRODUCTION

Some of the data recorded in the following pages were obtained as part of an investigation of the visibility of airplanes. Owing to the necessity for hastening the completion of the work which was primarily in mind, many interesting details could not be given much attention. However, many data were obtained during sixty hours in the air with facilities for making flights of any character as to altitude, distance, terrain, etc., which could not be obtained as satisfactorily in any other manner. Various types of instruments were used, but most of the measurements were made with a compact instrument of special design which need not be described. It is proposed to present in this paper only certain photometric measurements and allied observations, which may be of interest from various viewpoints aside from the original one. In order that the reader may have a clearer conception of the details described in later sections, a few introductory paragraphs are included which deal with the general characteristics of natural lighting.

Natural light in the daytime reaches the earth directly from the sun and also indirectly through scattering by the atmosphere, clouds, haze, etc. For the sake of brevity the former will be termed "sunlight" and the latter "skylight." It will be understood that skylight will mean all the light which reaches a given point from not only the blue sky but also from clouds and haze. If the sky were perfectly clear and if space were a perfect void the illumination on a horizontal surface due to direct sunlight would be proportional to the sine of the sun's altitude and the illumination due to skylight would be zero. Owing to the scattering of light by the atmosphere some skylight is always present. On a clear day the illumination on a horizontal surface due to the sun roughly approximates a sine curve and that due to skylight is approximately constant[1] for a large portion of the day.

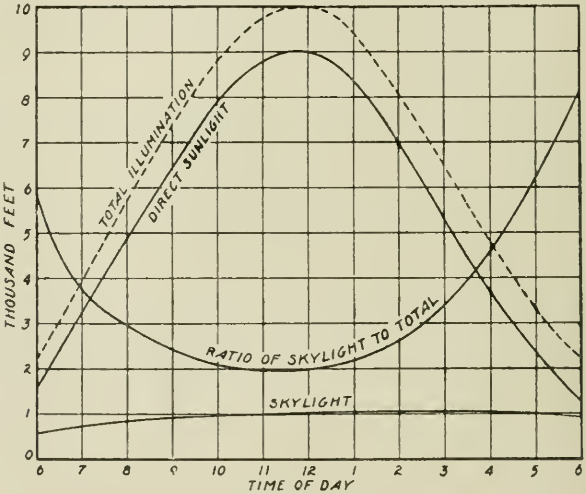

FIG. 1.—Relations of sunlight and skylight on a horizontal surface on a clear day

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[1] M. Luckiesh, *Light and Shade and Their Applications*, 1916, chap. vii.

In Fig. 1 are presented the results obtained at the earth's surface on an exceedingly clear, cloudless day in June when the maximum altitude of the sun was about seventy-four degrees. The measurements represent illumination on a horizontal surface. It is seen that the illumination due to direct sunlight throughout the portion of the day represented approximates a sine curve. The illumination due to skylight is nearly constant, although it increases slightly with the sun's altitude. The ratio of skylight illumination to total illumination at noon is 0.1, which represents about the minimum determined at the earth's surface. This ratio generally decreases as the sun's altitude increases. As haziness increases the values of total illumination decrease somewhat and those for skylight illumination increase considerably, with the result that the ratio of skylight to total illumination increases very much.

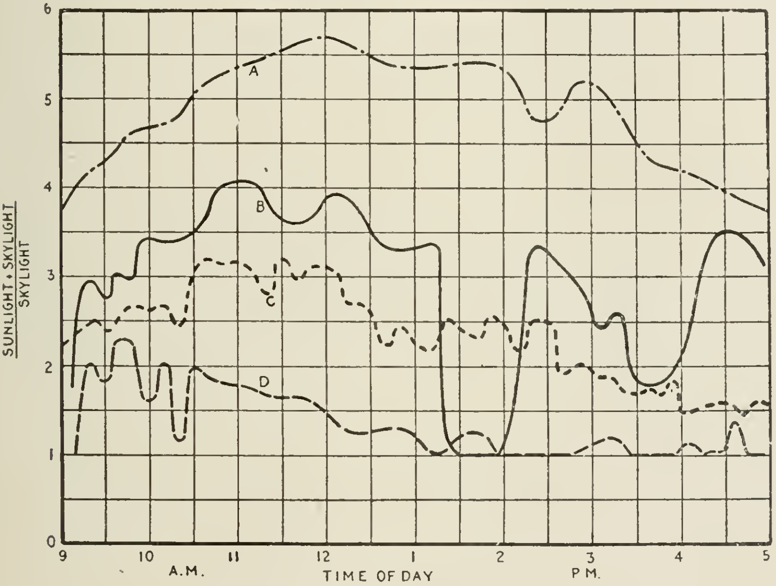

FIG. 2.—Relations of sunlight and skylight on four typical days

In Fig. 1 the data represent the results on an ideal, extremely clear and cloudless day. In Fig. 2 the ratios of skylight to total illumination[1] are shown for four common days. Of course, there are many variations owing to clouds, haze, etc. Many such records have been made, but it appears unnecessary to present examples in this paper; however, it is well to visualize the causes for the extreme variability in natural lighting and in the sky aspect.

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[1] *Ibid.*

On perfectly clear days in this country the maximum total illumination on a horizontal surface at the earth at noon varies from about 10,000 foot-candles in June to about 3500 foot-candles in January. According to H. H. Kimball[1] it is less than the direct solar illumination of a normal surface from September to February, inclusive, but exceeds the latter from May to August, inclusive, for a period of from four to eight hours in the middle of the day. On thinly overcast days when the sun is just invisible the total illumination is about one-half that on a clear day with the sun at the same altitude. This value decreases to one-third or one-fourth on thickly overcast days and occasionally reaches a very low value during storms. According to measurements made in various places in this country, on clear days in summer the illumination due to skylight on a horizontal surface varies from one-third to one-tenth of the total illumination on the same surface at noon, and in winter from one-half to one-sixth. A common value or approximate mean for a large part of the year is about one-fifth for the outskirts of Cleveland, Ohio.

H. N. Russell[2] has grouped some of Kimball's data[1] according to the amount of skylight illumination and by extrapolation to the zenith (air mass unity) arrives at a mean transmission factor of the atmosphere equal to 0.77 for 15 clear days. The mean maximum intensity of sunlight from the zenith sun is found by extrapolation to be 9600 foot-candles (103,000 meter-candles) for these same days. The skylight illumination at noon on these days averaged 1000 foot-candles approximately. Assuming the above value of atmospheric transmission factor for visible relation, Russell computes the illumination on a horizontal surface due to zenith sun would be 12,500 foot-candles (134,500 meter-candles) outside our atmosphere.

It is interesting to plot Kimball's value for 22 days, which were grouped by Russell in accordance with the illumination due to skylight alone, that is, according to the haziness of the weather. By plotting values of skylight illumination against those for sunlight

[1] *Monthly Weather Review*, 42, 648, 1914. [2] *Astrophysical Journal*, 43, 127, 1916.

illumination it is found by extrapolation that the value for zenith sun illumination on a horizontal surface becomes about 11,700 foot-candles (126,000 meter-candles) when the illumination due to the sky alone reaches zero. This appears to be a method worthy of consideration for determining the maximum possible illumination at the earth that can be expected from the zenith sun.

Notwithstanding the excellent work which has been done, the data pertaining to natural lighting are incomplete. However, it is not the purpose of this paper to discuss the foregoing aspects or to include data pertaining to them. They have been mentioned in order that the reader may appreciate the variables involved in some of the values presented in the sections which follow.

APPARENT REFLECTION FACTORS OF EARTH AREAS

To apply the term reflection factor (commonly called reflection coefficient) to the earth's surface which has "depth" is somewhat misleading; however, as viewed from an appreciable distance above the earth, this term is not misapplied from a practical standpoint. The term "relative brightness" will be used occasionally and, unless otherwise stated, will mean the brightness relative to a horizontal, perfectly reflecting and diffusing opaque white surface under the same illumination. Obviously the reflection factor of this theoretical white surface is equal to unity so the values of "relative brightness" of various earth areas, clouds, sky, etc., are in reality "apparent" reflection factors.

In Table I values of reflection factor are presented for various characteristic earth areas as obtained on various days during late summer. The individual values are means of many observations, and a sufficient number of these are presented to show the consistency of the results. All were made within 150 miles (240 km) of Langley Field at Hampton, Virginia. Further comment on these various types of areas will be found in following sections. Measurements, unless otherwise stated, were made vertically downward from the airplane and usually at altitudes from 1000 to 5000 feet (300 to 1500 meters). The brightness measurements were made with different types of instruments in terms of the brightness of a white diffusely reflecting (or transmitting) surface receiving the same illumination. These values were then reduced to values in terms of a perfect white. Obviously such measurements had to be made below the clouds when the latter were present. Many measurements were made at various altitudes up to 20,000 feet, although for earth measurements the effect of the low-lying haze was reduced to minimum by making the measurements vertically downward and usually at low altitudes. Most of the measurements were made during the middle of the day because instrumental errors were less at this time. However, no extensive differences were

TABLE I

PERCENTAGE OF RELATIVE BRIGHTNESSES OR APPARENT REFLECTION FACTORS OF VARIOUS TYPES OF EARTH AREAS DURING LATE SUMMER

(These were determined in late summer in eastern Virginia within 150 miles of Hampton. In all cases the surfaces were viewed from points directly above them varying from an altitude of 1000 feet upward. The individual values in the following table are means of numerous measurements. In some cases hundreds of measurements were made during a single flight.)

Date		Fields	Barren Land	Woods	Inland Water
Aug. 30, 11:00–12:00	Generally clear..............	7.4	10.4	4.9	5.7
Aug. 30, 2:00– 3:00	Sunny with large clouds scattered....................	9.3	10.3	4.9	9.0
Aug. 31, 11:00–12:00	Generally clear, some thin hazy clouds....................	6.1	10.1	4.4	6.7
Aug. 31, 1:00– 2:00	Fairly clear, some clouds.....	5.8	11.1	3.8	6.2
Sept. 3, 11:00–12:00	Sunny with large scattered clouds....................	7.9	15.9	3.9	6.3
Sept. 3, 1:00– 2:00	Generally clear..............	7.0	19.0	4.6	7.1
Sept. 3, 3:00– 4:30	125-mile trip across country, east and south...........	6.8	17.1	5.0	6.2
Sept. 4, 11:00–12:00	175-mile trip to Richmond, Petersburg, and return.....	6.5	13.8	4.5	7.4
Sept. 6, 11:00–12:00	Overcast sky................	6.7	13.6	4.4	9.1
Sept. 7, 1:00– 2:00	Clear day..................	6.3	......	......	7.6
Sept. 9, 11:00–12:00	Clear day..................	......	11.2	3.6	6.7
Sept. 9, 2:00– 3:00	Clear day..................	6.7	14.2	3.9	4.4
Sept. 10, 9:00	On trip to Washington and return....................	5.2	8.7	3.5	5.2
Mean..........	....	6.8	13.0	4.3	6.8

Dense clouds, 55 to 78 per cent, for upper sides

Deep ocean water 50 miles from shore, 3.5 per cent

Shadow of cloud on earth's surface is less than 1 per cent of the brightness of the theoretically white surface receiving sunlight and skylight.

obtained in the results at various times of the day or even on sunny and overcast days.

Owing to the object of the original work four different types of surfaces were chosen which inclusively represented the entire earth's surface with the exception of cities, towns, and villages, which cover a negligibly small portion. The data in Table I are for summer landscape, and the water areas were entirely inland waters such as Chesapeake Bay, Hampton Roads, and rivers such as the James, the York, the Rappahannock, and the Potomac. Barren land included plowed ground and waste land on which there was no vegetation. Fields included pastures and growing crops.

Most of the observations which are represented by the foregoing mean values on various days were included in the following ranges:

	Percentage
Fields	5–10
Barren land	10–20
Woods	3– 5
Water (inland)	5–10
Deep ocean water	3– 5

A number of flights were made from Langley Field during the middle of October in order to obtain similar measurements for the autumn landscape. The colors were those typical of autumn when the woods were variegated and the fields were dull yellow or brown. These observations were not as exhaustive as those recorded in Table I and on other pages; therefore not much stress is placed on the values. The average values obtained for the apparent reflection factors or the brightnesses relative to a perfectly white surface under the same illumination are as follows:

	Percentage
Fields	6.0
Barren land	11.3
Woods	4.3
Water (inland)	5.5

The value for water in the foregoing does not include many measurements of rivers, as is the case in Table I. In a later section water is discussed further. There is reason to believe that the apparent reflection factors of these earth areas are practically the same in autumn as in summer. Obviously there should be no difference in the values for water. The landscape during this autumn period was decidedly brownish and generally dull in character. Few green areas were present and the woods displayed a variety of tints. It is noteworthy that the landscape in general appeared more dull and dead from high altitudes than was expected. Although the number of observations during this autumn period was limited to hundreds instead of thousands as in the summer period, it will be noted that the results are not widely different.

It is interesting to note that Abbot[1] from energy measurements concludes that the albedo of a cloud surface is about 0.65, and after allowing for atmospheric absorption computes the albedo of the earth to be 0.60 if it were covered with clouds, 0.14 if it were cloudless, and 0.37 as matters actually are. These values cannot be directly compared with the values of apparent reflection factor given in this paper, however. Abbot's discussion of the reflection factors of the earth is interesting, although the data in this paper will probably modify some of his estimates. It is interesting to compare the values with certain qualifications in mind. For a discussion of albedo two references[2] are recommended.

LANDSCAPE

During the summer period in which the observations whose mean values are presented in Table I were made, the vegetation was predominantly green, although in a few areas there was a slight tinge of yellow common to middle and late summer. There were many plowed fields barren of vegetation and others which were not entirely covered by green foliage. In eastern Virginia there is a great deal of wooded country including wooded swamps, so it appeared, among other reasons, best to keep separate records for the different types of earth areas. Furthermore, there are various kinds of soil, so that the variations in the values in Table I are to be expected. However, the consistency of the observations on woods, water, and fields covered with crops is surprising.

The mean relative brightness or apparent reflection factor for a landscape will depend upon the proportions occupied by the various types of areas. Attempts were made to estimate the proportions of different types of areas present in the landscapes studied. This could be fairly accurately determined for any given locality by cross-country flights during which a record of time in passing over the various types of areas could be kept. If the airplane speed were approximately constant this would give a fairly satisfactory mean value for the relative brightness or the apparent reflection factor of the terrain. Another plan was to make observations continually and take the average value. This was done a number of times. For example, the mean value obtained during a 175-mile trip from Langley Field to Richmond to Petersburg and return was about 7 per cent. However, it should be noted that this portion of the country contained much wooded area although the course was generally maintained over the more cultivated regions.

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[1] *Annals of the Smithsonian Astrophysical Observatory*, **2**, 161.

[2] H. N. Russell, *Astrophysical Journal*, **43**, 103, 173, 1916; Louis Bell, *Ibid.*, **45**, 1, 1917.

Notwithstanding the writer's familiarity with brightness measurements, the generally low values obtained in this investigation were surprising. However, these may be accounted for by considering the earth's surface more in detail. On looking down such areas as fields, woods, etc., one sees a mixture of highlights and shadows. Much of the area receives only skylight, which is a small part of the total light reaching the earth on sunny days. Even on overcast days the shadows receive only a small portion of the total light reaching the earth's surface. Furthermore, the trapping of light is a powerful influence in reducing the reflection factor. For example, a grass area possesses "depth" similar to velvet. Black velvet has a very low reflection factor (about 0.004), but ordinary blotting paper colored with the same dye will possess a diffuse reflection factor of about 2 per cent. The depth of the velvet fiber provides light-traps and shadows, with the result that the reflection factor as a whole is much lower than that of a single fiber. In a similar manner grass plots, corn fields, plowed ground, woods, etc., provide light-traps and shadows so that the mean brightness or apparent reflection factor is materially reduced.

For example, let us assume that the diffuse reflection factor of a blade of grass is 0.16; that the sun contributes 0.8 of the total light; and that the normal aspect of the grass plot is one-half shadow and one-half highlight. If the brightness of the sky is equal to B, then the brightness of a perfectly white horizontal reflecting surface will equal B due to skylight alone and equal 5 B due to sunlight and skylight. The brightness of the blades of grass receiving both skylight and sunlight will be $5\ B \times 0.16 = 0.8\ B$. The brightness of the grass in shadow, assuming the shadows to receive full skylight (they actually receive much less), will equal $0.16\ B$. The mean apparent brightness will be equal to $\frac{1}{2}(0.8 + 0.16)\ B = 0.48\ B$. The mean apparent reflection factor would then equal $\frac{0.48\ B}{5\ B}$, or 0.1 approximately. Thus it is seen, with these assumptions, the apparent reflection factor of the grass plot is about 0.1 notwithstanding the fact that the actual reflection factor of an individual blade was assumed to be 0.16.

It will also be noted that the trapping of light by virtue of the "depth" of such a surface was not considered in the foregoing analysis, and also that the blades of grass are oriented in all directions. Both of these influences would tend materially to reduce the value of reflection factor. Woods are strikingly dark owing to these reasons and on the whole these considerations prepare us to expect a surprisingly low mean apparent reflection factor of the earth. Bare earth which has been packed by rain and baked by the sun is conspicuously brighter than the same soil freshly plowed even after the latter has dried. It is commonly noted that wet dirt is darker than the same dirt when dry. For example, a piece of blotting paper which, when dry, reflected 0.74 of the incident light reflected only 0.54 of the incident light when wet.

The brightnesses of all earth areas do not vary much with the angle at which they are viewed. This is not the case with the surface of water, as will be noted later. Owing to haze a photometer will generally yield greater values for very oblique measurements than for vertical measurements. In making the thousands of observations upon which Table I is based it is interesting to note that an overwhelming majority of the values of apparent reflection factor as obtained indiscriminately over the landscape are included within a range from 3.5 to 10 per cent.

Many attempts were made to obtain the average relative brightness or reflection factor of landscapes as a whole by measuring the ratio of the illumination of the lower side of a horizontal surface to that of the upper side. This measurement was anticipated in designing the photometer, but inasmuch as it was necessary with this small instrument to lean out of the airplane, the latter intercepted some of the light reflected by the earth. Precautions were taken to reduce errors to a minimum and the values obtained were considered sufficiently accurate for their intended purpose. They were included well within the range from 5 to 10 per cent. The mean value of the reflection factor of the landscapes as a whole obtained in this manner was slightly greater than the mean obtained by weighing the values in Table I, but it will be noted that the latter are for perpendicular viewing and that the brightness of the landscape generally increases for oblique viewing owing to haze, to distant clouds, and to increased specular reflection from water. It is a simple matter to obtain the ratio of upward to downward light from an airplane by placing a ground opal glass at each end of a long vertical tube and determining the brightness of one in terms of the other by means of an instrument between them. This was done, but inasmuch as the mean reflection factor of the earth was of little importance in solving the problems primarily in mind, few readings were made. The values of the four types of areas were much more adaptable to the solution of the primary problem.

CLOUDS

Some measurements were made in the course of these investigations in order to establish the magnitude of the relative brightness or apparent reflection factor of clouds. These values vary considerably with the character of the cloud, the lighting conditions, etc. The apparent reflection factor of cumulus clouds receiving full skylight and sunlight as measured vertically above them on various days varied with the density of the clouds. The mean values of the upper surface of clouds, determined on different days, were found to be as follows:

Very dense clouds of extensive area and great depth	0.78
Dense clouds, quite opaque	.62
Dense clouds, quite opaque	.59
Dense clouds, quite opaque	.55
Dense clouds, nearly opaque	.44
Thin clouds	.40
Thin clouds	.36

One day when the sky was very heavily overcast with the exception of a small rift which admitted the airplane with safety to the upper region, a great many measurements of the brightness of the upper surface of the clouds were made. These observations were made near the sunny side of the hole in the extended massive layer of clouds and the values of relative brightness obtained varied from 0.7 to 0.8 with a mean of 0.78. It was somewhat surprising to obtain these high values, but they are perhaps accounted for by mass effect. Inasmuch as the upper side of a cumulus cloud was found to be six times as bright as the lower side we must conclude that there is a "mass" effect unless we choose to conclude that there is a tremendous absorption. In other words, great masses of clouds can be relatively much brighter than a purely diffusing, non-absorbing medium in a thin layer which theoretically would possess a transmission factor equal to 0.5 and a reflection factor of the same value. Of course white paint owes its high reflection factor chiefly to body or "mass" effect, but the depth to which the light penetrates is very small. The sunlit surfaces of dense clouds as viewed from below were found to be commonly 5 to 10 times the brightness of the adjacent patches of clear blue sky.

HAZE

The term "haze" is rather indefinite, for it may be taken to mean any atmospheric condition which increases the brightness of the sky (with the exception of definite clouds) or which intercepts light in any manner. There appear to be two general types of haze. One is the smoke and dust haze; the other is similar in appearance to very thin diffused cloud and is ofttimes perceptible at high altitudes beyond which the "dust" haze is not ordinarily found. The writer is not familiar with any definite distinction in nomenclature between these different types of haze but will adopt the term "earth" haze for the absorbing smoke or dust haze in order to distinguish it from the vapor haze, which is less absorbing and which apparently extends to much higher altitudes than the former. This haze will be termed "cloud" haze for the present, but with the understanding that no definite cloud shapes are apparent. Probably the classification could be carried farther to the higher regions of the atmosphere which account for the deep blue sky beyond the heights attained by cirrus clouds, by supplying a distinct name for this scattering of light at extremely high altitudes. During the progress of these investigations the earth haze extended ordinarily to an altitude of about a mile, although on a few occasions it extended to altitudes as high as 11,000 feet. This haze is somewhat absorbing, as is readily seen on viewing the earth obliquely, and is also fairly luminous. On some days at altitudes of 10,000 to 15,000 feet the sky appeared quite bright and unsaturated in color. As already stated, this upper haze often appears to be of the nature of a very thin, uniform, diffused cloud.

No very satisfactory measurements of the luminosity of this low-lying earth haze were obtained from above. Such an experiment requires a large, uniform, horizontal area at the earth's surface. However, a deep channel in the Chesapeake Bay was selected and measurements of its brightness were made as the airplane ascended over this area from 50 to 10,000 feet. Unfortunately the haze was unusually thin and little effect was detected. The greatest increase in brightness that could possibly be accepted from the brightness measurements was 10 per cent. Some measurements of the total illumination upon the upper surface of a ground opal glass at various altitudes were made without obtaining differences greater than the possible experimental errors in this case. This day was strikingly clear and cloudless.

The earth haze which absorbs as well as scatters light decreases the brightness of the earth, as viewed from high altitudes. Scattered light renders the haze luminous as viewed from above, which brightness tends to offset the decrease in the brightness of the earth. Viewing the matter in this manner it appears possible that the brightness of the earth plus the haze as viewed vertically does not vary appreciably for altitudes as high as 10,000 feet when there are no clouds or conspicuous vapor haze.

On another cloudless day similar measurements were made of the brightness of water at altitudes of 20,000 feet. The experiment was not as definitely planned as the preceding one, so that the measurements were less reliable; however, it appears possible that the brightness of the water areas increased as much as 25 to 50 per cent.

The presence and the effect of the earth haze, which ordinarily possesses a fairly definite upper surface, is easily apparent to the eye. The "false" horizon due to this dust haze is clearly visible, often with a few cumulus clouds partially submerged in it like icebergs. Many of the effects of air currents, the formations of clouds, the operation of winds and storms, can be viewed and studied so easily from the airplane capable of attaining high altitudes that it appears certain the airplane will be utilized by the scientist in studying these and many other problems.

SKY

During this and previous work many measurements of the sky brightness were made. The brightness of the sky generally varies from 0.5 to 2 lamberts. One lambert is a fair average as determined at the earth's surface. Very deep blue sky and dark overcast skies are approximately of the same brightness. The brighter skies are those that are very hazy or fairly heavily overcast. The brightest sky is that which is thinly overcast. On cloudless days the sky contributes from one-half to one-tenth of the total light, reaching a horizontal surface at noon. The larger value is usually encountered in winter, when the altitude of the sun is less than in summer. Measurements of this value should be for the same altitude of the sun in order to be strictly comparable. It is obvious that as the haze increases, the ratio of skylight to total light on the horizontal surfaces increases.

At low altitudes the sky is rarely so clear that only one-tenth of the total light which reaches the earth comes from the sky. Many measurements of this ratio were made at various altitudes with the obvious results. It is interesting to note in one case that at an altitude of 20,000 feet only 5 per cent of the total downward light was skylight notwithstanding the presence of a few thin cirrus clouds at altitudes of about forty-five degrees. The sun was at an altitude of perhaps fifty degrees, so that when corrected to zenith sun the value would be nearer 4 per cent. On this day at the earth's surface the sky contributed about 20 per cent of the total light. In other words, the sky was not more than one-fourth as bright when viewed from the high altitude as when viewed from the earth's surface on this particular day. It was interesting to note how strikingly dark the sky appeared and how extremely harsh the lighting effect. That is, the shadows were noticeably very dark, especially if one is used to such observations.

On a very hazy but cloudless day measurements were made of the brightness of the zenith sky at altitudes from 0 to 15,000 feet. At the high altitudes the sky was extremely free from "vapor" haze and was very dark. In Fig. 3 are shown the results obtained in the morning and afternoon respectively. The region where the curve flattens out indicates the altitude where the earth haze ends. These regions are seen to be at about 10,000 feet and 7,000 feet, respectively, for morning and afternoon on this particular day, which was an exceptionally hazy one. Ordinarily there would not be such a great difference between the sky brightness as viewed from the earth's surface and from high altitudes, although the difference is usually considerable.

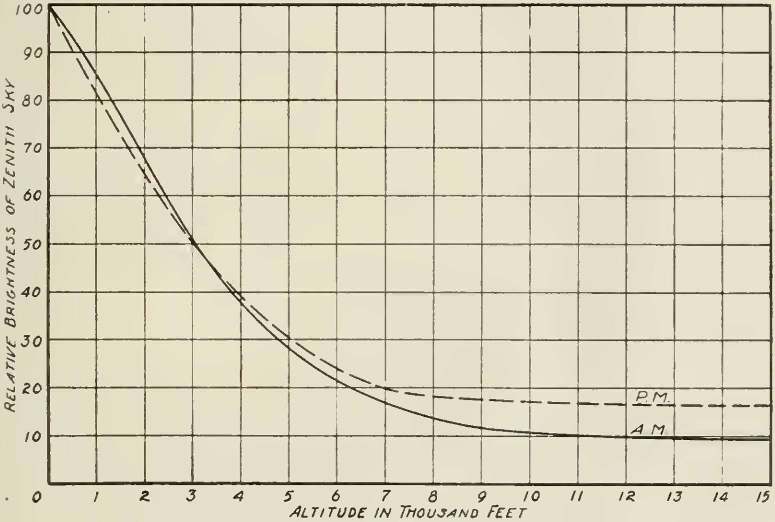

FIG. 3.—Relation of sky-brightness and altitude on a cloudless hazy day

Although the brightness of clear sky is quite variable, it is of interest to know the brightnesses of various areas in terms of the sky brightness B, but these values should not be confused with those values of "relative brightness" which are in reality "apparent reflection factors." The mean values of many observations obtained on various days for various areas are presented in Table II,

TABLE II

Relative Brightnesses of Various Types of Areas in Terms of Clear Blue Sky as Determined Usually at Low Altitudes
(Zenith sky = *B*)

Woods	Water	Fields	Plowed or Barren Land
0.44 *B*	0.53 *B*	1.02 *B*	....
.36	....	.67	....
.45	.58	.86	....
.50	.73	....	....
.40	.80	.77	1.08 *B*
.53	.91	.93	1.19
	.72	.62	....
Mean 0.45 *B*	0.71 *B*	0.81 *B*	1.14 *B*

Landscape covered with hazy clouds, 3.5 *B*
Upper sunlit sides of cumulus clouds, 5–10 *B*
Under side of massive cumulus clouds, 1.4 *B*

the sky brightness *B* being unity in arbitrary units. The measurements were made simultaneously and usually at low altitudes. That is, the brightness of an earth area relative to the zenith sky was made by comparing the two brightnesses simultaneously.

The values for clouds and for water are discussed in other sections under their respective headings. It is interesting to note that the data in Table II were obtained independently from those in Table I with an instrument of a different type, and in terms of the blue sky which varies from hour to hour and from day to day. However, when reduced to the same value for water they compare thus:

	Fields	Barren Lands	Woods	Water
Table I......	6.8	13.0	4.3	6.8
Table II......	7.7	10.9	4.3	6.8

This is an excellent check when all aspects are considered.

A number of determinations of the portion of the total light contributed by the sky on clear days were made at the earth's surface during this period. These averaged about 0.16. In other words, the sky contributed about one-sixth and the sun about five-sixths of the total light on the earth's surface for the clear days of this period and in this locality. However, the values varied considerably. Let us assume the mean sky brightness equal to B and the brightness of a perfectly reflecting and diffusing horizontal surface due to sunlight equal to $5\,B$. By weighing the values of earth brightness in terms of the mean sky brightness (which is greater than the brightness of the zenith sky) let us assume the mean brightness of the earth equal to $0.4\,B$. This value is lower than those in Table II because it is not uncommon for the sky near the horizon to be several times brighter than the zenith sky brightness. The values in Table II would be considerably smaller if they were in terms of average sky brightness. Based on these facts and assumptions,

$$R\,B\,(5+1)=0.4\,B$$

$$R=\frac{0.4}{6}=0.066$$

where R is the mean apparent reflection factor of the earth. This is approximately the weighted mean arrived at by measurement. Inasmuch as the assumptions cannot be far from correct this serves as an approximate check for the data in Table I.

Some measurements at altitudes of 10,000 to 20,000 feet on a clear day yielded the following brightnesses in terms of the zenith sky brightness B viewed from these altitudes.

Fields, etc.	$1.1\,B$
Water	$1.4\,B$
Landscape covered with thin clouds	$3.5\,B$
Dense sunlit clouds	$11.0\,B$

A mean of a number of measurements at 18,000 feet indicated that the lower horizontal surface of a perfectly reflecting and diffusing opaque object (reflection factor = 1) due to light reflected upward by the earth was 1.7 times the brightness of the zenith sky as observed at that altitude. In accordance with the preceding computation, taking account of the fact already stated that at this altitude the sky was of uniform brightness and contributed 0.05 of the total light,

$$R\,B\,(19+1)=1.7\,B.$$

Therefore R, the mean apparent reflection factor of the earth, would equal 0.085. Considering that the earth haze is responsible for some of the light which reaches the lower horizontal surface of the theoretical perfectly reflecting and diffusing object, the value 0.085 appears to check well with preceding values.

Various observations of the brightness of the lower horizontal surface of a perfectly reflecting and diffusing object were determined in terms of the brightness B of the zenith sky. A mean of observations of this value made at altitudes from 7000 to 13,000 feet with a few clouds at 5000 feet was found to be 0.9 B. A mean of observations of this value made on another day at an altitude of 7000 feet with some clouds at 5000 feet (2000 feet below) was found to be 1.6 B. A mean of observations of this value at altitudes from 10,000 to 18,000 feet on a fairly clear day was found to be 1.7 B. In these cases there were no clouds directly below. The differences in the values presented above are due doubtless to different brightnesses of the sky, to clouds, and to various other lighting conditions. From the mean of the three mean values and assuming an average apparent reflection factor of the earth equal to 0.085 (including the contribution of the haze) we compute that the upper horizontal surface of a perfectly white, opaque diffusing object would appear about 12 times brighter than the lower surface of such an object above the clouds and haze on a day representing a mean of these three days.

From the data in Table I, assuming a mean apparent reflection factor of the earth equal to 0.07, the upper surface would be about 14 times brighter than the lower surface when the opaque object is a few thousand feet above the earth. Under these conditions the brightness of the lower side of the opaque object would equal the brightness of the sky (assuming it to be uniformly bright) on a clear day if the sky contributed one-fourth of the total light. This latter condition is perhaps never obtained at very low altitudes. At low altitudes up to 5000 feet the lower horizontal surface of a perfectly reflecting and diffusing object is usually only a fraction of the brightness of the zenith sky. It should be noted that this fraction increases with altitude due chiefly to the decrease in the sky brightness.

It is interesting to note that Kimball and Thiessen[1] obtain the following ratios of illumination on a horizontal surface for clear and smoky atmosphere:

$$\frac{\text{Total illumination on smoky day}}{\text{Total illumination on clear day}} = 0.64$$

$$\frac{\text{Direct sunlight on smoky day}}{\text{Direct sunlight on clear day}} = 0.48$$

$$\frac{\text{Skylight on smoky day}}{\text{Skylight on clear day}} = 1.64$$

This shows how the sky brightness is increased by the presence of reflecting particles.

WATER

As seen in Table I the mean relative brightness or apparent reflection factor of inland waters as obtained by measurements perpendicular to its surface was 0.068. Practically all the values obtained on different days were within the range from 0.05 to 0.1 and most of them were close to the mean value. If the surface of water is perfectly smooth it will reflect at its surface about 0.02 of perpendicularly incident light; in other words, its brightness will be 0.02 of the brightness of the sky or clouds vertically above. This reflection factor varies only slightly with increasing angle of incidence until about fifty degrees is reached. It begins to increase more rapidly from this angle until obviously at an angle of incidence of ninety degrees it reflects all the light.

Owing to the fact that the brightness of an area of water, viewed from above, is due partially to specular reflection, its brightness will depend partially upon the brightness of the objects whose images are reflected. These objects are blue sky, haze, clouds, and the sun; however, the latter will be neglected in this discussion because it is a very special case. It will be seen in Table I that the mean value of the relative brightness of water determined perpendicularly to its surface is fairly constant on different days. In fact this value varies surprisingly little for deep water which would be considered fairly clear. An increased brightness is noted for shallow water and for water which holds bubbles or fine particles of solid matter in suspension. For example, measurements on the James River about 30 miles from its mouth gave values of 0.10 for shallow water near the shore and 0.075 in midstream. The latter value was very constant for its deep channel. Near Richmond, where the river was very yellow, due to suspended particles of earth, the relative brightness rose to 0.20, which is very unusual.

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[1] *Monthly Weather Review*, 45, 205, 1917.

The foregoing values were obtained for inland waters within a radius of 150 miles of Hampton, Va., and included Chesapeake Bay, Hampton Roads, and various rivers. In order to obtain values of apparent reflection factor or relative brightness of deep ocean waters far from shore, a trip over the Atlantic was arranged. A series of measurements was made during a six-hour flight over Hampton Roads, Chesapeake Bay, through the Virginia Capes and 50 miles to sea, at altitudes averaging about 1000 feet. As usual the measurements were made approximately at normal incidence with the surface of the water. The mean values of apparent reflection factor follow:

	Percentage
Shallow water in Hampton Roads..............	7.4
Deep channel in Chesapeake Bay..............	4.9
Atlantic Ocean 10 to 50 miles from land........	3.5

These represent means of many observations. The lowest value obtained for deep water far from land was about 2 per cent and the highest 5.6 per cent. Most of the values were confined to a range from 3 to 5 per cent. Of course the brightness increased for oblique angles of view and varied with the position of the sun.

The difference in the hues of shallow turbid water and deep clear water is quite apparent. When the water is extremely shallow the hue partakes partially of the color of the bottom. In moderately deep inland waters which are in motion and consequently more or less turbid the water appeared a dirty greenish hue. The deep clear ocean water is by comparison a much purer blue-green hue.

In Table III are presented some values of the brightness of inland waters in terms of the brightness of the zenith sky. These measurements were made vertically and simultaneously.

Similar measurements were made on one day for deep ocean water which resulted in values from 0.16 to 1.0 times the brightness of the zenith sky. This extreme range was due largely to the changing brightness of the sky on this day. Early in the day the sky was a clear deep blue, but gradually became more hazy and finally was covered with thin cirrus clouds.

TABLE III

Relative Brightnesses of Inland Waters on Various Days Viewed Perpendicularly, in Terms of the Brightness of the Zenith Sky *B* as Determined from Altitudes up to 20,000 feet

4:30 P.M.	Sunny day, large clouds	0.58 *B*
10:30 A.M.	Clear day	.63
10:00 A.M.	Clear day, some clouds	.80
9:30 A.M.	Clear day, some clouds	.87
2:00 P.M.	Clear day	.99
1:30 P.M.	Clear day, deep blue sky	.79
1:30 P.M.	Clear day, some clouds, high altitude	1.40
Mean		0.87 *B*

From the position of the observer directly overhead on clear days the surface of the water reflected only images of portions of the blue sky regardless of the character of the waves. Inasmuch as surface reflection could account for only a small portion of the mean value in Table III, it is obvious that most of the brightness of water is due to light diffused within it. This is well verified by the presence of cloud shadows commonly seen upon the water. These are especially conspicuous in aërial photographs of water areas when cumulus clouds are present to cast deep shadows and are strikingly evident when the water is viewed at an oblique angle from high altitudes.

In this connection it is interesting to note that some small lakes and ponds are strikingly dark amid their surroundings, sometimes appearing almost black. In fact these are often the darkest areas in a landscape. If a pool is quiescent, clear, and deep, with a porous bottom of low reflection factor such as is often the case, practically the entire brightness is that due to specular reflection from the surface. A smooth surface of water reflects only about 0.02 of normally incident light, and as viewed vertically its brightness due to surface reflection is only 0.02 that of the zenith sky. On a clear day, if the sky contributes only one-fifth of the total light, the brightness of such a quiet, clear, deep pool (assuming a sky of uniform brightness) would be only 0.004 or 0.4 per cent of a perfectly white surface. In other words, under these conditions it would appear of about the same brightness as a black velvet fabric. As seen in Table I, inland water is about as bright as grassy plots of ground, which shows that most of the brightness of water is due to its turbidity, that is, to the light diffused within it. On the foregoing assumptions about 90 per cent of the brightness of water viewed perpendicularly is due to light diffused within it.

Some values of the brightness of areas of water viewed at forty-five degrees in terms of the brightness of the zenith sky varied from 1.5 to 3.0. The mean of the few values obtained is about 2.2. These values varied considerably at any given time, depending upon the direction of observation with respect to the sun. This is one of the reasons which led to making practically all the observations perpendicularly to the earth's surface.

In such measurements as have been presented in this paper, the variability in natural lighting, in sky brightness, etc., causes considerable uncertainty in some cases. The values of reflection factor of earth areas, water, and clouds represent closely the true values for the summer landscapes. The values involving the brightness of the sky indicate no more than limits or approximate magnitudes. Owing to the variation in sky brightness an exhaustive research extending over a long period would be necessary in order to establish mean values.

Nela Research Laboratory
National Lamp Works of General Electric Co.
Nela Park, Cleveland, Ohio
November 1918

Reviews

The Binary Stars. By Robert Grant Aitken. New York: Douglas C. McMurtrie, 1918. 8vo. Pp. xiv+316, plates 5, figs. 12. $3.15.

This volume is an important addition to our literature on stellar astronomy. It is especially welcome because written by a man who is in a better situation than anyone else for treating the subject. His personal contributions to the field covered by the book are very important. His sphere of activity has been for years one where the newer branches of double-star astronomy, as developed by the use of the spectroscope, have been cultivated successfully and intensely. Through these circumstances the different angles of the subject have been given an adequate attention and the whole field is covered in a well-balanced way.

Some forty years ago the *Handbook of Double Stars*, by Crossley, Gledhill, and Wilson (London, 1879), could be considered as embodying our essential knowledge in the field of binary stars at that time. Nothing can make us realize better the progress made between the publication of this book and the present time than to compare the *Handbook* with Mr. Aitken's volume. How the field has been enlarged! What unexpected developments have come to side-lines of the subject in the meantime!

The first two chapters of the book outline the historical evolution of the study of double stars. Beginning with the first isolated discoveries, it is followed by the exploring work of the Herschels and Struves and the standardizing of the visual methods of observing. Next we come to the startling discoveries by Burnham in the seventies, the extension of the field of the binaries through stars revealed by the spectroscope and those that have become known as eclipsing variables until, with the systematic survey of the northern sky at the Lick Observatory, we arrive at the present epoch.

The author considers next the technical side of the observations of visual binaries with the micrometer. The instrument used by him is described in detail and also the methods of measuring that he has practiced. He strongly advocates the advantages of having only two parallel wires, a fixed and a movable one, in the field. In the reviewer's opinion Mr. Aitken is going too far when he adds, on page 41: " . . . . and even for comets, asteroids, satellites, or other forms of micrometric work I regard it as superior to the more complicated forms. . . . . " In the case of comets or asteroids it is generally not possible to connect those moving objects with a catalogue star by using direct differential measures. Generally they have to be referred to some fainter star whose position is found by connecting it to a known star by transits. Here the observations with only two wires are decidedly too slow as compared with what is done when the micrometer has three or five transverse wires so that both co-ordinates can be obtained simultaneously with about the same accuracy; this disadvantage cannot be overlooked when we bear in mind what the author says on page 55: "An hour in the dome on a good night is more valuable than half a dozen hours at the desk in daylight." Also in the determination of the parallel a long transverse wire gives quicker and more accurate results than the short parallel wires. In turning off the illumination of the transverse wires, we can use the micrometer for double-star work as if there were only two parallel wires.

As to the determination of the value of the revolution of the micrometer screw (page 43) we would make this comment: One of the methods advised consists in taking transits of circumpolar stars over the movable wires in successive positions in the field. "Theoretically a correction for refraction should also be introduced, but if all of the measures are made near the meridian this correction will rarely be appreciable." That this correction is quite appreciable even *on the meridian* can be seen from the following table, corresponding to a latitude of 42°.

Declination	Correction in Right Ascension
+20°	1 part in 4200
40	1 part in 3600
60	1 part in 2300
80	1 part in 1900

The corrections are fully of the same order as the corrections in declination as we find them on page 47 in Mr. Barnard's measures of Electra and Celaeno taken about two hours past the meridian: 0.020 in 65.57 Rev. or 1 in 3300.

This same chapter condenses a great amount of useful information about the work with the micrometer. Every new student in this field should carefully read the sound remarks about the choice of an observing program, in which Mr. Aitken shows himself a worthy disciple of Mr. Burnham, to whom he dedicates this book.

In chapter iv we find the problem of computing double-star orbits clearly set forth. No original views are presented but the more generally used methods are made very plain. Graphical methods are considered more especially in detail and a numerical example of an application of the Glasenapp-Kowalsky method is given as an illustration of the determination of a preliminary orbit, that of the rapid binary A 88 with a period of but 12 years. When the observing material justifies it, a further approximation is obtained by differential corrections. No mention is made here of Doberck, who does away with the complicated analytical expression for the differential coefficients but finds these by changing arbitrarily the successive elliptic elements and computing the representation of the normal observations. He computed tables for this many years ago giving the values of the true anomalies and radii vectores in function of the mean anomalies for different values of *e;* these tables, which are much handier than Åstrand's, giving eccentric anomalies to which the author refers (p. 74), have, however, been superseded recently by the so-called Allegheny tables whose importance cannot be overestimated by the double-star computers. Since these are at hand no computer should go any longer to the trouble of calculating the analytical formulae for the differential coefficients.

Further paragraphs are devoted to the special cases in the orbits: the case of an orbit inclined at 90° so that the apparent oscillating motion is rectilinear, with an original contribution of F. R. Moulton (p. 96) on the subject; the case of an apsidal line coinciding with the nodal line, as treated by H. N. Russell (p. 99); the systems where only one component is visible, and, finally, the case of rectilinear motion.

The next chapter, written by J. H. Moore of the Lick Observatory, gives with regard to the spectroscopic binaries what the first part of the book has brought in regard to the visual ones. To find these two fields brought so closely together and treated as far as possible along parallel lines, will be gratifying to most readers. After explaining the principle of the Doppler-Fizeau method, Mr. Moore discusses the instrumental devices that are necessary for bringing the spectrograms to the high standard that they have reached in a relatively small number of years, and also the methods used in measuring and reducing the spectra. A plate giving some typical spectra is found on p. 114, but some readers will miss an indication of wave-length of the principal lines.

Chapter vi corresponds for the spectroscopic binaries to chapter iv for the visual ones. Mr. Aitken gives here the methods generally used in computing the orbits and the many new ones that have been proposed in recent years. The principal method is again illustrated by a numerical application to κ Velorum.

The stars where the binary character is found from periodic variations of the apparent brightness are dealt with in chapter vii. Mr. Aitken follows closely the theoretical investigation of the problem as it was worked out by H. N. Russell and applied with great success by H. Shapley to the large number of stars already found in this class. The principal numerical tables used in Russell's work are reprinted here so that it is not necessary to refer to the original memoir for using the method. Here also we find a numerical illustration, that of the well-known Algol star, W Delphini. No description is given of the instruments or methods used in observing these eclipsing binaries. This subject would cover the whole field of astronomical photometry for which there is already an abundant literature.

With chapter viii we come to the second part of the book and the discussion of the facts that have been collected in the various ways described in the first part. The valuable list of orbits for spectroscopic binaries (137) as well as visual binaries (87) tabulated at the end of the volume forms the basis of this investigation. The close relations between the length of the periods, the size of the systems, and the degree of ellipticity of the orbits are clearly brought out. The preference of the binaries for certain spectral classes is another striking feature: the visual ones clustering in class G, the spectroscopic just as decidedly in classes B and A, and also the increasing masses of the systems as we come to earlier types.

Next the author considers individually a selection of stars that are interesting from different points of view. The history of such stars as α Centauri, Sirius, Polaris, Algol, etc., is always full of interest and the author has made this chapter ix one of the most attractive for reading.

In chapter x, as a result of the systematic survey of the northern sky, recently completed by the author, he discusses in a statistical way the number and distribution of the visual doubles. The main conclusion, that at least 1 in every 18 of the stars north of the equator that are at least as bright as $9\overset{\mathrm{M}}{.}0$ in the BD is a close double in the 36-inch telescope, has already been announced previously by the author. It is supplemented by the fact that close visual double stars are relatively more abundant in the Milky Way. Many other facts in connection with the spectral types are also discussed here.

The final chapter is devoted to the cosmogonic theories that have been proposed for explaining the formation of so many binary stars: the capture theory, the fission theory, and the theory of independent nuclei. Each of these hypotheses explains a certain number of the facts that have been established by the study of double stars. But none of them is entirely satisfactory when all the facts are considered. It is very difficult to decide yet in favor of one of these. In the opinion of the author, the fission theory would perhaps lead to the most natural explanation of the observed facts.

To co-ordinate in one comparatively small volume such a large variety of lines of research was not an easy task. Mr. Aitken must be congratulated for doing it so successfully and for having presented the subject in such an attractive way.

G. VAN BIESBROECK

YERKES OBSERVATORY

EDWARD CHARLES PICKERING

With the deepest regret we record the death on February 3, 1919, of EDWARD CHARLES PICKERING, in the seventy-third year of his life and the forty-second year of his imperishable service to science as director of the Harvard Observatory.

He had been associated with this Journal as Collaborator since its foundation.

A biographical sketch will appear in a later number.

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A PHOTO-ELECTRIC DETERMINATION OF THE BRIGHTNESS OF THE CORONA AT THE ECLIPSE OF JUNE 8, 1918

By JACOB KUNZ and JOEL STEBBINS

A general account of the eclipse expedition to Rock Springs from the University of Illinois has been given in *Popular Astronomy*, **26**, 665, 1918. It was our purpose to secure accurate measures of the light of the corona as it affects a photo-electric cell, which if successful would form the basis of further studies of the corona at subsequent eclipses, an excellent opportunity for which is probably coming in 1923. A glance at the results of photometric work at past eclipses shows a curious discrepancy between the work of different investigators, and the reported values of the coronal light indicated either a great change between one eclipse and the next, or that the methods and instruments used were so different that it is doubtful whether the observers were all measuring the same thing. It was also true that in most cases the photometry of the eclipse was only an incidental feature of the program, and no great effort was made to get the best possible results. For these reasons it seemed to us that it would be worth while to devote our entire energies to measuring the total light of the corona, and the use of the photo-electric cell promised distinct advantages over visual or photographic methods.

Tests made some months in advance indicated that several photo-electric cells which we had on hand would give satisfactory measures of lights as faint as the crescent moon, an illumination at the cell equal to only a few hundredths of a candle-meter. These measures were possible with a d'Arsonval galvanometer of no great sensitivity, and it seemed probable that the corona would be bright enough to be measured with such equipment as we could set up at a temporary station, far from the conveniences of a laboratory.

Our procedure was to expose a photo-electric cell directly to the corona at the time of totality, and compare the galvanometer-deflection then obtained with the deflection caused by standard lights at known distances before and after the eclipse. As the cell was constant in its action and would give the same current for the same light for several days in succession, there was no need for manipulating the lamps during the precious seconds of totality. It was also proposed to compare the corona with the moon, which would serve as a permanent standard of light, and also with a definite area of the sky both during the eclipse and in full sunshine. This last comparison would give some estimate of the possibility of using the photo-electric cell in attempts to detect the corona without an eclipse.

Each of the photo-electric cells which we ordinarily employ has a glass bulb of about 25 mm diameter, and as the sensitive surface covers nearly a hemisphere we may take a circular area of 25 mm diameter as the portion sensitive to light. The two cells actually used, however, had a diameter of 40 mm and were mounted in exactly the same way in two similar tubes, which made a double-barreled arrangement. Each cell was placed in a box with a 25-mm circular opening, which was central at the end of a tube 100 mm in diameter and about 122 cm long. At the upper end of this tube there was a diaphragm 89 mm in aperture, which was at an effective distance of 127 cm from the sensitive cell. From these dimensions it follows that any point on the cell was illuminated by a circle of sky with radius $= \text{arc tan}\,\frac{4\cdot 45}{127} = 2^{\circ}.0$. It also may be derived that the clear field of the apparatus was 3° in diameter, which was enough to include all of the corona and to allow for the diurnal motion during the minute and a half of totality. It was therefore unnecessary to provide any driving arrangement, and the tubes were simply pointed at the sun's position at the middle of the eclipse. A series of diaphragms were inserted at intervals of 15 cm in each tube, carefully arranged, so that no part of the cell would receive light from the side of the tube.

The twin tubes were fastened to an equatorial mounting, two stops provided in declination, which allowed just 8° motion. During totality the tubes were turned back and forth from the sun to the sky 8° north, which was ample to take the corona safely out of the field.

Each cell-box was readily detached from its tube and could be placed on a photometer bench for measures of the standard lamps. No lenses or windows of any kind were interposed between the cell and the light-source to be measured, though of course there was the glass wall of the cell, which cannot be eliminated.

The photometer bench at the station was a rack fastened to the side of the building, and the distance from light to cell could be varied from 1 to 3 meters. Between the light and the cell was a set of telescoping tubes with proper diaphragms, which kept out all stray light.

For source of potential for the photo-electric cells we used some seventy-five small flash-light batteries of three cells each, giving a total of about 300 volts. These commercial batteries are quite good enough for many testing purposes, and we prefer them to storage cells, which require constant attention. For the needs at the temporary station they served the purpose admirably.

Altogether we had four galvanometers available, three of Leeds & Northrup and one by Weston. The two definitely used were a Leeds & Northrup Type R instrument with a constant of 6×10^{-10} amperes for 1 mm deflection at 1-meter scale-distance, and the other with a similar constant of 2×10^{-10} amperes. As used with damping shunts, the times to secure full deflection were 6 and 10 seconds respectively. Each galvanometer has a plane mirror and was used with a view telescope and scale at 2 meters. The circuit for each cell consisted simply of the battery, photo-electric cell, galvanometer, and a guard resistance of 100,000 ohms, all connected in series. Exposure of the cell to light gives the current to be measured, and the high resistance in circuit obviates the danger of spoiling a cell by too high a potential. The critical or glowing voltagé of a cell is some ten volts higher in the dark than in a strong light.

Various investigators have had difficulty with a non-linear relation between light and current for the photo-electric cell. One of us has already shown[1] that for our cells the current is very nearly proportional to the light, and for faint sources like the stars this is especially the case. We give herewith the calibration of the cell, which was made on June 8 shortly after the eclipse, and which was actually used for determining the light of the corona. In Fig. 1 the intensities are on the basis of the inverse square of the distance from lamp to cell.

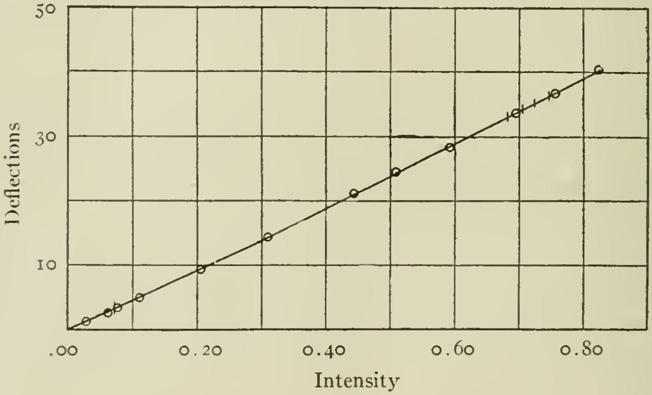

FIG. 1.—Calibration of photo-electric cell

The color-sensibilities of photo-electric cells have not been thoroughly studied in general, but we have definite results for the potassium cell which was used at the eclipse. Fig. 2 shows the curve as determined by Mr. T. Shinomiya in our physics laboratory. The energy of the beam of light which affected the cell was measured with a thermo-couple, and the results are all reduced to equal energies. The curve is not yet complete in the ultra-violet, as the energy becomes so small as to require a more sensitive arrangement for its measurement. From rough measures and various considerations we know that this cell is very insensitive to radiation of wave-length shorter than 4000 A, and the effective maximum sensibility of the cell is therefore toward the red from the maximum of the ordinary unstained photographic plate. The "color-equation" of the potassium cell used in our stellar photometry is 0.86 magnitude.

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[1] *Astrophysical Journal*, 45, 69, 1917.

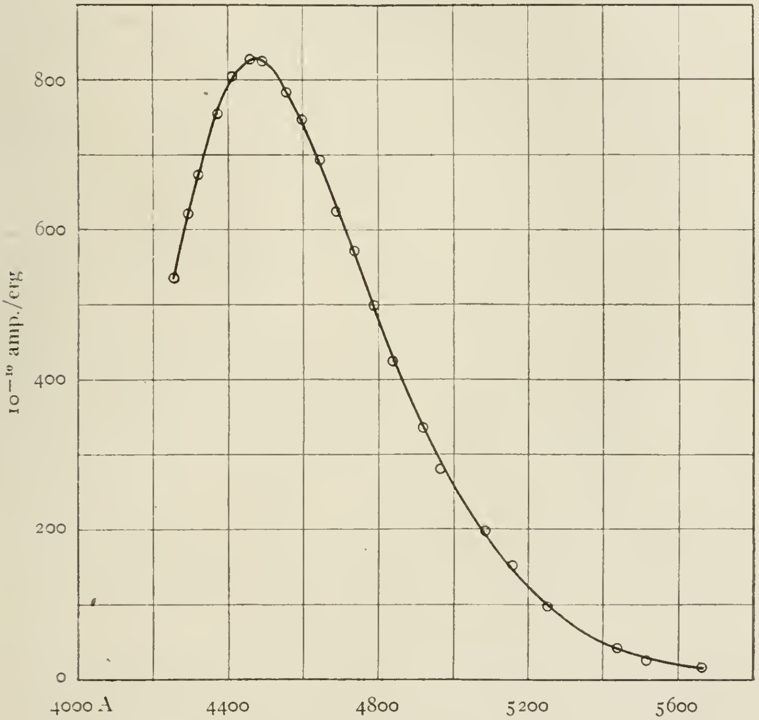

FIG. 2.—Color-sensibility curve of a potassium photo-electric cell

Our main reliance for a standard of light was an amyl-acetate candle, or Hefner lamp, the same as used by Stebbins and Brown in measures of the light of the moon by means of a selenium cell.[1]

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[1] *Astrophysical Journal*, **26**, 326, 1907.

The wick is of 8 mm diameter, and the height of the flame is regulated to 40 mm. The amyl acetate was purified by distillation between 137° and 143° C. Although this candle is much more troublesome to use than a low-voltage electric lamp, nevertheless it is easy to reproduce after a long period of time, and seems to be the best source to rely upon for comparisons of the corona at different eclipses. No measures were taken with the candle unless it had been burning for some time, preferably for half an hour.

We also had two electric pyrometer lamps, secured from the Nela Research Laboratory, which, although not designed for this kind of work, served very well for tests and calibrations of the photo-electric cells and as checks upon the standard candle. Another lamp, which had been calibrated and sent to us by the Bureau of Standards, was unfortunately burned out by accident before we had used it at the station.

Since much of the preliminary work was experimental, we shall give only the intercomparisons of the lamps which were made at the station, as these were under the exact conditions at the time of the eclipse. The electric lamps are designated by N_1 and N_2, require about 4 volts, and each was burned at 0.60 ampere, as determined with a precision ammeter. The comparison of the lamps with the candle by means of the potassium cells gave the ratios presented in Table I. We adopt 0.93 candle-power for N_1 as measured by a potassium cell, while the light of N_2 was 0.79 that of N_1. As far as accidental errors are concerned, these measures are quite satisfactory, but we have not made an exhaustive study of the candle at widely different dates. For the electric lamps, N_1 and N_2, the measures were really only a check upon the readings of the ammeter, as a small change in the current was as readily determined by the light-effect shown by the galvanometer as by the motion of the ammeter needle. We expect to keep all of these standards of light for the next eclipse.

We were able to compare our candle with an electric lamp which Mr. Parkhurst brought over from Green River to our station on June 5. Measured with cell K113 his lamp at 5.4 volts gave 10.4 candles, as compared with the Bureau of Standards visual tests, which gave 4.65 candles at the same voltage. The difference is due to the filament of this electric lamp being much whiter than the amyl-acetate flame, and the blue-sensitive photo-electric cell naturally gives a higher value than the eye does when a white light is expressed in terms of a yellow one.

TABLE I

Comparison of Lamps, Cell K113

Date	N_1/Candle	Residual	Date	N_2/N_1	Residual
June 4......	0.92	−0.01	June 6.....	0.78	−0.01
June 6......	.93	.00	June 8.....	.79	.00
	.92	− .01		.79	.00
	.93	.00		.78	− .01
June 8......	.93	.00		.78	− .01
	.92	− .01		.78	− .01
	.94	+ .01	June 9.....	.80	+ .01
	.96	+ .03		.79	.00
June 9......	.93	.00		.79	.00
	.92	− .01		.80	+ .01
Mean...	0.93	±0.008	Mean...	0.79	±0.006

Comparison of Lamps, Cell K59

	N_1/Candle	N_2/N_1
June 9........	0.94	0.80
	0.92	.80
	....	.80
	....	.80
Mean.....	0.93	0.80

The program during totality provided for a first exposure to the corona and sky background, then to the sky 8° north of the sun, back to the corona, and so on, with one stop to check the zero of the galvanometer. As full galvanometer deflection was secured in ten seconds or less, it was convenient to take readings every ten seconds, and the photometer tubes were moved at each count of ten and then left undisturbed. The metronome used for timing was regulated to beat 100 times in the 95 seconds of totality, and we thus got in ten counts of ten each, the half-second lost in each ten being unnoticed. The start at second contact was probably quite accurate, as the sun reappeared within one second of the hundredth count.

The exposures to the sky 8° north of the sun required that there should be no clouds in that direction, and we were very fortunate in that the clear space just before totality came from the northwest, and during the critical time the sky was perfect for our purpose. It would have been impossible to change at the last minute and turn the instrument in any direction but north. As very little of the sky illumination during a total eclipse comes from the corona, it is safe to consider the sky background close to the sun to be the same as at some distance away.

The two cells which were to be used were the potassium cell, K113, in a circuit of moderate sensitiveness, and a rubidium cell which had a voltage applied almost up to the glowing-point, so as to get the maximum effect. A false start just before second contact flashed the rubidium cell, and it could not be brought back into adjustment in the short time, so that this half of the program was lost. The safe side of the combination worked perfectly, however, and Mr. Kunz found that the readings were made with perfect ease. Our advance estimate was that the initial deflection from the corona plus sky background would be 5 scale-divisions with the potassium cell and 12 with the rubidium. We were prepared to observe deflections say ten times smaller and up to fifty times larger than the estimate, and we felt safe that the readings would not go off the scale. The actual result was a deflection of 33 divisions with the potassium cell, or nearly seven times the amount estimated.

Table II gives the readings and results during the eclipse. In the first column are the times of reading, counting 100 to the duration of totality. The second column is self-explanatory. The readings in the third column are just as they were put down at the time, the observer noting afterward that where no tenths were recorded the whole division was exact within one- or two-tenths. These readings show a gradual shift of the zero from 90.0 to 89.5 during the course of the measures, and this has been allowed for in the deflections in the next column. This would have been called a large shift of the zero under the preliminary conditions, but altogether it was fortunate that nothing worse came up during totality. The fifth, sixth, and seventh columns were derived from the calibration shown in Fig. 1, where the four deflections from the corona and sky, also the sky alone, are marked on the curve. Considering the first deflection of 33.0 divisions, it is found that this corresponded to an illumination of 0.682, the unit being the light from N_1 at 1 meter. Subtracting the sky effect of 0.072 in the same unit, we have the corona equal to 0.610 of N_1 at a meter. Multiplying this by 0.93 and rounding off one place we get 0.57 candle-meters for the corona.

TABLE II

OBSERVATIONS DURING TOTALITY

TIME	EXPOSURE TO	GALVANOMETER READING	DEFLECTION	UNIT = N_1 AT 1 METER			CORONA IN CANDLE-METERS
				Corona + Sky	Sky	Corona Alone	
		div.	div.				
0...	Dark	90	0.0	....	....	....	....
10...	Corona + Sky	123	33.0	0.682	0.072	0.610	0.57
20...	Sky	93	3.1	....	....	....	....
30...	Corona + Sky	125	35.1	0.723	0.072	0.651	0.61
40...	Sky	93	3.1	....	....	....	....
50...	Corona + Sky	124	34.2	0.705	0.072	0.633	0.59
60...	Dark	89.8	0.0	....	....	....	....
70...	Sky	92.8	3.1	....	....	....	....
80...	Corona + Sky	126	36.3	0.746	0.077	0.669	0.62
90...	Sky	93.2	3.6	....	....	....	....
100...	Dark	89.5	0.0	....	....	....	....
Mean	....	....	....	....	....	....	0.60

The differences between the four values in the last column are considered to be real, but we are unable to say with certainty whether or not the variation is due to the motion of the moon over the inner corona and chromosphere. From the report of our assistants, who were in the open, there was absolutely no haze near the sun during totality, so we think it improbable that the differences are due to variable transparency of the atmosphere. As photometric measures go, however, the measures agree well enough, with a range of less than 10 per cent. If it had been possible to carry out our complete program we should have had eight readings on the corona instead of four, and these slight variations would have been checked; but for the mean value in candle-meters the four readings are quite sufficient.

We may now consider additional comparison measures which were made at the station before and after the eclipse. It was intended to take a long series of readings on the brightness of the sky as the eclipse came on, but the presence of clouds prevented this. We did obtain, however, a few exposures to the clear sky shortly after first contact. With an aperture of 4.57 mm at the upper end of the photometer tube. the potassium cell was exposed at G.M.T. 10^h16^m to the sky 8° north of the sun. The mean of several deflections was 47.2 divisions, as compared with 31.5 divisions, the average deflection from the corona. For this purpose we may assume a linear relation between light and current. It is convenient to reduce the sky measure to that of a circle of $\frac{1}{2}°$ diameter, and a factor 0.93 is introduced because about 7 per cent of the sun's disk was covered by the moon at the time of measurement. We have then

$$\frac{\text{Corona}}{\text{Sky circle } \frac{1}{2}^\circ \text{ diameter}} = 0.93 \; \frac{31.5}{47.2}\left(\frac{0.457}{127 \tan \frac{1}{2}^\circ}\right)^2 = 0.105,$$

or the corona gave one-tenth the light of an area of daylight sky of the apparent size of the sun and 8° distant. This refers to the Wyoming sky, which was clear and blue at the time, and the altitude of the station was about 6500 ft. above sea-level.

From the readings in Table II the 4° circle of sky gave a deflection of 3.1 divisions, or one-tenth the effect of the corona. Reducing to a circle of $\frac{1}{2}°$ diameter, this makes the corona 640 times as bright as that area of the sky during totality. It follows, therefore, that the sky intensity was reduced $\frac{640}{0.105} = 6100$-fold by the moon's shadow. As the decrease from sun to full moon is something like one hundred times greater than this, and the corona is fainter than the moon, it follows that less than 1 per cent of the sky illumination during totality came from the corona.

It was planned to get measures of the full moon before leaving Urbana for the West, but bad weather prevailed. We did succeed in getting test measures at Rock Springs on the waning moon on two mornings just before dawn. However, we were especially fortunate in having two clear nights on our return to Urbana two weeks after the eclipse, and the moon was measured on June 23 just before it entered the penumbra of the earth's shadow. This was as nearly an uneclipsed full moon as can ever be observed.

Table III gives the measures of the moon and the results. The fourth column contains the observed values of the moon's light, the fifth the reduction to mean distance of moon and sun, and the sixth the correction for atmospheric absorption. The numbers in the seventh column are derived from the fourth column by applying the logarithmic corrections in the fifth and sixth. In the eighth column the phase is counted each way from the point opposite the sun, positive after full moon. At the foot of the table are given the measure on the corona and the comparison with the full moon.

TABLE III

Observations of the Moon

Date	G.M.T.	Zenith Distance	Moon's Light in Candle-Meters	Log. Reduction to Mean Distance	Log. Correction to No Atmosphere	Corrected Moon's Light in Candle-Meters	Mean in Candle-Meters	Phase	Absorption Factor
1918 June 3	$22^h 30^m$	69°.5	0.017	9.967	0.635	0.068	.....	.....	....
	22 35	68.6	.020	9.967	0.608	.075	.....	.....	....
	22 39.5	67.7	.018	9.967	0.582	.064	.....	.....	....
	22 46	66.6	.017	9.967	0.550	.056	0.066	+120°	2.5
June 4	22 18	78.2	.005	9.968	0.747	.026	.....	.....	....
	22 28	76.3	.005	9.968	0.662	.021	.....	.....	....
	22 59	70.7	.011	9.968	0.484	0.031	0.026	+134	1.8
June 22	18 07	66.6	.58	0.015	0.444	1.67	.....	.....	....
	18 52	70.2	.51	0.015	0.524	1.77	.....	.....	....
	19 14	72.1	.45	0.015	0.578	1.76	1.73	− 14	2.0
June 23	16 10	67.4	.81	0.005	0.414	2.12	.....	.....	....
	16 58	64.6	.88	0.005	0.367	2.07	.....	.....	....
	17 32	63.8	.96	0.005	0.356	2.20	2.13	− 3	1.8
Corona on June 8......	11 18	53.3	0.60	....	0.252	....	1.07	......	1.8

Ratio of corona to full moon, 0.50

The first three nights in Table III are included only as a general check upon the measures at full moon. The correction for atmospheric absorption, which increases the values as much as fivefold, shows that it is useless to measure the moon at low altitudes unless a star near by can be used for comparison. The measures of the full moon on June 23 were quite satisfactory, and the sky was estimated to be fully as good as at the time of the eclipse. The application of the absorption correction changes the ratio corona to moon from about 0.6 to 0.50, and it is evident that no great error has been introduced here by improper allowance for the atmosphere.

The method of allowing for the atmospheric absorption in our photometric work on stars at this observatory is to take the reduction to zenith from Müller's[1] table and multiply by an arbitrary factor, based upon the observer's judgment, but this factor is often determined by measures of suitably placed stars, and with constant instrumental conditions is fixed by the absolute amount of photo-electric effect produced by any star which has been observed on many nights. If the reduction to outside the atmosphere is desired, an additional allowance of 0.20 magnitude times the factor is made. Since for a potassium cell our usual factor for a good night is 1.8, this gives 0.36 magnitude as the absorption at the zenith. For the work on stars, where the correction is always a differential one, it is not a great matter if the absolute value of the assumed absorption is considerably in error, as no theory can allow for the sudden and irregular changes in the transparency of the air at any ordinary station. There is of course a great difference in the absorption for light of different wave-lengths, but the foregoing estimate is considered to be about right for the integrated effect with a potassium cell. Although it is quite unsatisfactory to guess in this way at the effect of our atmosphere on the brightness of the corona, it is better than to ignore the effect altogether, as has been done in most cases.

We give in Table IV other photometric measures of the corona subsequent to 1870 and 1878, when only first rude attempts were made.

The photo-electric value ought to be fairly comparable with the photographic results, considering the color-curve of the potassium cell. Our result does fit in between the extremes, but it is evident that there are large outstanding differences between observers which are probably not due to changes in the corona.

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[1] *Photometrie der Gestirne*, p. 515.

The values ascribed to Pickering and Holden are not given explicitly in their papers but may be derived from their data, using Pickering's figure for the moon for both cases.

The photographic results by Turner apparently depend upon a value of 0.02 foot-candles for the moon, which is all right for visual light, but, we suspect, may be five or ten times too small for the moon's photographic effect. If this surmise is correct, then Turner's values should be considerably reduced.

The volumes of the British Astronomical Association, edited by Maunder, are not available to us, and we are indebted to Mr. Parkhurst for the reference to them. We are quite at a loss to account for the high values obtained in 1898 and 1900.

The independent measures of Graff and Schwarzschild in 1905 seem to be in good agreement.

Perrine states that most of the light which goes into his value of 0.11 is additional to what is shown on an ordinary photograph, as he allows for the part of the corona which is hidden by the moon. He considers that most of the coronal light comes from the inner ring about the sun's limb, only 1 minute of arc in apparent width.

Of the visual determinations, those published by Abney and Thorpe were made with the same instrument, the observers being Douglas in 1886 and Forbes in 1893. A direct comparison with the moon was not made, and the corona was expressed in Siemens' units at 1 foot, 0.0197 in 1886 and 0.026 in 1893, the conditions being poor at the first date. Following Turner's discussion of these measures, the values in terms of the full moon are 0.8 and 1.1 respectively, which are inserted in Table IV.

Leuschner in 1889 found 0.0086 units of a wax candle at 1 foot. We arbitrarily assume the moon to be 0.02 in the same unit, and derive 0.4 moon for the corona.

The results of Fabry and Knopf are as given in the originals.

As far as we can make out, all of the photographic determinations are for the corona exclusive of the sky background. The values of Abney and Thorpe apparently include the sky, an area of perhaps 12° diameter according to Turner. The results of Leuschner in 1889 exclude the sky, and the same is presumably true of the careful measure of Fabry in 1905, while Knopf included the background in the immediate vicinity of the corona.

There are large outstanding corrections for atmospheric absorption which should be applied to the results in Table IV to make them homogeneous, but we have not the data at hand to hazard such a procedure. In a general way it is evident that the visual values run about ten times larger than the photographic, except for the British observers ,and the photo-electric result lies between, but nearer the visual.

TABLE IV

TOTAL LIGHT OF THE CORONA

ECLIPSE	OBSERVER	IN TERMS OF FULL MOON	REFERENCE
	Photographic Determinations		
1886	W. H. Pickering	0.025	*Harvard Annals*, **18**, 104, 1890
1889 (January)	Holden	0.04	*Lick Contributions*, **1**, 14, 1889
1889 (December)	Holden	0.02	*Ibid.*, **2**, 14, 1891
1893	Turner	0.6	*Proc. Roy. Soc.*, **68**, 36, 1901
1898	Turner	1.1	*Ibid.*
1898	Bacon and Gare	2.7	*The Indian Eclipse of 1898* (E. Walter Maunder, ed.), p. 129
1900	Gare and Johnston	10.(?)*	*The Total Solar Eclipse of 1900* (E. Walter Maunder, ed.), p. 154
1905	Graff	0.26	*Hamburger Abhandlungen*, **3**, 48, 1913
1905	Schwarzschild	0.17	*Astronomische Mitteilungen*, Göttingen, No. 13, p. 36
1908	Perrine	0.11	*Lick Obs. Bull.*, **5**, 98, 1909
	Visual Determinations		
1886	Abney and Thorpe	0.8	*Phil. Trans.*, **108A**, 363, 1889
1889 (January)	Leuschner	0.4	*Lick Contributions*, **1**, 100, 1889
1893	Abney and Thorpe	1.1	*Phil. Trans.*, **187A**, 433, 1896
1905	Fabry	0.75	*Comptes rendus*, **141**, 870, 1905
1905	Knopf	0.85	*Hamburger Abhandlungen*, **3**, 80, 1913
	Photo-Electric Determination		
1918	Kunz and Stebbins	0.50	This paper

* Measured as 6 c.p.

The question naturally arises as to how much of the total light which we assume to be from the corona in 1918 was due to the prominences, which were unusually bright at this eclipse. The short-exposure photographs show the prominences to be intrinsically more intense than adjacent regions of the inner corona, and can it be that what we measured was the prominences and not the corona at all? The answer is that in the photographic spectrum of the prominences the most prominent features are the H and K calcium lines, which have almost no effect upon the photo-electric cell. From a print of the spectrum obtained at Green River and kindly sent to us by Mr. Parkhurst, it is seen that the prominence radiations which would affect the cell are principally $H\beta$ and $H\gamma$; but it seems that the selective effect of cutting out the H and K light, combined with the long stretch of continuous coronal spectrum to which the cell is sensitive, would make the prominences much less effective on the cell than on the photographs. It is possible that a study of the 1918 plates will give a measure of the photographic light of the prominences as compared with the corona, but until this is accomplished we cannot give a numerical estimate of the effect of the prominences.

SUMMARY

By means of a potassium photo-electric cell the corona of June 8, 1918, was compared with a standard candle, two electric lamps, the full moon, and with an area of sky during totality and in full sunshine. The numerical results are as follows:

Observed total light of corona	0.60 candle-meters
Same, corrected to outside atmosphere	1.07 candle-meters
Observed ratio of corona to full moon	0.6
Same corrected to outside atmosphere	0.50
Observed ratio of corona to sky circle ½° diameter 8° from uneclipsed sun	0.105
Same during totality	640.
Ratio of intensity of daylight sky 8° from sun to intensity of same area during totality	6100

All of the foregoing measures refer to the light to which the photo-electric cell is sensitive (see Fig. 2). It is probable that the light from the prominences had a small effect upon the measures, which, however, can be evaluated from the photographs.

We are indebted to Dean David Kinley for a grant from research funds of the Graduate School which made the expedition possible; also to Dr. Elmer Dershem, instructor in astronomy, and to Mr. J. B. Hayes, mechanician in the department of physics, for efficient assistance in the preparations for the eclipse. We are also glad to record that the work here described was supported in part by a grant from the Draper Fund of the National Academy of Sciences.

University of Illinois
January 1919

THE MAGNETIC POLARITY OF SUN-SPOTS[1]

BY GEORGE E. HALE, FERDINAND ELLERMAN,
S. B. NICHOLSON, AND A. H. JOY

Whirling storms in the earth's atmosphere, whether cyclones or tornadoes, follow a well-known law which is said to have no exceptions: the direction of whirl in the Northern Hemisphere is left-handed or counterclockwise, while in the Southern Hemisphere it is right-handed or clockwise. The theory of terrestrial cyclones is still very obscure, but the direction of whirl is evidently determined by the increase in linear velocity of the air from pole to equator, due to the earth's rotation. The question naturally arises whether storms in the solar atmosphere are also whirlwinds, and, if so, what law governs their direction of whirl in the Northern and Southern hemispheres.

The first definite evidence bearing on this question was obtained with the spectroheliograph in 1908.[2] Photographs of the hydrogen flocculi made with the Hα line showed clearly marked vortical structure in regions centering in sun-spots. This structure was found to be repeated in hundreds of spots, leaving no doubt as to the generality of the phenomenon. Furthermore, photographs were obtained showing masses of hydrogen in the act of being drawn from a great distance toward the center of sun-spots, as though sucked into a vortex.

These photographs suggested the hypothesis that a sun-spot is a vortex, in which electrified particles, produced by ionization in the solar atmosphere, are whirled at high velocity. This might give rise to magnetic fields in sun-spots, regarded as electric vortices. A search for the Zeeman effect led to its immediate detection, and abundant proofs were soon found of the existence of a magnetic field in every sun-spot observed.[3]

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 165.

[2] Hale, "Solar Vortices," *Mt. Wilson Contr.*, No. 26; *Astrophysical Journal*, **28**, 100, 1908.

[3] Hale, "On the Probable Existence of a Magnetic Field in Sun-Spots," *Mt. Wilson Contr.*, No. 30; *Astrophysical Journal*, **28**, 315, 1908.

Subsequent investigations have led to the view that two classes of vortices are involved:

1. High-level hydrogen vortices, centering in sun-spots, which are revealed by the spectroheliograph when monochromatic images of the sun are photographed with the light of the central part of the Hα line. In these vortices, which sometimes cover vast areas of the solar surface, the motion of the hydrogen appears to be spirally inward and downward. Little is known, except by inference, as to the form of the stream-lines in the lower levels close to the photosphere. There is reason to infer, however, that the hydrogen gas, descending toward the solar surface, moves spirally outward in the lower chromosphere above the spots.[1]

2. Low-level electric vortices, formed in the photosphere, which constitute the sun-spots themselves. In these vortices the motion of the gases appears to be spirally upward from within the photosphere and outward along its surface.[2] It is easy to show by laboratory experiments that such a primary vortex formed in water may set up a secondary vortex in a gaseous atmosphere above it, closely analogous to the hydrogen vortices above sun-spots.

The present paper deals with the magnetic polarity of sun-spots, though some reference will also be made to the closely related phenomena exhibited by the vortex structure of the hydrogen flocculi. The purpose in view is to discover the law of magnetic polarities, if such exists, and to consider whether this law is subject to variation in the course of the sun-spot cycle. Subsequent papers will deal with the various magnetic peculiarities of sun-spots and the structure of the hydrogen flocculi, with special reference to the nature of the vortices associated with sun-spots.

METHOD OF OBSERVATION

It is a well-known fact that when a normal Zeeman triplet is observed along the lines of force of a magnetic field the central (p) component is absent and the two side (n) components are circularly polarized in opposite directions. A quarter-wave plate and Nicol prism mounted over the slit of the spectroscope permit either *n*-component to be cut off at will by rotating the Nicol. Furthermore, if the polarizing apparatus be adjusted so as to extinguish one component, reversal of the current through the coils of the magnet will cause this component to reappear, while the other will be extinguished. The method thus offers a simple means of determining the polarity of a magnetic field, which can still be used when the angle between the line of sight and the lines of force is as great as sixty or seventy degrees. In this case, however, the *p*-component of the triplet is present, and the elliptically polarized light of the *n*-components can be only partially extinguished.

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[1] St. John, *Mt. Wilson Contr.*, No. 69; *Astrophysical Journal*, **37**, 322, 1913; Evershed, *Kodaikanal Observatory Bulletin*, No. 15; Memoirs, **1**, Pt. 1, 1909.

[2] St. John, *loc. cit.*

This method is employed daily for the study of the magnetic polarity of sun-spots with the aid of the 75-foot spectrograph of the 150-foot tower telescope on Mount Wilson. A 12-inch (30.5 cm) visual objective of 150 feet focal length, mounted just below the second mirror of the coelostat near the summit of the tower, produces at the base of the tower an image of the sun about 43 cm in diameter, any portion of which may be brought upon the slit of the spectrograph below by means of the electric motors that control the slow motions of the coelostat and second mirror. The massive circular head of the spectrograph, which carries the slit in a horizontal plane, can be rotated to any desired position angle. After passing through the slit, the light from the sun-spot, or other portion of the solar image, descends vertically into a subterranean well, 10 feet (3.0 m) in diameter, with concrete walls. The collimating lens of 6-inch (152 mm) aperture is 75 feet (22.9 m) below the slit. This lens and the large Michelson grating below it are mounted on a heavy support at the bottom of the well, where the temperature is essentially constant throughout the year. After falling on the grating, the light is returned through the collimating lens, which forms an image of the spectrum at a point near the slit of the spectrograph. Here it can be observed visually, or photographed if desired. There is no connection (other than the walls of the well) between the head of the spectrograph, which carries the slit, and the support for the collimator-camera lens and grating at the bottom of the well. When the slit is rotated in position angle, the grating support is rotated through the same angle by means of an electric motor, thus bringing the lines of the grating again into parallelism with the slit. All of the necessary adjustments, including the rotation of the dome at the summit of the tower, the slow motions of the coelostat and second mirror, the focusing of the sun's image on the slit, the focusing of the collimator-camera lens, the inclination of the grating to bring different orders of spectra into view, and the rotation in azimuth of the grating support, are accomplished by electric motors controlled by push buttons near the hand of the observer.

Most of the observations, both visual and photographic, of the Zeeman effect in sun-spots are made in the second-order spectrum, where the linear dispersion at the region in question is 1 A = 2.96 mm. For the daily determination of spot polarities the sharp iron triplet λ 6173.553 is usually employed.

In work of this nature, as already stated, it is necessary to use a Nicol prism, which is supported just over the slit of the spectrograph, and surmounted by a compound quarter-wave plate, so constructed that the principal sections of the successive mica strips (2 mm wide) are normal to one another. The Nicol prism, 130 mm in length, 18 mm high, and 10 mm wide (effective width 5 mm), was built by Werlein (from four sections each 32.5 mm long) for use with the 75-foot spectrograph (see Plate III*a*, *Contribution* No. 71). The compound quarter-wave plate, there shown swung to one side, can be turned into position above the Nicol. In view of the considerable focal length of the spectrograph, the distance of the quarter-wave plate from the slit (65 mm) is not sufficient to reduce materially the sharpness of the dividing lines between adjoining strips of spectra. For the study of plane polarization phenomena, a compound half-wave plate is substitued for the compound quarter-wave plate. Circular half-wave and quarter-wave plates, mounted so that they can be rotated in position angle, are also used for special purposes.

With this apparatus the magnetic polarities and the strength of the magnetic field in all sun-spots are recorded daily (see Fig. 1). A sheet of paper is placed in the focal plane of the telescope and all spots visible are traced in outline upon it. The polarity of each spot is then indicated by a "V" or an "R," according as the violet

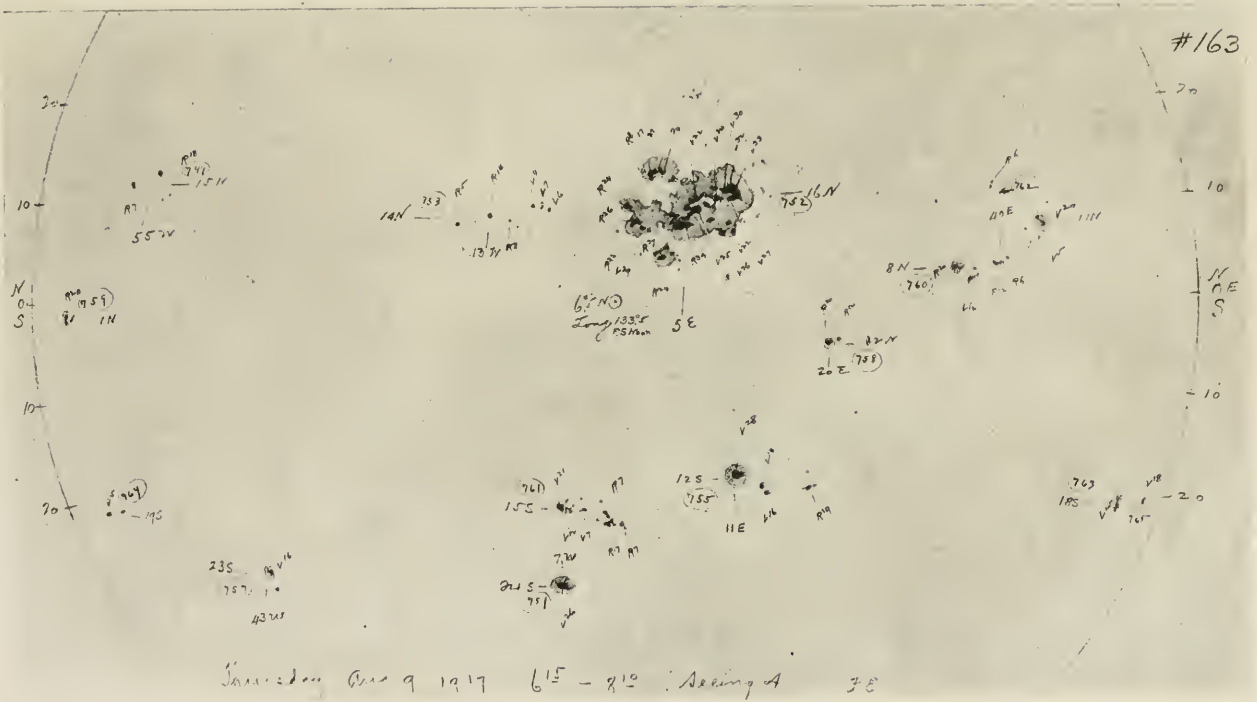

Fig 1.—Reproduction of sketch showing daily record of observations of sun-spot polarities

or red component of the Zeeman triplet is transmitted by a given strip of the quarter-wave plate. As the diameter of the focal image of the sun is 43 cm, it is possible in this way to observe very small spots. In addition to the record of polarities, the strength of the field in each spot is measured by means of a parallel-plate micrometer. The scale of this micrometer has been carefully calibrated by Ellerman and Nicholson, and when used to measure a triplet whose separation for a given field-strength has been determined in the laboratory its readings are readily convertible into gausses. In the case of the sharp triplet λ 6173, the *n*-components may still be distinguished separately in sun-spots when the field-strength is as low as 1000 gausses. For greater field-strengths the observed values are therefore fairly reliable, but below this limit they must be regarded as merely approximate. The readings with the parallel-plate micrometer are recorded in degrees; and since for λ 6173 one degree corresponds approximately to 100 gausses, the designation "R 28" (see Fig. 2) at a certain point on a sun-spot indicates that the polarity at this point corresponds to that of the north magnetic pole of the earth, and that the field-strength is about 2800 gausses. "V 7" would mean that the polarity corresponds to that of the south magnetic pole of the earth, while the field-strength is of the order of 700 gausses.

THE DETERMINATION OF SUN-SPOT POLARITIES; INCLINATION OF THE LINES OF FORCE

The determination of the magnetic polarity of a sun-spot depends upon an estimate of the relative intensities of the red and violet components of a Zeeman triplet. The zinc triplet λ 4680 in the spectrum of a spark between the poles of a large Weiss magnet, when photographed with a Nicol prism and compound quarter-wave plate at angles of 0°, 60°, and 90° with the lines of force, presents the appearance shown in Plate V*a*, *b*, and *c*. When the observation is made parallel to the lines of force, the *p*-component of the triplet is absent and one of the *n*-components is completely cut off. At right angles to the lines of force the *p*-component is twice as strong as the *n*-components (in the case of a normal triplet), while the two *n*-components are of equal intensity. At intermediate angles the plane polarized p-component is of intermediate intensity, while the elliptically polarized n-components are no longer completely cut off by the Nicol prism and quarter-wave plate. The relative intensities of the three components of a normal triplet for one set of alternate strips, when observed at any angle from 0° to 180° with the lines of force, are given by

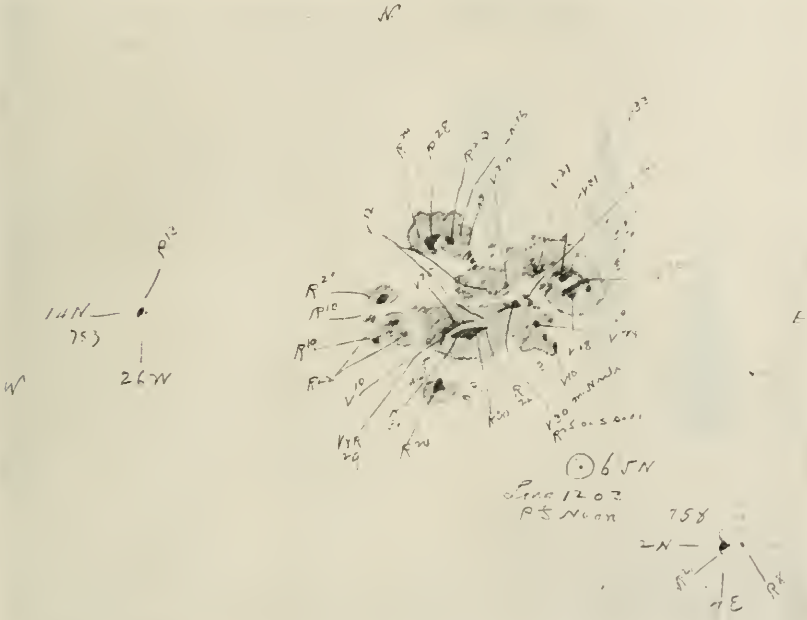

FIG. 2.—Sketch of large sun-spot group from the daily record for August 10, 1917, showing the manner of indicating the polarity and field-strength at different points in the group.

$$n_V=\frac{1}{4}\,(1-\cos\gamma)^2 \quad p=\frac{1}{2}\sin^2\gamma \quad n_R=\frac{1}{4}\,(1+\cos\gamma)^2$$

where n_V, p, and n_R are the intensities of the violet, central, and red components respectively; γ is the angle between the line of sight and the lines of force, and the unit of intensity is such that $1=n_V+p+n_R$.[1] The variation of intensity in the three components with the angle γ is illustrated by Fig. 3. We thus have a means of determining the angle between the line of sight and the lines of force in observations of sun-spots. The important bearing of this on polarity determinations will shortly appear.

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[1] Seares, *Mt. Wilson Contr.*, No. 72, p. 5; *Astrophysical Journal*, 38, 99, 1913.

Repeating the foregoing observations, at the same angles of 0°, 60°, and 90°, but substituting the compound half-wave plate for the compound quarter-wave plate previously used with the Nicol, we have the appearances illustrated in Plate V*d*, *e*, and *f*. Parallel to the lines of force, when no plane polarized light is present, we naturally find that the half-wave plate and Nicol are without effect on the circularly polarized *n*-components. At 60°, where the plane polarized *p*-component is absent and the *n*-components are elliptically polarized, the appearance is as shown in Plate V*e*, the central component being completely cut off by alternate half-wave strips, while the *n*-components are materially reduced in intensity.

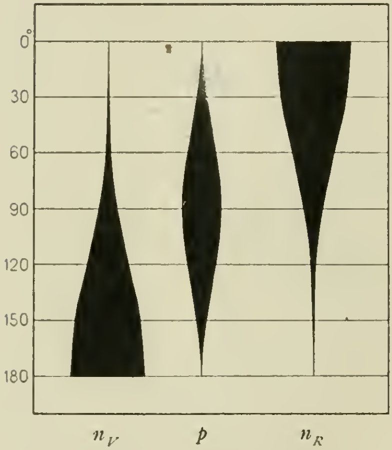

FIG. 3.—Relative intensities of the three components of a normal Zeeman triplet for inclinations of 0° to 180° between the line of sight and lines of force when observed with a Nicol prism and quarter-wave plate placed in front of the slit of the spectrograph. Thus for 0° the relative intensities are 0, 0, 1, respectively; for 90°: 0.25, 0.50, 0.25; for 180°: 1, 0, 0.

Consider now the bearing of these observations on the determination of the polarity of a sun-spot when close to the sun's limb. Let us assume that the lines of force at the center of the spot are parallel to a radius from the sun's center passing through this point, and that at other parts of the umbra and penumbra the lines of force are inclined approximately as indicated in Plate V*g*. This drawing relates to a spot photographed near the west limb of the sun on October 1, 1915, and reproduced in Plate V*h*, where the scale is that of the image of the sun at the focus of the 150-foot tower telescope. The triplets λ 6302.709 and λ 6301.718 were observed at various parts of the spot, the successive positions of the slit on the image of the spot being indicated on the illustrations by the lines numbered 1 to 7. The observations were made visually by Hale in the second-order spectrum of the 75-foot spectrograph, using the Nicol and the compound quarter-wave plate over the slit. As these observations are typical of all fairly symmetrical spots when near the sun's limb, the phenomena observed at each position of the slit may be indicated here. Reference is made to the appearance of the triplet λ 6173.

1. Slight "zigzag" effect, red *n*-component (R) stronger than violet *n*-component (V). Diffuse.

2. Widened, diffuse. R stronger than V. Sharp at one point in spot, where *p* and R are separated. R stronger than *p*. V absent or very faint.

3. Wide, diffuse. R stronger than V. Three components of triplet sometimes visible.

Between (3) and (4). *p* visible but faint. V strong and separated from *p*. R apparently absent.

4. More diffuse. *p* hardly separated from V, which is very strong Marked "zigzag."

5. Similar, but less "zigzag." Diffuse.

The chief point to be noticed is that, whereas the red *n*-component (R) was the stronger on the side of the spot toward the limb, the violet *n*-component (V) was the stronger on the opposite side of the spot. In other words, opposite polarities were indicated by the observations on the opposite sides of the spot. This is easily explained when the inclination of the lines of force at various points in the spot is considered.

As already stated, Plate V*h* represents the spot at the point of observation near the west limb of the sun. The direction of the lines of force corresponding to the various parts of the spot is hypothetically indicated in Plate V*g*. Thus when the slit occupied positions (2) and (3) the lines of force were directed away from the observer, and *R* was stronger than *V*.[1] Between (3) and (4) the *n*-components became of equal intensity, indicating that the line of sight was nearly at right angles to the lines of force. In subsequent positions of the slit the violet *n*-component, which had been weaker than the red component in positions (2) and (3), was invariably the stronger, showing that the lines of force from the inner side of the spot were directed toward the observer. Thus the true polarity of a sun-spot, corresponding with that given by observations made on the umbra when the spot is near the center of the sun, may also be determined near the limb by observing that portion of the penumbra which lies toward the center of the sun.

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[1] Assuming the lines of force at the center of the spot to be directed radially outward from the sun's center.

The observations indicate that at some position of the slit between (3) and (4) the lines of force were nearly at right angles to the line of sight. At this point the compound half-wave plate gave strong evidence of plane polarization (Plate V*k*), while the the quarter-wave plate indicated the same result by showing that maximum intensity shifted from *R* to *V*.

The position on the spot occupied by the slit when the red and violet components of a Zeeman triplet are of equal intensity may be called the "neutral line" (see Fig. 3). By determining the position of this line in a large number of spots we may obtain data which will enable us to fix, with some precision, the average inclination of the lines of force in a plane passing through the center of the sun and the line of sight.

The photographs of a large bipolar spot group made on February 9, 10, 12, 13, 14, 1917, and reproduced in Plate VI, will serve to illustrate how the position of the neutral line changes with the longitude. Observations were made on both of the principal spots as the group was carried by the solar rotation toward the west limb from a point near the center of the sun. One day's observations are missing, because of clouds. The gradual displacement with longitude of the neutral line illustrated by these observations is perfectly typical, and may be seen in any spot of sufficient size as it is carried by the solar rotation away from the eastern or toward the western limb.

Irregularities in the form of spots, the presence of small components opposite in polarity to the chief umbra, and other magnetic peculiarities such as are noted below, frequently complicate the observations and affect the form and position of the neutral line. But its average shift with longitude, determined from observations of a considerable number of spots, gives the approximate inclination of the lines of force in various parts of the umbra and penumbra. Thus we find, from 105 observations of 61 spots made by Joy and Nicholson, the positions of the neutral line and the corresponding inclination of the lines of force (taken as normal to the lines of sight passing through the neutral line) given in Table I.

The position of the neutral line is given in tenths of the spot's radius, measured from the center of the spot toward the limb. In the average spot of moderate size the radius of the umbra is 0.4 that of the penumbra.

TABLE I

INCLINATION OF LINES OF FORCE

Longitude	Position of Neutral Line	Angle Between Lines of Force and Solar Radius
30°–40°	8.0	55°
40 –50	7.0	45
50 –60	4.7	35
60 –70	3.2	25
70 –90	2.0	10

These results relate to the lines of force that lie in a plane passing through the center of the spot, the center of the sun, and the line of sight. They do not show, however, the inclination of the lines of force in the plane at right angles. These can be very accurately determined by a method first applied by Nicholson, as follows:

When light from a Zeeman triplet is examined in a direction normal to the lines of force the vibrations producing the *p*-component are parallel to the lines of force. When a sun-spot is observed near the sun's limb the lines of force lying in a meridional plane through the center of the spot are nearly normal to the line of sight, and their inclination to the north or south can be found by determining the direction in which the light of the *p*-component is vibrating. The simplest way to do this would be to pass the light through a Nicol and observe in what part of the spot the *p*-component is completely extinguished for a given position of the Nicol. For that region the lines of force would be parallel to the long diagonal. A modification of this method has been used because the grating over the 75-foot spectrograph almost completely plane-polarizes the light in the region generally observed, with the result that for certain positions of the Nicol it would be almost completely extinguished. The difficulty could be avoided by leaving the Nicol fixed with reference to the slit and rotating the whole spectrograph. In practice the same result is accomplished by using a half-wave plate placed over a fixed Nicol. The half-wave plate is mounted on a scale calibrated in degrees so that the angle between its principal axis and that of the Nicol can be observed. By observing this angle the direction of vibration of the *p*-component projected on a plane perpendicular to the line of sight can be determined in all parts of the spot. Since the plane through the sun's axis and the center of the spot is nearly perpendicular to the line of sight, observations of points in the spot lying in the plane give directly, with good approximation, the inclination of the lines of force toward the north or south. The results of 24 observations by Nicholson on three typical unipolar spots are as follows, the unit of distance, as before, being one-tenth of the radius of the spot.

Distance from Center of Spot	Angle Between Lines of Force and Solar Radius
0	0°
3	18
6	49
9	73

The inclinations found by the two methods are shown in Fig. 4, where the ordinates are angles between the lines of force and the solar radius and the abscissae are distances from the center of the spot. The results of the first method are indicated by crosses and of the second by circles. Considering the difficulties and uncertainties of observation, the agreement is excellent.

Additional information can be obtained by determining the relative intensities of the *n*- and *p*-components of a Zeeman triplet in different parts of a spot observed near the center of the sun. The results of such observations vary among themselves, but in many spots the *p*-component is strong even at the center of the umbra when the spot is near the center of the sun (see Plate V*i, j*). This investigation, the results of which are not essential in the present study of polarities, is being continued by Nicholson.

By taking into account the position of the neutral line and avoiding the observation of spots at the extreme limb of the sun (where the lines of force along the line of sight are frequently not visible at the outer edge of the spot), the polarities of sun-spots may be determined with certainty. It is to be understood that for this purpose a quarter-wave plate is always used with a Nicol prism over the slit of the spectroscope. Serious errors in the determination of the relative intensities of the lines of spot triplets may enter, as Zeeman has pointed out, when the observations are made without polarizing apparatus. In the case of the 75-foot spectrograph, for example, the plane polarization due to the grating varies greatly for different wave-lengths and for different orders of the spectrum. It is therefore entirely insufficient to determine the relative intensities of the lines without complete knowledge of the polarization phenomena of the grating and also those of the tower telescope. With a Nicol and quarter-wave plate, however, the results of polarity observations are essentially unaffected by these causes.

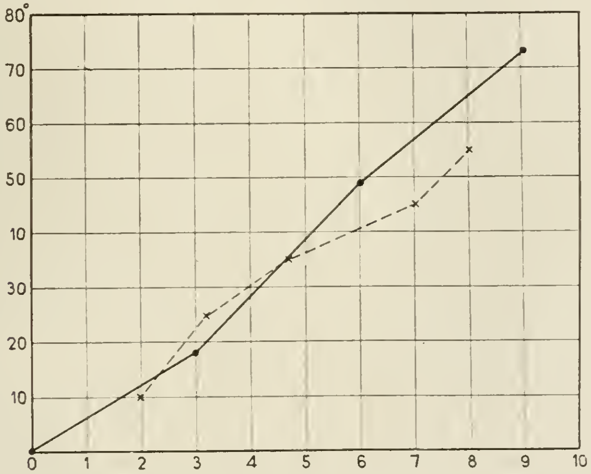

FIG. 4.—Inclination (ordinates) of lines of force to sun's radius at different distances from the center (abscissae, in tenths of radius of spot) of sun-spots. Crosses indicate results from observations of the position of the neutral line. Points are from measures of the orientation of the plane of polarization for the p-components in spots observed near the sun's limb.

BIPOLAR SUN-SPOTS

In Hale's first paper "On the Probable Existence of a Magnetic Field in Sun-Spots,"[1] photographs were reproduced of two large spots, of opposite polarity, lying on opposite sides of the solar equator. Other cases which at first seemed to suggest a relationship between polarity and hemisphere were subsequently found. But it soon appeared that the simple law of terrestrial cyclones does not apply to the sun; for spots of opposite polarity were detected, not only in the same hemisphere, but also in the same spot-group.

Thus the large spot (Greenwich No. 6728) observed in the focal image of the 60-foot tower telescope on September 24, 1908, was seen to have at least three well-defined smaller umbrae, in addition to the principal umbra, within the boundary of the large penumbra. Photographs of the iron triplet λ 6303, made with the 30-foot spectrograph, showed the three small umbrae to have the same polarity, opposite to that of the principal umbra and penumbra. Many similar cases have been found, while cases of small companion spots, completely separated from a larger member of the group, and differing from it in polarity, are also common.

In one very remarkable instance, illustrated in Plate VII*a* and *b*, two large umbrae of the same spot, separated only by a bridge, were of opposite polarity, while the polarities of other members of the same group were as indicated in the sketch, Plate VII*c*, accompanying the photograph.

It fortunately happens, however, that cases of mixed polarity are not so common as to obscure our perception of a simple unifying principle leading directly to the detection of a general law of polarities. This has its origin in a well-known peculiarity of sun-spots, indicated in the records of the earliest observers, and commented upon by Carrington.

Sun-spots frequently occur in pairs, the principal members of which may be several degrees apart. The western or preceding member of such a group is often the first to be formed, but sooner or later a second spot, comparable with the first in size, but frequently smaller, or split into several components, is likely to appear behind it. Sometimes both members of the group appear simultaneously, and in other cases the following member is formed first. Many minor spots usually accompany the larger ones, either clustered about them or lying in the space between the principal spots. It is probably a significant fact that the axis of the group usually makes only a small angle with the equator.

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[1] *Mt. Wilson Contr.*, No. 30; *Astrophysical Journal*, 28, 315, 1908.

TABLE II

INCLINATION OF AXIS OF SUN-SPOT GROUPS

Semicycle	Latitude						
	0°–4°	5°–9°	10°–14°	15°–19°	20°–24°	25°–29°	30°–34°
Min.–Max. 1856–1860	7°(3)*	5°(27)	10°(69)	7°(94)	9°(103)	8°(60)	13°(20)
Max.–Min. 1860–1867	1 (24)	3 (117)	4 (148)	6 (73)	9 (40)	8 (9)	14 (3)
Min.–Max. 1867–1871	4 (4)	5 (15)	5 (55)	6 (80)	6 (97)	9 (45)	8 (23)
Max.–Min. 1871–1879	7 (35)	3 (139)	6 (147)	6 (90)	8 (65)	10 (13)	10 (13)
Min.–Max. 1879–1884	7 (11)	2 (42)	4 (105)	3 (123)	9 (85)	9 (31)	14 (9)
Max.–Min. 1885–1889	3 (34)	−1 (72)	6 (110)	7 (43)	10 (7)	17 (2)	(0)
Min.–Max. 1889–1893	−2 (7)	0 (21)	6 (81)	6 (93)	10 (88)	10 (46)	10 (12)
Weighted mean inclination and number of spots	3.7 (118)	2.4 (433)	5.6 (715)	5.8 (596)	8.7 (485)	9.3 (206)	10.8 (80)

*Quantities in parenthesis are numbers of spots.

A study by Joy of the sun-spot drawings of Carrington (1856–1861)[1] and Spörer (1861–1893)[2] shows that there is little change in this angle during the life of the group, but that in the mean the angle bears a definite relation to the latitude of the group. Twenty-six hundred and thirty-three bipolar and multiple groups, covering three and one-half sun-spot cycles, were examined. The following spot of the pair tends to appear farther from the equator than the preceding spot, and the higher the latitude, the greater is the inclination of the axis to the equator. This relation holds for both hemispheres. The details are shown in Table II and the results are illustrated in Fig. 5.

[1] *Observations of Spots on the Sun*, London, 1863.

[2] *Publicationender Astronomischen Gesellschaft*, 13; *Pub. Astrophys. Obs. zu Potsdam*, No. 1, 1878; No. 5, 1880; No. 17, 1886; No. 32, 1894.

In general, the angle of inclination was found to depend entirely on the latitude of the group, without reference to the number of the cycle or the time within the cycle. A knowledge of the polarities of the spots would have aided greatly in determining the position of the axes of the groups.

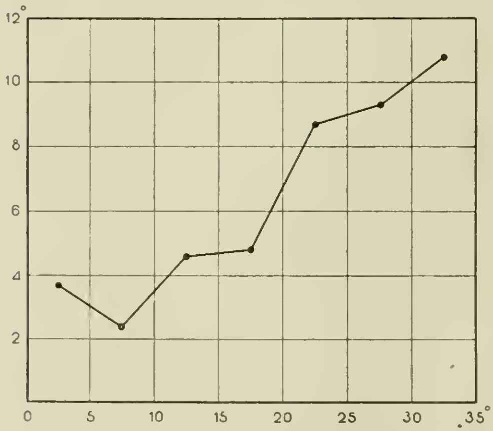

FIG. 5.—Summary of a statistical study of the sun-spot drawings of Carrington and Spörer showing the variation with latitude (abscissae) in the preferential inclination (ordinates) of the axis of bipolar sun-spot groups. In low latitudes the axes are nearly parallel to the sun's equator, but with increasing latitude the mean inclination increases to a maximum of about 11°.

The most significant characteristic of these binary spot-groups lies in the fact that the two principal members, whether single or multiple, are almost invariably of opposite magnetic polarity. A photograph made in the second-order spectrum of the 75-foot spectrograph will serve to illustrate this point, and at the same time to indicate how these polarity phenomena may be recorded photographically. The long Nicol prism and compound quarter-wave plate are used over the slit and the spectrograph is rotated in position angle until the slit passes through the two principal members of the group. A single exposure on a wide Zeeman triplet such as λ 6173 or λ 6303 then gives the polarity of the two members, as shown in Plate VII The spot-group reproduced in Plate VII*d* from a direct photograph made with the Snow telescope on June 19, 1914, consisted of a larger preceding spot and two smaller spots to the east of it, with various minor companions between them. Plate VII*e*, from a photograph of the spectrum made with the long Nicol and single quarter-wave plate, shows that the two smaller spots were of the same polarity, but opposite to that of the largest spot. Plate VII*f*, taken with the compound quarter-wave plate, not only confirms this result, but also serves to better advantage for the study of the weaker fields of the small companion spots lying between the preceding and following members of the group. It will be seen from this figure that magnetic displacements occur at points where no spots are shown, though faint indications of them appear in the original negative.

Analyzing these spectra, we find that there are apparently two "spheres of influence" of opposite polarity which meet near the center of the group. The first of these is dominated by the large preceding spot, while the other comprises the oppositely directed magnetic fields of the two following spots. Minor spots of opposite polarity sometimes occur, as already stated, within these "spheres of influence," but the generality of the bipolar effect is very strikingly shown by an examination of hundreds of spot-groups.

The preceding and following members of bipolar groups may be split into several components, often well separated, and small companion spots, of either polarity, may be present. Furthermore, the strength of the field and the area over which it can be detected may be very different for the preceding and following members of the group. But if magnetic fields of opposite polarity are distinctly shown by the preceding and following members of the pair, the essential characteristics of a bipolar group are present.

The tendency toward bipolar structure is so strongly marked that hardly more than 10 per cent of all spots observed are wholly free from it. In the case of spots which are apparently single, some traces of asymmetry, more or less suggestive of the structure of bipolar groups, can usually be detected. Sometimes such evidence of asymmetry is afforded by faculae following or preceding the spot. More commonly, however, especially in the central part of the sun, it is necessary to have recourse to spectroheliographic plates for the purpose of detecting the asymmetrical structure. In such photographs it is usually found that a single spot, or a group of small spots all having the same magnetic polarity, is near the preceding end of a mass of calcium flocculi elongated in a direction not greatly inclined to the solar equator. Much less frequently the spot occurs near the following end of such a group of flocculi. In about 10 per cent of all cases hitherto observed here the distribution of the flocculi is fairly symmetrical to the east and west of single spots.

Thus in the magnetic classification of sun-spots it is highly desirable to investigate the magnetic records in the light of information afforded by the distribution of the flocculi. The calcium flocculi serve admirably for the purpose just stated, but in order to study the characteristic vortex structure associated with bipolar spot groups we must have recourse to spectroheliograms of the hydrogen flocculi, taken under high dispersion with the light from the center of the Hα line.

In a later paper the characteristics of these bipolar groups will be discussed more in detail, and a variety of evidence bearing on their nature will be presented. Our immediate purpose is to present a scheme of magnetic classification and to bring out the fact that the recognition of this typical structure, even in the rudimentary form where the second spot is absent[1] and represented only by a train of flocculi, is essential to an intelligent discussion of the distribution of spots of different polarities in the northern and southern hemispheres of the sun.

The methods described above for recording polarities can of course be applied to the more complicated groups as well as to those of the simple bipolar type. A case of unusual complexity is illustrated and described in Plate VIII.

MAGNETIC CLASSIFICATION OF SUN-SPOTS

Our scheme of classifying sun-spots is based primarily upon the determination of their magnetic polarities. Supplementary evidence is frequently needed, however, and this is supplied, as already stated, by calcium and hydrogen spectroheliograms. Three classes of spots are included in the scheme: (α) unipolar, (β) bipolar, and (γ) multipolar. These may be subdivided as indicated below.

(α) *Unipolar spots.*—Single spots, or groups of small spots, having the same magnetic polarity. It should be noted, however, that unipolar spots, and the chief components of bipolar groups, may occasionally have small companions of opposite polarity, which play such a minor and sporadic part in the group that they are disregarded in the classification.

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[1] In view of the fact that the general absorption in the spectrum of a spot increases greatly in passing from red to violet, it may perhaps become possible to photograph very faint spots, invisible to the eye, by using ultra-violet light of the shortest possible wave-length.

Unipolar spots, whether single or multiple, may be divided into three groups, which are illustrated in Plate IX:

(α) Those in which the distribution of the calcium flocculi is fairly symmetrical preceding and following the center of the group.

(αp) Those in which the center of the spot-group precedes the center of the surrounding calcium flocculi.

(αf) Those in which the center of the spot-group follows the center of the surrounding calcium flocculi.

(β) *Bipolar spots.*—The simplest and most characteristic bipolar spot-group consists of two spots of opposite polarity. The line joining the two spots generally makes only a small angle with the solar equator. Each member of the group may be accompanied or replaced by many small spots, but the great majority of the spots constituting the preceding and following members of the group are of opposite magnetic polarity. One or more companion spots, of polarity opposite to that which characterizes the corresponding region of the group, sometimes occur in association with either the preceding or following member.

Bipolar spots may be divided into four groups, illustrated in Plates X and XI$\beta\gamma$:

(β) Those in which the leading and following members, whether single or multiple, are approximately equal in area.

(βp) Those in which the leading member is the principal member of the group.

(βf) Those in which the following member is the principal member of the group.

($\beta\gamma$) Those in which the preceding or following members are accompanied by minor companions of opposite polarity.

(γ) *Multipolar spots.* Groups of this character, comprising hardly more than 1 per cent of the total number of spots observed, contain spots of both polarities so irregularly distributed as to prevent classification as bipolar groups (see Plates VIII*c* and XIγ).

MAGNETIC CLASSIFICATION OF 970 SUN-SPOTS OBSERVED DURING 1915–1917

While it is obvious that the changes that occur during the life of a spot must preclude a hard-and-fast determination of type, it may be interesting to give the preliminary results of a classification by Nicholson of the spots observed since our present method of recording was put into effect in 1915. In presenting the following figures it should be stated that they are provisional, and therefore subject to revision in the light of further study. Table III is self-explanatory, giving the number of spots observed in both hemispheres during 1915, 1916, and 1917, classified according to the system already explained.

Several interesting facts are brought out by this table. Most notable of these is the strong tendency toward the bipolar type, indicated not only by the large percentage of bipolar groups, but also by the small proportion of symmetrical unipolar spots. Another striking fact is the dominance of preceding spots, shown by the high percentage of βp spots as contrasted with βf spots, and also indicated by a similar preponderance of αp over αf spots. The constancy of the percentages for the successive years, and the very small number of multipolar (γ) spots, should also be noted.

The results of magnetic observation, revealing the great preponderance of bipolar groups, are closely in harmony with the conclusion reached by Father Cortie in his valuable paper "On the Types of Sun-Spot Disturbances."[1] After describing the various types, he remarks:

> The chief type, however, of which the above mentioned are in most, possibly in all, cases but phases, is the double-spot formation, with a train of smaller spots between the two principal spots of the group, the whole group generally drifting into more or less parallelism with the solar equator. In this form the principal spot, which eventually becomes a normal spot of regular outline, is generally the leading spot, but in many cases it is the following spot, while sometimes the preponderance in area alternates between the two, as the group traverses the disk. In yet rarer instances both the chief spots develop as regular spots.

A discussion of the details of sun-spot formation and dissolution, and of the relationship between spot polarities and the structure of the accompanying hydrogen flocculi, must be reserved for another paper. Here it is our purpose to consider only such phases of these and other questions as bear directly upon the derivation of a law of sun-spot polarities.

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[1] *Astrophysical Journal*, 13, 260, 1901.

TABLE III

MAGNETIC CLASSIFICATION OF 970 SPOTS

		α	αp	αf	β	βp	βf	$\beta\gamma$	γ	Unclassified	Total
1915	N.	10	17	5	17	29	3	4	3	11	99
	S.	7	16	4	21	22	6	4	0	3	83
	Total	17	33	9	38	51	9	8	3	14	182
	Percentages	9	18	5	21	28	5	4	2	8	
Total percentages		32			58						
1916	N.	22	26	4	49	63	12	7	0	8	191
	S.	31	28	8	17	37	13	6	0	7	147
	Total	53	54	12	66	100	25	13	0	15	338
	Percentages	16	16	4	20	30	7	4	0	4	
Total percentages		35			61						
1917	N.	25	54	2	66	62	17	9	3	3	241
	S.	16	41	7	53	69	11	7	0	5	209
	Total	41	95	9	119	131	28	16	3	8	450
	Percentages	9	21	2	26	29	6	4	1	2	
Total percentages		32			65						

SUMMARY (PERCENTAGES)

	α	αp	αf	β	βp	βf	$\beta\gamma$	γ	Unclassified
1915	9	18	5	21	28	5	4	2	8
1916	16	16	4	20	30	7	4	0	4
1917	9	21	2	26	29	6	4	1	2
Mean	11	18	4	22	29	6	4	1	5
Total number of spots	111	182	30	223	282	62	37	6	37

POLARITIES OF SPOTS IN THE NORTHERN AND SOUTHERN HEMISPHERES

For the purpose of deriving a law of sun-spot polarities we may regard unipolar spots (αp) followed by a train of flocculi as preceding spots of incomplete bipolar groups, and those (αf) which are preceded by a train of flocculi as following spots of such groups. During the last sun-spot cycle (observations from June 1908 to December 1912) it was found in the great majority of spots observed that the "marked strip" of the compound quarter-wave plate transmitted the violet component of preceding spots in the Northern Hemisphere and of following spots in the Southern Hemisphere.[1] It naturally follows, from the characteristics of bipolar groups, that during the same period the marked strip transmitted the red component of following spots in the Northern Hemisphere and of preceding spots in the Southern Hemisphere. The numbers of spots observed are given in Table IV.

TABLE IV

	REGULAR			IRREGULAR			RANGE IN LATITUDE	AVERAGE LATITUDE
	N V_p-R_f	S R_p-V_f	Total	N	S	Total		
Last cycle........... (June 1908 to December 1912)	7	17	24	0	2	2	18° to 3°	9°

Thus while the bipolar characteristic of sun-spots introduces an element not encountered in the case of terrestrial storms, the opposition in polarity north and south of the solar equator is analogous to the opposite direction of whirl, invariably observed in the case of northern and southern cyclones and tornadoes.

After the sun-spot minimum, which occurred in December 1912, we found, to our surprise, that the polarity of the members of bipolar groups was opposite to that observed before the minimum. That is to say, the marked strip of the compound quarter-wave plate now transmits the red component of the preceding spots of bipolar groups in the Northern Hemisphere and the violet component of the preceding spots of bipolar groups in the Southern Hemisphere.

This sudden change was so remarkable that it was feared some observational error had been made. The results have been checked repeatedly by different observers, however, and in all cases the conclusion has been the same. The first observations of the former cycle were made with the 60-foot tower telescope and 30-foot spectrograph, but before the end of the cycle the 150-foot tower telescope and 75-foot spectrograph were put into commission, and this work was transferred from the old to the new tower. Thus the danger of any confusion due to the change of spectrographs and polarizing apparatus was eliminated, as the results obtained with the 150-foot tower telescope during the former cycle were in harmony with those secured with the 60-foot tower telescope during the same period. During the former cycle the observations were both visual and photographic, but since the beginning of the present cycle most of the polarities have been determined visually, though the results have frequently been checked photographically.

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[1] Few spots were observed during this cycle, because attention was then concentrated upon a small number of very large spots.

TABLE V

Present Cycle	Regular			Irregular			Range in Latitude	Average Latitude
	N R_p–V_f	S V_p–R_f	Total	N	S	Total		
1913, 1914.....	18	14	32	1	1	2	34° to 13°	22°
1915..........	73	68	141	2	0	2	29 " 2	19
1916..........	166	120	286	7	7	14	37 " 4	17
1917..........	221	193	414	8	6	14	30 " 1	14

The observations of the present cycle are given in Table V, which corresponds with Table IV except for the fact that in Table V the polarities are classed as "regular" when the marked strip transmits the red component in preceding spots of the Northern Hemisphere, whereas during the former cycle it transmitted the violet component in such spots.

In a preliminary paper describing the observed change of polarity at the last sun-spot minimum,[1] it was suggested that the explanation might be connected with the difference in latitude of the spots of the old and new cycles. The mean latitude of the spots observed during the old cycle was 9°, while that of the spots observed up to that date during the new cycle was 23°. It therefore seemed possible that there might be two zones on the sun, of low and high latitude, in which the polarities of spots were of opposite sign.

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[1] Hale, *Mount Wilson Communication*, No. 10; *Proceedings National Academy of Sciences*, 1, 385, 1915.

This view of the case, however, has not been borne out by time, as the results given in Table V indicate. The percentage of irregular spots has not increased materially during the present cycle, although their mean latitude has decreased from 22° in 1913–1914 to 14° in 1917. The maximum of solar activity has also definitely passed, so that it can hardly be supposed that any reversal of polarity will be observed before the close of the cycle, unless it should be found to lag considerably behind the maximum.

DISCUSSION OF RESULTS

The present investigation should ultimately lead to the formulation of a definite law of sun-spot polarities, from which the polarities of normal spots of any type can be predicted for either hemisphere and for any epoch in the sun-spot cycle. We already know that the preceding and following spots of binary groups, with few exceptions, are of opposite polarity, and that the corresponding spots of such groups in the Northern and Southern hemispheres are also opposite in sign. Furthermore, the spots of the present cycle are opposite in polarity to those of the last cycle (see Fig. 6, in which these results are expressed graphically). It is evident, however, that the formulation of a law of polarities cannot be undertaken until after the close of the present cycle, when it will be learned whether a reversal of sign is actually characteristic of the sun-spot minimum.

After an empirical law of spot polarities has been formulated, the problem of its interpretation will remain. The possibility of expressing such a law in the same terms that apply in the case of terrestrial storms depends upon the possession of data still beyond our reach. Thus the view that sun-spots are vortices depends upon the assumption that the magnetic field must be caused by electrically charged particles whirling in a vortex. Supposing that the existence of a true hydrodynamical vortex and the direction of the whirl within it can be determined by observations of the Evershed effect, or in some other way, the sign of the dominant charge will follow at once from our knowledge of the magnetic polarity of the spot in question. Or, if the sign of the charge can be independently found, the direction of the whirl will follow.

In Hale's first paper on the Zeeman effect in sun-spots, the sign of the dominant charge was found to be negative by proceeding on the assumption that the direction of whirl in the spot vortex coincides with that in the hydrogen vortex above it. A long series of observations of the hydrogen flocculi has since shown that this assumption is probably unwarranted. A good majority of the hydrogen whirls associated with preceding spots are right-handed in the Southern Hemisphere and left-handed in the Northern Hemisphere, as in the case of terrestrial storms, but there are many exceptions to this rule. Moreover, the hydrogen whirls showed no reversal of direction at the sun-spot minimum, and during the present cycle we have found cases in which either right-handed or left-handed hydrogen whirls are associated with spots of a given polarity. The whole subject is so complex that it will be advantageous to postpone further discussion until the great mass of observational material at our disposal can be more exhaustively studied.

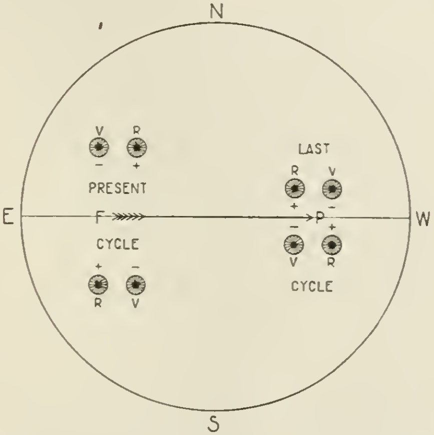

FIG. 6.—Diagram summarizing the results of polarity observations of sun-spots during the present and last cycles. The arrow indicates the direction of the sun's rotation; the letters *R* and *V*, the components of a normal triplet transmitted by the "marked strip" of the compound quarter-wave plate; and the algebraic signs, the distribution of the polarities between the preceding and following members of a bipolar group. Unipolar spots are normally of the same polarity as the preceding members of bipolar groups in the same hemisphere.

SUMMARY

1. The magnetic polarity of a sun-spot can be determined by observations, made with Nicol and quarter-wave plate, of the relative intensities of the *n*-components of a Zeeman triplet in its spectrum.

2. Such determinations of polarity can be made at almost any position on the sun's disk, but certain precautions must be taken to avoid error in the case of spots near the limb.

3. The inclination of the lines of force in sun-spots can be measured with considerable precision.

4. About 60 per cent of all sun-spots are binary groups, the single or multiple members of which are of opposite magnetic polarity.

5. Unipolar spots usually exhibit some of the characteristics of bipolar groups.

6. Before the last sun-spot minimum, the magnetic polarity of unipolar spots and of the preceding members of bipolar spots was positive in the southern and negative in the northern hemispheres of the sun.

7. Since the minimum these signs have been reversed.

8. The paper describes a scheme of classifying sun-spots on the basis of their magnetic properties.

9. The results of a magnetic classification of 970 spots observed during the years 1915–1917 are briefly summarized.

MOUNT WILSON SOLAR OBSERVATORY
November 11, 1918

THE MOTIONS IN SPACE OF SOME STARS OF HIGH RADIAL VELOCITY[1]

By WALTER S. ADAMS AND ALFRED H. JOY

The determinations of radial velocity made at the Lick and Mount Wilson observatories during the past few years have added greatly to the number of stars known to have large velocities in the line of sight. Most of these stars have well-determined proper motions and for many of them parallaxes have been measured by numerous observers. The motions in space of these stars are of interest because of the velocities with which they move and the wide range in spectral type and absolute magnitude which is found among them.

A list of all stars with radial velocities exceeding 80 km for which proper motions and parallaxes (derived either by the trigonometric or the spectroscopic method) are known is given in Table I. The stars are numbered consecutively, and the successive columns give the positions for 1900, their apparent visual magnitudes, spectral types, proper motions and direction of proper motion, radial velocities and radial velocities corrected for the solar motion. The latter are derived from the apex determined by Strömberg,[2] $A_0 = 270.^\circ 9$, $D_0 = +29.^\circ 2$, $V_0 = 21.5$ km. The apparent magnitudes are from Harvard determinations, the spectral types from Mount Wilson observations, and the proper motions have in most cases been taken from the catalogues of Boss and Porter.[3] The radial velocities are from the Lick and Mount Wilson determinations, the values being combined when the same star has been investigated at both observatories.

The parallaxes of all of the stars in the list with the exception of five have been derived from the absolute magnitudes determined spectroscopically at Mount Wilson. Small systematic corrections

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 163

[2] *Mt. Wilson Contr.*, No. 144; *Astrophysical Journal*, **47**, 7, 1918.

[3] *Publications of the Cincinnati Observatory*, No. 18, Parts 1–4, 1915–1918.

TABLE I

No.	Name	α (1900)	δ (1900)	m	Spec.	μ	ψ	V	V
								km	km
1	ε Andromedae	0^h33^m3	+28°46′	4.5	G5	0″336	222°	− 84	− 81
2	μ Cassiopeiae	1 1.6	+54 26	5.3	G5	3.760	114.5	− 97	− 91
3	Lalande 1966	1 3.3	+61 1	7.9	F5	0.638	86	−325	−318
4	Lalande 4855	2 32.6	+30 24	7.2	F8	0.625	231	− 99	−103
5	Lalande 5761	3 2.5	+25 58	8.0	A3p	0.861	194	−144	−151
6	W.B. 3^h617	3 35.3	− 3 32	7.2	F6	0.745	107	+114	+ 99
7	A.G.Berl. 1366	4 8.6	+22 6	8.9	F0	0.54	125	+339	+328
8	Groom. 864	4 34.5	+41 56	7.3	G2	0.690	127	+105	+ 99
9	Cordoba 5^h243	5 7.7	−44 59	9.2	G-K	8.75	131	+242	+222
10	Lalande 9960	5 14.1	− 3 11	8.5	K5	0.721	78	+ 86	+ 68
11	ϕ^2Orionis	5 31.4	+ 9 14	4.4	G7	0.321	163	+ 99	+ 82
12	δ Leporis	5 47.0	−20 53	3.9	G7	0.696	160	+ 99	+ 78
13	A.G.Berl. 1866	5 57.3	+19 23	9.0	F9	0.76	129	−190	−204
14	Boss 1511	5 59.2	−26 17	5.2	K3p	0.104	24	+183	+162
15	θ Can. Maj.	6 49.5	−11 55	4.2	K4	0.137	264	+ 97	+ 77
16	Groom. 1281	7 8.4	+47 25	5.6	G0	0.184	168	+ 88	+ 83
17	Lalande 15290	7 47.2	+30 55	8.2	F7	1.962	158	−242	−251
18	Lalande 19821	10 6.3	+24 15	8.5	G2	0.414	279	+ 81	+ 77
19	Lalande 21185	10 57.9	+36 38	7.6	Ma	4.779	187	− 87	− 85
20	Groom. 1830	11 47.2	+38 26	6.5	G8	7.047	145	− 97	− 92
21	Lalande 23995	12 47.9	−17 57	8.2	F3	0.877	159	+144	+144
22	Lalande 27274	14 54.2	−21 36	8.3	F4	0.785	229	+160	+168
23	A. Oe. 14318. A. Oe. 14320	15 4.7 15 4.7	−15 59 −15 54	9.6 9.2	K0 G8	3.682	196	+300	+310
24	Lalande 28607	15 37.7	−10 36	7.3	A2p	1.178	256	−170	−157
25	A.G.Leid. 5734	16 11.4	+32 24	(8.3)	K4	0.035	145	−164	−144
26	Boss 4188	16 22.3	− 7 22	5.4	Mb	0.176	168	+ 97	+112
27	Σ 2173	17 25.2	− 0 59	5.9	G6	0.214	215	− 81	− 63
28	W.B. 17^h514	17 29.9	+ 6 4	8.6	F5	0.623	311	−148	−129
29	Barnard's Star	17 53.0	+ 4 28	9.7	Mb	10.27	356	−106	− 87
30	31 b Aquilae	19 20.2	+11 44	5.2	G7	0.961	49	− 98	− 78
31	α Vulpeculae	19 24.5	+24 28	4.6	Ma	0.170	227	− 86	− 66
32	Lalande 37120	19 29.7	+32 59	6.6	F9	0.517	294	−162	−142
33	A. Oe. 20452	20 17.7	−21 40	8.1	F5	1.182	154	−179	−168
34	η Cephei	20 43.3	+61 27	3.6	G8	0.826	7	− 87	− 71
35	Boss 5482	21 16.6	+23 26	5.8	G9	0.252	123	− 88	− 73
36	W.B. 21^h502	21 24 5	−12 56	9.1	K5	1.052	105	− 90	− 81
37	W.B. 21^h594	21 26.0	+45 27	7.5	G8	0.56	50	− 83	− 67

have been applied to these values in accordance with an unpublished investigation by Strömberg. These corrections are as yet provisional and in nearly all cases their influence is slight. The directly measured parallaxes have been used in the case of stars numbered 5, 9, 23, 24, and 29. For star No. 27, which is a close visual binary, the apparent magnitude of the brighter component has been employed. The derivation of the parallaxes is from the formula $\log \pi = 0.2(M-m)-1$, and the values of the absolute magnitude and parallax are given in the second and third columns of Table II.

TABLE II

No.	M	π	A	D	x	y	z	v	L	B
					km	km	km	km		
1	+0.9	+0.019	211°	−29°	+26	−70	+44	87	290°	+30°
2	+5.7	0.120	112	−42	−115	−103	−28	156	222	−10
3	+3.9	0.016	152	−47	−155	−326	+49	364	245	+8
4	+4.4	0.028	275	−40	+117	−92	−35	153	322	−13
5	+6.0	0.039	247	−54	+89	−155	−13	179	300	−4
6	+4.4	0.028	103	−13	−148	−34	−11	152	193	−4
7	+3.0	0.007	101	−8	−489	−63	−28	494	187	−3
8	+6.0	0.055	94	+29	−93	+52	+14	108	151	+7
9	+11.7	0.319	123	−59	−121	−219	−57	257	241	−13
10	+7.0	0.050	121	+12	−87	+3	+38	95	178	+24
11	+1.0	0.021	94	−26	−92	−35	−29	102	200	−16
12	+1.4	0.032	126	−66	−45	−116	−31	128	249	−14
13	+5.6	0.021	230	−36	+134	−212	+74	262	302	+16
14	+0.6	0.012	95	−13	−163	−26	−30	168	189	−10
15	−0.4	0.012	69	−10	−77	+7	−49	91	175	−33
16	+5.5	0.095	116	+54	−47	+54	+43	84	131	+31
17	+5.0	0.023	217	−70	+73	−457	−70	467	279	−9
18	+6.0	0.032	115	+35	−73	+38	+41	91	153	+27
19	+10.8	0.437	328	−61	+31	−58	−68	95	298	−46
20	+6.5	0.100	243	−50	+180	−301	+2	351	301	0
21	+3.6	0.012	266	−66	+143	−321	−122	372	294	−19
22	+3.8	0.013	156	−45	−128	−289	+62	322	246	+11
23	{+7.8 7.4}	0.044	200	−64	+38	−489	−10	491	274	−1
24	+4.9	0.033	98	0	−221	+4	−8	221	179	−2
25	+0.1	0.002	33	−51	−40	−71	−144	166	241	−60
26	−0.6	0.006	266	−51	+101	−128	−35	167	308	−12
27	+5.1	0.069	86	−1	−61	+6	−14	63	174	−13
28	+4.4	0.014	130	+31	−182	+68	+148	245	159	+37
29	+13.4	0.540	92	+45	−92	+89	+30	132	136	+13
30	+5.4	0.110	94	+11	−86	+20	−2	89	167	−1
31	+0.4	0.014	158	−42	−36	−78	+23	89	245	+15
32	+5.1	0.050	134	−24	−116	−89	+40	152	217	+15
33	+4.0	0.015	96	−36	−314	−194	−130	391	212	−19
34	+3.0	0.076	136	−29	−60	−56	+19	84	223	+13
35	0.0	0.007	63	−39	−107	−60	−123	174	209	−45
36	+8.7	0.083	112	+9	−88	+5	+23	91	177	+15
37	+5.9	0 048	120	−17	−73	−33	+12	81	204	+9

The computation of the equatorial velocity-components referred to the sun, ξ, η, and ζ, has been made from the formulae

$$\xi = V \cos \alpha \cos \delta - \frac{k}{\pi}(\mu_1 \sin \alpha + \mu_2 \cos \alpha \sin \delta)$$

$$\eta = V \sin \alpha \cos \delta + \frac{k}{\pi}(\mu_1 \cos \alpha - \mu_2 \sin \alpha \sin \delta)$$

$$\zeta = V \sin \delta + \frac{k}{\pi}\mu_2 \cos \delta$$

In these expressions V is the radial velocity, $k=4.737$ and $\mu_1 = \mu_\alpha \cos \delta$, $\mu_2 = \mu_\delta$ where μ_α and μ_δ are the components of the proper motion. To reduce these values to the usually adopted centroid of the stars by eliminating the solar motion, we put:

$$\xi_0 = V_0 \cos A_0 \cos D_0$$
$$\eta_0 = V_0 \sin A_0 \cos D_0$$
$$\zeta_0 = V_0 \sin D_0 .$$

in which V_0 is the solar motion, and A_0 and D_0 the right ascension and declination of its apex. We then have for the equatorial components of the star's space-velocity,

$$\xi_1 = \xi + \xi_0 = v \cos A \cos D$$
$$\eta_1 = \eta + \eta_0 = v \sin A \cos D$$
$$\zeta_1 = \zeta + \zeta_0 = v \sin D$$

From these equations we calculate not only ξ_1, η_1, and ζ_1, but also v, A, and D, the space-velocity of the star and the co-ordinates of its apex. Using the values already referred to we have

$$\xi_0 = 0, \qquad \eta_0 = -19, \qquad \zeta_0 = +10$$

The components of velocity with reference to the galactic system are found from the equations

$$x = v \cos L \cos B = +0.1846\ \xi_1 - 0.9828\ \eta_1$$
$$y = v \sin L \cos B = +0.4494\ \xi_1 + 0.0844\ \eta_1 + 0.8893\ \zeta_1$$
$$z = v \sin B \qquad = -0.8740\ \xi_1 - 0.1642\ \eta_1 + 0.4573\ \zeta_1$$
$$v^2 = x^2 + y^2 + z^2.$$

The numerical coefficients result from the assumed values $\alpha = 190.^\circ 6$, $\delta = +27.^\circ 2$ for the pole of the galaxy.

The results of the computations are given in Table II. A and D are the right ascensions and declinations of the apices, x, y, and z the components of velocity referred to the galaxy, v is the total velocity, and L and B the galactic co-ordinates of the apices. The values are shown graphically in Fig. 1, the plane used being that of the galaxy. The apices are shown in projection, and the vectors drawn from the origin represent the direction and amount of motion in this plane.

Some of the conclusions to be drawn from these results may be summarized as follows:

1. The components of velocity normal to the plane of the galaxy are much smaller than those in the plane. Thus we find from Table II the average values:

$$x = 115 \text{ km}, \qquad y = 119 \text{ km}, \qquad z = 46 \text{ km}.$$

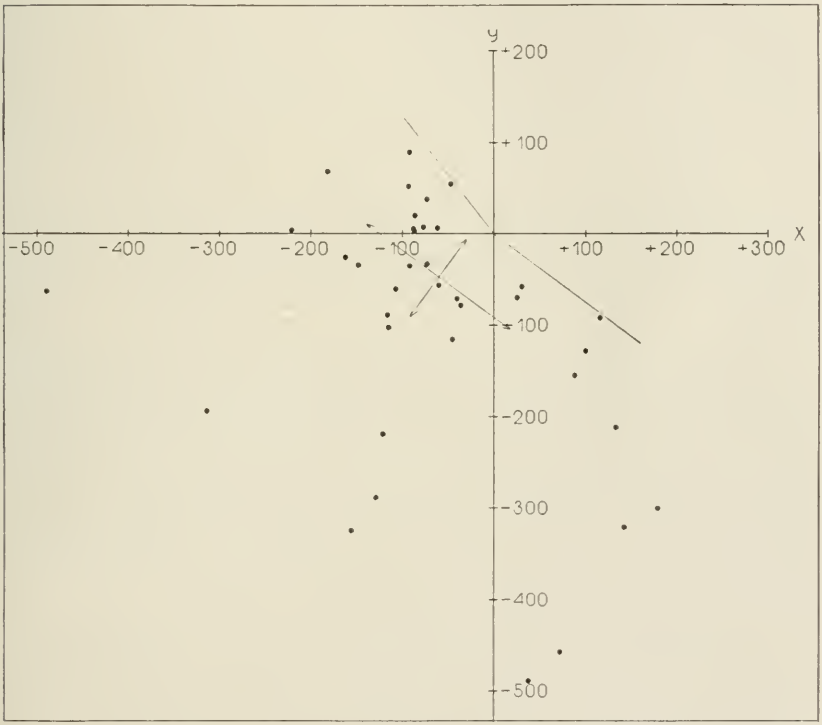

FIG. 1.—Projection on galactic plane of apices of motion of thirty-seven stars of high radial velocity. The vectors drawn from the origin represent the projected velocities in km per sec. The axes of the ellipse of intersection of the velocity-ellipsoid with the galactic plane are indicated by the two arrow-headed lines.

The same result follows from a consideration of the latitudes B of the apices. The latitudes of only six of the thirty-seven stars exceed 30°.

2. Nearly an entire hemisphere is devoid of apices, the galactic longitudes all lying between 131° and 322°.

3. The velocity of the centroid of these stars is remarkably high. Thus omitting those with total velocities greater than 300 km we find from the remaining twenty-nine stars:

$\bar{x}=-59.5$ km $\bar{y}=-44.5$ km $\bar{z}=-4.4$ km
$v=74.4$ km $L=217°$ $B=-3°$

4. If we project the velocities of these twenty-nine stars upon axes through this center such that the sum of the squares of the projected velocities is a maximum and a minimum we find the following directions[1] and the mean square dispersion σ:

Maximum: $L=141°$ $B=+9°$ $\sigma=100.6$ km
Minimum: $L=61°$ $B=-49°$ $\sigma=46.1$ km
Intermediate: $L=43°$ $B=+39°$ $\sigma=61.4$ km

The directions of the smaller axes are necessarily uncertain. Thus it is clear that the effect of stream motion among these stars is very marked.

5. It is of interest to note that the direction of the major axis as derived from these stars is in close agreement with that found by Strömberg from a discussion of the radial velocities of 260 dwarf stars ($L=148°$, $B=+14°$), and that of Raymond[2] from 559 stars of large proper motion ($L=146°$, $B=+8°$). All three of these investigations agree in indicating that the galactic longitude of the principal vertex for the stars of high velocity is considerably less than that for stars in general, which is about 170°.

6. An interesting feature of the results, but one which must be accepted with some reserve, is the apparent tendency of the stars to move along a line of galactic longitude about 260°, the assumption being made of a motion of the centroid of $\bar{x}=-100$ km, $\bar{y}=-20$ km. This direction coincides nearly with that of the greatest star-density as determined by Charlier, Walkey, Nort, and Plummer.

7. Reference may perhaps also be made to the tendency shown by the stars of the very highest velocity (over 300 km) to move along a line parallel to that of the major axis, referring the motion

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[1] The intersection of this velocity-ellipsoid with the galactic plane is indicated in Fig. 1 by straight lines which show the axes of the ellipse of intersection.

[2] *Astronomical Journal*, 30, 191, 1917.

to their common center. Thus all of the stars omitted from the previous discussion show a very fair degree of alignment with the direction of the principal axis as given in Fig. 1.

8. A marked difference is seen in the average galactic latitudes of the apices of the stars of high and low luminosity, the latter lying more nearly in the plane of the galaxy. Thus if we select the stars of absolute magnitude brighter and fainter than 3.0 we find:

	Average *M*	Average *B*
9 stars	+0.4	26°
28 stars	5.9	14°

The galactic latitudes of the apices of only four of the fainter stars exceed 26°.

9. A similar result is found for the average velocities of the stars of high and low luminosity, the latter stars moving more rapidly.

	Average *M*	Average *v*
9 stars	+0.4	130 km
28 stars	5.9	216 km

10. An examination of the spectral types of these stars shows that an extraordinarily large proportion, twenty-six out of thirty-seven, are of types F and G. The stars of type F have by far the largest average space motion, nearly double that of the stars of type G. The comparison, which is of little weight for types A, K and M, gives:

No. of Stars	Type	Average *v*	Average *V′*
		km	km
2	Ap	200	115
11	F	307	187
15	G	156	104
5	K	122	106
4	M	121	88

Among the stars of type F it is of interest to note that those of earlier spectrum show the largest velocities. Thus the six stars with spectra between F0 and F5 inclusive have space motions of 364, 494, 372, 322, 245, and 391 km, an average value of 365 km. The average value of V′ for these stars is 209 km.

Mount Wilson Solar Observatory
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THE ORBITS OF THREE SPECTROSCOPIC BINARIES[1]

By WALTER S. ADAMS and ALFRED H. JOY

THE ORBIT OF BOSS 593

The two stars Boss 592 and 593 ($\alpha = 2^h\ 31^m.2$; $\delta = +24^\circ\ 13'$; 1900) form the well-known double star 30 Arietis, or Σ 5, with a distance of 38″ and a common proper motion of $0''.146$ annually. The visual magnitudes of the stars are 7.4 and 6.6 respectively, and the spectral types are very similar, being F5 and F4 according to our most recent determinations. The brighter star of the pair, Boss 593, was found in 1916 to be a spectroscopic binary, and the number of spectrograms is now sufficient to make a determination of its orbit readily possible.

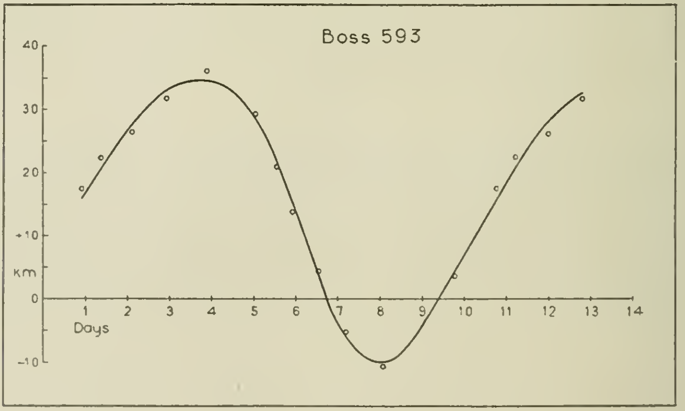

FIG. 1

All of the spectrograms were secured with the Cassegrain spectrograph and the 60-inch reflector, a dispersion of one prism and a camera of 18 in. (45.7 cm) focal length being used throughout the observations. The spectrum is not of the best quality for measurement, and the lines are broad and somewhat hazy. For this reason, and because of the low dispersion employed, the computation of the orbit has been based upon normal positions, the radial velocities for spectrograms taken at nearly the same phase being combined to give mean values.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 164.

Since the observations extend over an interval of three years, it has been possible to determine the period with considerable accuracy. This is found to be closely 9.851 days. With the aid of this period the phases have been determined and the radial velocities combined into the normal values given in Table I. The epoch selected is December 20.0, 1917, Julian day 2421583.0 G.M.T.

TABLE I

No.	No. of Plates	Phase	V	$O-C_1$	$O-C_2$	$O-C_3$
		Days	km	km	km	km
1.........	2	0.910	+36.2	+1.1	+2.1	+1.5
2.........	2	2.008	+29.4	+0.8	+1.0	+0.9
3.........	2	2.531	+21.0	−0.4	−0.9	−0.8
4.........	3	2.931	+14.2	−0.4	−1.1	−0.9
5.........	4	3.551	+ 4.3	+0.6	+0.3	+0.5
6.........	4	4.188	− 5.3	0.0	+0.2	+0.5
7.........	1	5.081	−10.5	−0.5	−1.1	−0.4
8.........	2	6.778	+ 3.8	−1.2	−1.3	−0.9
9.........	2	7.759	+17.6	+1.0	+1.6	+1.5
10.........	1	8.207	+22.3	+1.3	+1.7	+1.3
11.........	2	8.976	+26.0	−1.2	−1.2	−1.8
12.........	2	9.784	+31.5	−0.8	−0.7	−1.4

The method of calculation adopted is that involving the use of a Fourier's series, the details of which are given by Plummer,[1] Russell,[2] and others. The mean radial velocities and phases of Table I were plotted, and a smooth curve was drawn through the corresponding points. From this curve were read off the radial velocities V_0, V_1 . . . . V_{11} for each twelfth part of the period. The following equation was then assumed:

$$V=b_0+b_1\cos\tau+b_2\cos 2\tau+a_1\sin\tau+a_2\sin 2\tau.$$

The value of τ being 30°, the computation of the coefficients is made readily from the relationships

$$b_0=\tfrac{1}{12}\sum_{0}^{11}V_n;\quad b_1=\tfrac{1}{6}\sum_{0}^{11}V_n\cos n.30^\circ;\quad b_2=\tfrac{1}{6}\sum_{0}^{11}V_n\cos n.60^\circ;\ \text{etc.}$$

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[1] *Astrophysical Journal*, 28, 212, 1908.

[2] *Ibid.*, 15, 252, 1902.

The resulting value for the series is:

$$V=+14.42+21.31 \cos \tau-3.08 \cos 2\tau+4.96 \sin \tau+15.1 \sin 2\tau.$$

The normal values of the radial velocities are well represented by this expression, as the residuals given in Table I under $O-C_1$ show.

Approximate values of the elements may be derived from the coefficients of this series by means of the relationships given by Plummer. Neglecting the second and higher powers of the eccentricity we find:

$\gamma=+14.4$ km
$K=21.9$ km
$\omega=127^\circ.6$
$e=0.157$
$T=$ J.D. 2421586.86
P (assumed) $=9.851$ days
$a \sin i=2,925,000$ km
$$\frac{m_1^3 \sin^3 i}{(m+m_1)^2}=0.010 \odot$$

The values of the radial velocity calculated from these elements show the differences from the observed values given in Table I under $O-C_2$.

Although this representation is satisfactory in view of the degree of accuracy obtained in the measurements of the radial velocity, it seemed desirable to carry out a least-squares solution for the purpose of obtaining the probable errors of the various elements. In this solution the period P was assumed to be constant, and corrections were determined for γ_0, ω, e, K, and T. The final values of the resulting elements are as follows:

$\gamma=+14.42\pm0.71$ km
$K=22.48\pm0.47$ km
$\omega=129^\circ.6\pm7^\circ.3$
$e=0.1455\pm0.0185$
$T=$ J.D. 2421586.899 ± 0.196
$a \sin i=3,012,700$ km
$$\frac{m_1^3 \sin^3 i}{(m+m_1)^2}=0.0113 \odot$$

The differences between the observed radial velocities and those computed from these elements are shown in Table I under $O-C_3$.

The parallax of Boss 593 has been determined by van Maanen and by Miller. The weighted mean of the results gives an absolute magnitude of 4.2.

THE ORBIT AND ABSOLUTE DIMENSIONS OF W URSAE MAJORIS[1]

The variable star W Ursae Majoris ($\alpha=9^{\mathrm{h}}\,36^{\mathrm{m}}\!.7$; $\delta=+56^\circ\,25'$; 1900) has attracted much interest since its discovery in 1903 by Müller and Kempf, who found its light to vary in the unusually short period of four hours. The variation was found to be continuous, with very sharp minima and rather flat maxima. The peculiar shape of the light-curve led Russell to apply the theory of an eclipsing binary, and he thus found that the star's light-changes might be accounted for by assuming that the period should be doubled, and that the primary and secondary eclipses produced equal minima. This result has not been as widely accepted as the evidence would warrant, probably on account of the exceptionally short period and the spectral type, which was given by the Harvard observers as G0. Hence the star is still given in some lists as a Cepheid or cluster-type variable.

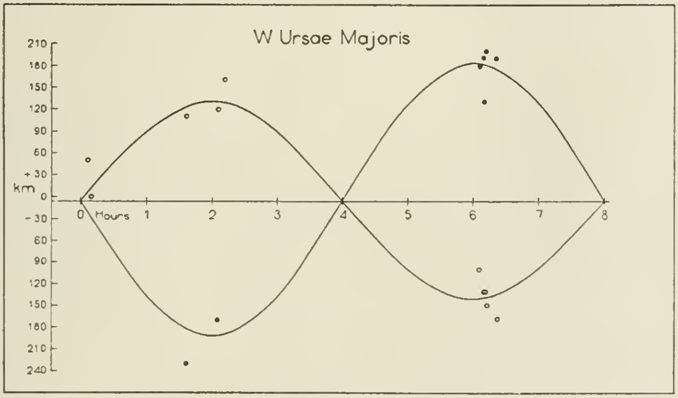

FIG. 2

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[1] An abstract of this paper was read at the meeting of the American Astronomical Society, August 20–22, 1918.

The star was placed on the Mount Wilson spectrographic observing list, together with other stars of the later types having measured trigonometrical parallaxes, for the determination of absolute magnitude and parallax. Ten plates were secured from December, 1917, to March, 1918, with exposures varying from 30^{m} with the 7-inch camera to 2^{h} 15^{m} with the 18-inch camera. The spectra of both components appear, the lines of the primary being slightly stronger than those of the secondary. The spectrum is most extraordinary. It appears to belong to the solar type and might be classified as F8p. The faint lines are entirely obliterated, and the strong lines are so widened and weakened that measures for velocity and estimates of line-intensity can be made only with the greatest difficulty. The results accordingly are given only as the best approximations which can be determined from the plates. The results of the spectrographic measures, together with the photometric orbits by Russell and Shapley, show that the unusual character of the spectral lines is due partly to the rapid change in velocity during even our shortest exposures but mainly to the rotational effect in each star, which may cause a difference of velocity in the line of sight of as much as 240 km between the two limbs of the star.

TABLE II

Plate	Date	G.M.T.	Exp.	Phase	Velocity of Primary	Velocity of Secondary
					km	km
6417.......	1917 Dec. 1	$22^{h}45^{m}$	135^{m}	$6^{h}.4$	−170	+190
6433.......	3	22 35	60	6.1	−130	+190
6443*......	5	0 13	115	0.1	.....	.....
6521.......	1918 Jan. 1	1 13	45	0.2	.....	.....
6524†......	1	19 15	30	2.2	+160	(−260)
6526.......	1	23 15	30	6.2	−150	+200
6538.......	2	23 11	38	6.1	−100	+180
6692.......	Feb. 1	19 50	56	2.1	+120	−170
6695.......	1	23 55	60	6.2	−130	+130
6820.......	March 31	20 35	70	1.6	+110	−230

* Lines of two spectra blended. Measured $V=+50$ km.

† Lines of two spectra blended. Measured $V=0$ km.

Table II gives a summary of the observations. Three plates were taken at maximum light when the primary star was receding, five at the opposite maximum when the primary was approaching, and two near the even minima. The phases were computed from the recent light-elements by Shapley and Van der Bilt,[1] using the period of 0.3336392 day. The velocities are corrected to give the effective velocity for the whole exposure and are rounded off to the nearest ten kilometers on account of the character of the spectrum. They are the means of the measures of three and in a few cases of four observers. On plate 6524 the second component was measured by only one observer. and the value is omitted.

The following orbital elements were determined after grouping the observations according to phase and assigning weights according to the character of the lines:

Eccentricity, e (assumed) $= 0.00$
Velocity of center of mass, $\gamma = -5 \pm 13$ km/sec.
Semi-velocity range of primary, $K_1 = 134 \pm 5$ km/sec.
Semi-velocity range of secondary, $K_2 = 188 \pm 15$ km/sec.
Radius of orbit of primary, $a_1 \sin i = 610{,}000$ km
Radius of orbit of secondary, $a_2 \sin i = 860{,}000$ km
Mass of primary, $m_1 \sin^3 i = 0.67 \odot$
Mass of secondary, $m_2 \sin^3 i = 0.48 \odot$
Ratio of masses, $\frac{m_2}{m_1} = 0.71 \odot$

With the aid of Russell's solution of the photometric orbit as given by Shapley[2] we may now compute the absolute dimensions of the orbit and the individual stars and find the masses and densities of each star, thus giving data which are known for only a very few stars.

Taking the "uniform" solution which Russell suggests as the most probable because of the color-indexes found by Shapley, we note that the stars are ellipsoids with the same dimensions. The inclination $i = 77° 35'$, and the ratio of the axes of the ellipsoids are given, whence we find:

Radius of the primary orbit $= 620{,}000$ km
Radius of the secondary orbit $= 880{,}000$ km

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[1] *Mt. Wilson Contr.*, No. 140; *Astrophysical Journal*, 46, 290, 1917.

[2] *Contributions from Princeton Observatory*, No. 3.

Radius of the relative orbit = 1,500,000 km
Greatest semi-axis of each star = 548,000 km = 0.78 sun's radius
Intermediate semi-axis of each star = 408,000 km = 0.58 sun's radius
Polar semi-axis of each star = 373,000 km = 0.54 sun's radius
Mass of primary = 0.69 $\odot$
Mass of secondary = 0.49 $\odot$
Density of primary = 2.8 $\odot$
Density of secondary = 1.9 $\odot$

The densities are remarkable as being the highest known in any stellar system. The projected surface area of the two stars is equal to about 90 per cent of the sun. If we should assume that the same surface luminosity prevails on the stars as on the sun, as we should expect in similar spectral types, the absolute magnitude would be about 4.8 (calling the sun 5.0) and the parallax 0ʺ024.

The trigonometric parallax has been measured at the Yerkes Observatory and found to be 0ʺ012, corresponding to an absolute magnitude of 3.3. and our rough estimates from the very poor spectral lines seem to agree with it. If then we accept the measured parallaxes, we find the surface luminosity of the stars to be four times that of our sun.

THE ORBIT AND ABSOLUTE DIMENSIONS OF Z HERCULIS

The Algol variable Z Herculis ($\alpha = 17^h\ 53^m\!.6$; $\delta = +15^\circ\ 9'$; 1900) is well known because its period is so nearly equal to four days that it has been found difficult to make complete observations of its light and velocity changes at any one observatory. In the course of the observation of stars with measured parallaxes nine spectrograms have been secured at Mount Wilson. These observations have been timed in such a way that they allow a determination of the spectrographic orbit, if it is assumed to be circular in accordance with the photometric orbit computed by Shapley.[1] Both the photometric and spectrographic results indicate that in general the Algol variables move in orbits which are nearly circular. The star R Canis Majoris,[2] $e = 0.138$, is the only one thus far found to have an eccentricity of more than 0.100. Plate 6942, which was taken near minimum of light, was of inferior quality and was not used in the solution.

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[1] *Contributions from Princeton Observatory*, No. 3, p. 100.

[2] Jordan, *Publications of the Allegheny Observatory*, 3, 49, 1912.

Two spectrograms taken by Frost[1] in 1905 give velocities which fall fairly well on our curve and show that the period has been accurately determined. Our photographs show the presence of two spectra of type F2. The lines of the secondary spectrum are only about one-half as strong as those of the primary, and many are poorly defined or not seen at all on some of the plates. The orbit of the secondary is consequently less accurately determined. Although the results show that there is a marked difference in surface luminosity between the two stars, the spectra appear to be essentially of the same type. The rotational effect is so small that the spectral lines are fairly sharp, and measures of the primary show good agreement. The velocity of the center of mass is unusually large.

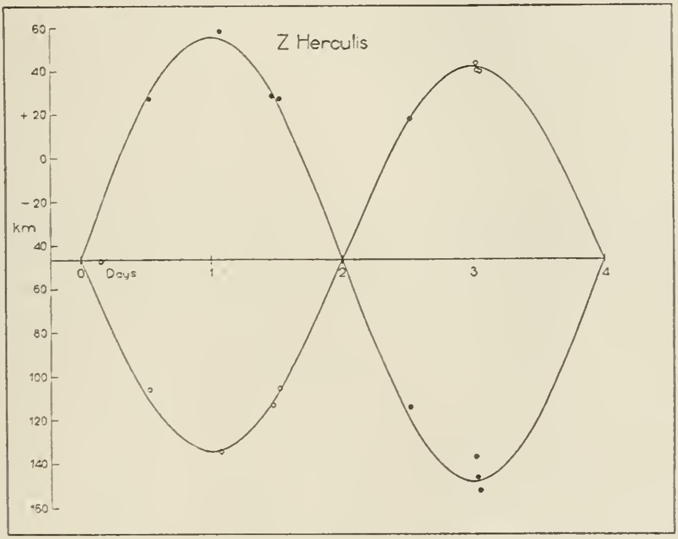

FIG. 3

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[1] *Astrophysical Journal*, 22, 214, 1905.

The following elements were determined, the orbit being assumed to be circular:

Photometric period, $P = 3.992775$ days (Lehnert)
Eccentricity, $e = 0.00$ (assumed)
Velocity of center of mass, $\gamma = -46.5 \pm 0.3$ km/sec
Semi-velocity range of primary, $K_1 = 88.2 \pm 0.4$ km/sec
Semi-velocity range of secondary, $K_2 = 101.8 \pm 1.4$ km/sec
Radius of primary orbit, $a_1 \sin i = 4{,}800{,}000$ km
Radius of secondary orbit, $a_2 \sin i = 5{,}600{,}000$ km
Mass of primary, $m_1 \sin^3 i = 1.5 \odot$
Mass of secondary, $m_2 \sin^3 i = 1.3 \odot$
Ratio of masses, $\frac{m_2}{m_1} = 0.86$

The photometric orbit has been computed by Shapley[1] from observations by Wendell at Harvard. On account of the peculiarity of the period the observations of different portions of the light-curve had to be made in different seasons, but the results were considered fairly reliable. The "uniform" solution gives a grazing total eclipse and the "darkened" solution a partial eclipse, with the larger star in front in both cases. The larger star is the fainter. The secondary eclipse is very shallow. Combining the results of the spectrographic observations with the photometric we obtain the absolute dimensions of the orbits and of the individual stars.

TABLE III

Plate	Date	G.M.T.	Phase	V_1	V_2
			Days	km	km
6265	1917 Oct. 5	15^h46^m	0.514	−106.0	+27.1
6273	6	15 44	1.512	−106.8	+27.5
6300	26	14 13	1.466	−113.0	+28.6
6357	Nov. 4	14 38	2.507	+17.4	−114.7
6933	1918 May 25	19 10	1.075	−134.9	+58.0
6939	27	18 40	3.054	+39.5	−152.7
6942	28	21 5	0.162	....	−47.2
7070	June 24	16 55	3.031	+43.7	−137.0
7134	July 18	16 17	3.049	+39.8	−146.4

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[1] *Contributions from Princeton Observatory*, No. 3, 1915.

The photometric observations are not sufficient to give the relation of the axes of the ellipsoids.

TABLE IV

	"Uniform" Solution	"Darkened" Solution
Inclination of orbit	83° 40′	82° 0′
Radius of primary orbit	4,870,000 km	4,880,000 km
Radius of secondary orbit	5,620,000 km	5,640,000 km
Radius of relative orbit or distance of centers	10,490,000 km	10,520,000 km
Radius of bright primary star	1,020,000 km	1,230,000 km
Radius of faint secondary star	2,170,000 km	2,290,000 km
Mass of primary	1.5 ⊙	1.6 ⊙
Mass of secondary	1.3 ⊙	1.3 ⊙
Density of primary	0.5 ⊙	0.3 ⊙
Density of secondary	0.05 ⊙	0.04 ⊙

The absolute magnitude of the brighter star determined from the spectra is +2.4, indicating a parallax of 0″.010, which is in good agreement with the hypothetical parallax. The trigonometric parallax as determined at the Yerkes Observatory is +0″.033.

Mount Wilson Solar Observatory
February 1919

MINOR CONTRIBUTIONS AND NOTES

THE SPECTRUM OF COMET MELLISH (1915*a*)

Two series of objective-prism plates were taken at this observatory, beginning June 5 and ending August 20, 1915. The series by Mr. R. Winter was taken with a 60°-prism attached to a two-inch lens of six inches focal length, giving a linear dispersion of 9 mm between λ 3600 and λ 5200.[1] The other series, by Mr. F. Symonds, was taken with a 20°-prism placed before a five-inch lens of 25 inches focal length, giving about three-fourths as much dispersion.[2] The plates are Capelli, extra rapid. Tables I and II give a brief record of the plates.

This series includes plates taken in July and August, but the images are weak and show practically nothing of the tail.

The spectrum of a star giving the usual hydrogen lines was selected as a standard of dispersion for the plate, and a bright knot at the violet end of the comet spectrum was assumed to be at λ 388. Simple estimation of the relative positions of the comet's radiations and the hydrogen lines gives the following approximate wave-lengths and their identification:

Description		Identification
λ 388	very bright knot	Third cyanogen band
400	tail band	Low pressure CO
403	knot	
422	faint knot	Second cyanogen band
427	tail band	Low pressure CO
436	knot	Fifth carbon band
456	tail band	Low pressure CO
468	bright knot	Fourth carbon band

It is at once evident that, except for minor differences in relative intensities, the spectrum of Comet Mellish is an excellent reproduction of the spectrum of Comet Brooks (1911*c*), which was studied in considerable detail by W. H. Wright.[1]

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[1] Attached to the astrographic telescope.

[2] From June 5 to July 8, inclusive, the apparatus was attached to the 12-inch equatorial, then it was put on the Heele photographic telescope.

TABLE I

SERIES BY F. SYMONDS

Plate	1915	Exp.	Description
6151	June 7	$1^h 20^m$	Image strong, but not sharp. 2° tail
6158	11	2 0	4° tail
6162	12	0 33	
6163	12	2 0	4° tail
6170	14	0 30	
6171	14	1 30	5½° tail
6194	July 6	1 50	Poor focus, plate fogged, no tail
6197	8	2 0	Poor focus, plate fogged, no tail
6198	17	1 50	Tail short
6202	20	2 30	Plate fogged
6205	21	1 27	2° tail
6214	Aug. 11	1 55	2° tail
6217	12	2 05	Plate fogged
6218	15	2 0	Plate fogged
6228	18	2 0	Tail short
6231	19	2 0	2° tail

TABLE II

SERIES BY R. WINTER

Plate	1915	Exp.	Description
6143	June 5	$1^h 0^m$	Short tail
6146	6	1 29	Short tail
6147	6	0 35	
6149	7	1 59	
6153	10	2 17	
6154	10	0 30	
6156	11	2 0	Tail longer
6157	11	0 31	
6162	12	1 34	
6163	12	0 31	
6167	14	1 44	
6169	14	0 31	
6172	16	1 43	9° tail
6174	16	0 43	
6175	17	1 30	9° tail
6177	17	0 33	
6178	20	1 06	9° tail
6180	22	2 0	Plate of same density as No. 6178 but tail is very short
6182	22	0 33	
6183	23	0 02	Very dense, tail very short
6186	24	1 34	Tail very short

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[1] *Lick Observatory Bulletin*, No. 7, 8–16, 1912.

There is a pronounced increase in intensity and length of the tail band from June 5 to June 17, and it might be expected that these series of plates would show the relation of the development of the tail bands to the comet's approach to the sun. Unfortunately they do not. Perihelion occurred on July 17, but from June 22 on the plates show little or no tail. Only two plates, one on August 11 and one on August 19, show as much as two degrees of tail. The general weakness of the images is largely due to the comet's increasing geocentric distance, and in the latter part of July to a fogging of plates by moonlight. Yet there seems to have been an abrupt decrease in intensity of the spectrum between June 20 and June 22, which is not confirmed on the direct photographs taken on these dates.

The continuous spectrum extends from about λ 3600 to λ 5200 A, but is very weak on either side of λ 388.

The knots at λ 388 and λ 468 are always considerably the strongest, but they vary in relative intensity. Early in June λ 388 was considerably the stronger. About the middle of June λ 388 was scarcely any brighter, if at all. On June 22 and following, λ 388 was always considerably the brighter.

The first few dates in June show the knot at λ 403 rather strong, but it gradually diminished in brightness until it was hardly distinguishable. The spectrum of Comet Mellish was observed at the Lowell Observatory by Slipher,[1] and he gives a group of lines in the region for which my measure is doubtless a mean. The lines which Slipher measures are also found in Comet Zlatinsky (1914*b*) and Comet Brooks (1911*c*). It seems not improbable that they can be identified with a new hydrocarbon band recently detected in the region λ 4107 and λ 4025 by C. W. Raffety.[2]

Between June 14 and June 16 the comet was subject to a pronounced outburst. From a straight quiescent type of tail there developed suddenly two bright tails, one long and divergent, the other short, curved, and well inclined to the main tail. The division of the tail is well marked on the spectrogram for the night.

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[1] *Lowell Observatory Bulletin*, No. 74, **2**, 151, 1916.

[2] *Phil. Mag.* (6), **32**, 555, 1916.

The bright cyanogen knot at λ 388 extends in the direction of both tails.

A similar action took place on June 20 and is faithfully recorded in the cyanogen band at λ 388.

A. ESTELLE GLANCY

OBSERVATORIO NACIONAL ARGENTINO, CÓRDOBA
February 20, 1918

SOME PECULIARITIES OF NOVA AQUILAE III OF 1918

At one stage of its decline Nova Persei of 1901 presented a perfectly planetary disk in the large telescope. Its light was also dull and planetary, resembling that of Neptune and some of the larger asteroids.[1] One would have instantly picked it out as differing from all the other stars.

Nova Aquilae III of 1918 has similarly shown in its decline a sharply defined planetary disk of measurable size wholly different from the image of any ordinary star. This disk was measured on a number of nights. Though in most cases the atmospheric conditions were not good for such work, the results, I believe, are fair. No one would have failed at a glance to recognize the nova as entirely different from the other stars. The striking difference between it and the ordinary star was apparent on each night by an examination of B.D.+0°4027 and 4 Aquilae, and frequently θ Serpentis. These stars showed the ordinary stellar image. Following are the measures of the diameter of the disk.

Date	C.S.T.		Diam.	
1918 Oct. 5	6^h	0^m	0″81	
12	5	40	0.54	
15	5	40	0.60	
Nov. 2	6	0	1.36	
5	5	20	1.26	
12	5	40	1.30	
23	5	15	1.02	
26	5	5	1.18	
Dec. 14	5	20	1.82	Very low
17	5	20	1.86	Very low
	Mean........		1.18	

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[1] See *Astrophysical Journal*, 14, 155, 1901.

This disk seemed to become larger as if growing, as shown by the measures. The conditions were seldom good enough for the best results. I think, however, that these diameters will fairly represent the size of the image. The following notes were made at the time of the measures, Central Standard Time being used.

1919. Sept. 17, 6^h40^m. The nova has a small planetary disk of a pale yellowish-white color. Clouds prevented measures.

Oct. 5, 6^h0^m. The light of the nova is dull and planetary and of a slightly yellowish cast. The image is a perfectly planetary disk entirely different from the image of B.D. $+0°4027$. The out-of-focus image is scarlet and well defined and smaller than the focal image. This scarlet image is surrounded by a greenish-yellow glow $2''$–$3''$ in diameter.

Oct. 12, 5^h40^m to 6^h0^m. The disk is planetary, pale white or slightly yellowish, and is imbedded in a broken yellow glow. Measures of the focus of the nova and of B.D.$+0°4027$ showed only 0.2 mm difference.

Oct. 15, 6^h0^m. The disk is beautifully planetary and sharply defined, dull white with a suggestion of yellow. It is surrounded by a close yellowish halo. The crimson image at the second focus is smaller than the regular image and well defined. It is surrounded by a greenish-white halo.

Nov. 2, 6^h0^m. Seeing bad. The image is planetary—a yellowish-white disk. The crimson image is strong. Clouds.

Nov. 5, 5^h20^m. Planetary; yellowish-white disk.

Nov. 12, 5^h35^m. The image is pale yellowish color with much strong light about it. Seeing very poor.

Nov. 23, 5^h15^m. Planetary disk. Pale white color with only a suggestion of yellow. Seeing very poor.

Nov. 26, 5^h10^m. Planetary disk well seen in moments of steadiness. Pale white with yellowish cast.

Dec. 14, 5^h20^m. Seeing very poor. The disk is planetary in moments of steadiness.

Dec. 17, 5^h20^m. The disk is planetary. Pale white or bluish white. Seeing poor.

The crimson image referred to is due to the α hydrogen line in the spectrum of the star, and seems to be always present in the novae at a certain stage of their decline. This image in the 40-inch telescope is 9 mm outside the regular focus.

I have observed the magnitude of the nova on about 130 nights. These observations were made with the naked eye, with Zeiss field-glass, with the 4-inch finder of the 40-inch telescope, and with the 5-inch guiding telescope of the Bruce Observatory.

In the instruments the nova and B.D. +0°4027 were visible at the same time. A striking peculiarity in its brightness has developed in the later observations. Very different results are obtained with the different instruments: apparently the smaller the instrument, the brighter the nova. Twilight and dawn also play an important part in the relative brightness of the star: the brighter the sky, the less bright the nova. Between the various instruments and the condition of the sky results may be obtained that differ by a large part of a magnitude.

These peculiarities have been more strongly marked since the morning observations began and especially since the position of the nova has permitted it to be seen on a dark sky. At the first glance in the telescope (on a dark sky) one gets the impression that the nova is decidedly brighter than B.D. +0°4027 ($6\overset{M}{.}3$), but a more careful inspection shows that it is really less bright than that star. The image of the nova is white and closely surrounded by a rather dense glow or haze and the star itself is very ill defined, while +0°4027 is sharply defined and free from glow and is slightly yellow. The smaller the telescope, the more this stray light enters into the image and hence the brighter the star. As the sky becomes brighter the effect of this extra light upon the eye becomes less and the star appears fainter. The peculiar appearance of the light of the nova, Professor Frost informs me, is doubtless due to the great strength of the hydrogen lines in its spectrum; this, in a refracting telescope, produces an image that is ill defined and hazy.

The planetary condition of the image must also enter to a certain extent into the relative brightness of the nova with different instruments. From my observations it would seem that from these various causes estimations of the brightness of this star may at one and the same time differ by nearly a whole magnitude. This was strongly shown on February 11, 1919. On this date, when I first examined the nova with the 5-inch Bruce telescope it seemed to be about half a magnitude brighter than +0°4027. It was surrounded by a close, rather dense, hazy glow and was very white and ill defined, while the other star was slightly yellowish and well defined. At 17^{h} 35^{m}, however, a closer examination showed that the image was of about the same brightness as that of +0°4027 or possibly a little fainter. At $17^{h}50^{m}$, with the field-glass, there was a striking difference in the brightness of the two stars, the nova being $0^{\mathrm{M}}.4$ the brighter. At $18^{h}10^{m}$ it was again examined with the 5-inch on a dawn-lit sky. The nova was decidedly the fainter of the two—the estimated difference in brightness being $0^{\mathrm{M}}.5$. These observations were repeated on February 17 at $16^{h}50^{m}$. The first glance at the nova in the 5-inch gave the impression that it was decidedly brighter than +0°4027, but a more careful inspection showed that it was about $0^{\mathrm{M}}.4$ less than that star. Immediately after, the field-glass showed the nova to be $0^{\mathrm{M}}.5$ the brighter of the two stars. It would seem, therefore, that from the nature of its light, estimates of the relative brightness of the nova are now likely to be discordant, depending on the size of the instrument, the brightness of the sky, etc. I have sometimes noticed with the five-inch telescope that if the nova was the fainter of the two stars for sharp focus, it became the brighter when they were out of focus.

Recent photographs with the 6-inch and 10-inch Bruce lenses make the nova slightly brighter than B.D.+0°4027. Sometimes the image with the 6-inch is a little brighter than that with the 10-inch. My last evening observations were made on December 19, 1918. The first morning observations were on January 7, 1919, near the time of its conjunction with the sun. It is an interesting fact that the star would have been observable at the time of its conjunction with the sun if weather conditions had permitted.

E. E. Barnard

Yerkes Observatory
February 21, 1919

A PROPERTY OF THE PHOTOGRAPHIC PLATE ANALOGOUS TO THE PURKINJE EFFECT

The "Purkinje Effect" in reference to visual photometry may be described thus: If two sources of light, one white, the other red, appear equal at a certain intensity, they will not remain equal if the intensity of both sources be changed in the same ratio. For example, if the intensity of each is doubled, the red will appear brighter than the white light; or, if the intensities are halved, the red will appear fainter than the white. This is equivalent to saying that the "contrast" appears greater in the red than in the white light. It also follows that as the intensities increase, the maximum sensitiveness of the eye shifts toward the red; or, if intensities decrease, the maximum shifts toward the blue.[1]

Numerous investigators have tested the photographic plate for a corresponding effect. Abney[2] finds that "the least steep gradation is that given by the monochromatic light to which the simple silver salt experimented with is most sensitive, and that the gradation becomes steeper as the wave-lengths of light employed depart in either direction in the spectrum from this point; the steepest gradation being given by the extreme red." His graphs in Fig. 5 of that paper show the gradation to be least steep at wave-length about 4600 A, and at 6000 A to be about 50 per cent steeper.

It is important to note that this change in gradation is in the same direction as occurs in visual comparisons; an increase in intensity favors the red light, both to the eye and on the photographic plate.

The implied shift in the wave-length of maximum sensitiveness on the plate has been independently confirmed by the work of a number of observers[3] on the "effective wave-length" of light from stars of different colors.

Lindblad has summarized his results in Fig. 2, page 26, of that paper and compared these with the accordant results of the other two authors quoted. With one exception his curves lead to the same conclusion as the work of Abney; that is, as the intensity of the light increases, the position of the maximum sensitiveness shifts toward the longer wave-lengths.

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[1] See Nutting's *Outlines of Applied Optics*, p. 122, Fig. 34 and p. 128.

[2] *Proceedings of the Royal Society*, **68**, 302, 312, 1904.

[3] O. Bergstrand, "Recherches sur les Couleurs des Étoiles fixes," *Comptes rendus*, **148**, 1079, 1909; E. Hertzsprung, "Über die Verwendung photograph. effektiver Wellenlängen zur Bestimmung von Farbenäquivalenten," *Pub. der astrophys. Obs. zu Potsdam*, **22**, 1, 1911; B. Lindblad, "Die photographisch effektive Wellenlänge als Farbenäquivalent der Sterne," *Arkiv för Matematik, Astronomi och Fysik*, **13**, 26, 1918.

It would thus appear that this property of the photographic plate is sufficiently confirmed in a qualitative way by these entirely independent investigations. It is the purpose of this paper to find provisional, or at least limiting, values for the amount of error which might arise from this property in photographic stellar photometry.

It is possible to derive approximate values for the difference in gradation which may be met in practice by combining Abney's results with Lindblad's. The former finds an increase of 30 to 50 per cent in gradation for an increase of 1400 A in the wave-length of the monochromatic light. Bergstrand and Hertzsprung find effective wave-lengths as follows:

	Bergstrand	Hertzsprung
White stars..............	4200 A	4200 A
Yellowish-red stars.......	4500	4300
Deepest red stars.........	4700	4400

Here is an extreme difference of 500 and 200 A, respectively, for white and deepest red light. We might therefore expect a change in gradation ranging between 11 and 18 per cent from Bergstrand's results, or from 4 to 7 per cent from Hertzsprung's values. Approaching from another direction, Lindblad finds for white stars an extreme shift in effective wave-length of 100 A for the greatest change in intensity of light on his plates. For stars of other colors the range is less. We might therefore look for an increase of from 2 to 4 per cent in gradation on account of change in intensity. It does not seem likely that these results are more divergent than would be expected from the use of different telescopes, plates, and developers.

A much better determination of this change in gradation can be derived from some of the extra-focal plates taken at this observatory with the Zeiss UV camera of 14.5 cm aperture and 81.4 cm focus. Two plates of the cluster in Coma, including an ideal range of star-colors, were used for this purpose.

No.	Date	G.M.T.	Exposures
UV 523.......	1909, May 6	$14^h 21^m$ to $14^h 40^m$	1^m, 2^m, 4^m, 8^m
UV 524.......	1909, May 7	15 00 to 15 17	15^s, 30^s, 1^m, 2^m, 4^m, 8^m

The measures and reductions were in all respects like those described by the writer in the "Yerkes Actinometry,"[1] except that only relative magnitudes were derived, all that were needed to fix the gradation of the plates. In Table I, which gives a summary of the measures of the plates, this "gradation" is expressed by the differences in apparent magnitude (ΔM) between the successive exposures.

TABLE I

Spectral Class	No. Stars	ΔM					Mean ΔM		
		UV 523			UV 524		523	524	Both
		8^m-4^m	4^m-2^m	2^m-1^m	8^m-4^m	4^m-2^m			
Ao to A5	7	$0^{\mathrm{M}}.48$	$0^{\mathrm{M}}.79$	$0^{\mathrm{M}}.58$	$0^{\mathrm{M}}.71$	$0^{\mathrm{M}}.72$	$0^{\mathrm{M}}.62$	$0^{\mathrm{M}}.71$	$0^{\mathrm{M}}.67$
F to G.........	3	0.41	0.78	0.42	0.64	0.71	0.60	0.68	0.65
K5............	1	0.45	0.84	0.63	0.73	0.66	0.64	0.70	0.68

In explanation of Table I it should be noticed that the values of ΔM for 8^m-4^m and 2^m-1^m on plate 523 are systematically smaller than for 4^m-2^m, while in the case of plate 524 no such difference occurs. This might arise from an actual difference in exposure-times for 523, or from differences in transparency; but in either case all stars were affected alike and the final results are not changed thereby. The last column of the table shows no systematic difference in the gradation depending on the color of the star, unless the slight increase of $1\frac{1}{2}$ per cent between the spectral classes A and K be considered real.[2] If so, the difference is in the direction indicated by the work of the authors already quoted. We therefore reach the conclusion that for Seed 27 plates (the brand used) and extra-focal images the slight systematic effect which may be present is nearly or quite masked by the accidental errors arising principally from the lack of uniformity in the film.

[1] *Astrophysical Journal*, **36**, 169–227, 1912.

[2] An indication that this increase may be real and an excellent confirmation of the amount of increase found in the Coma plates is furnished by Hertzsprung in his "Photographische Helligkeitsmessungen von T Vulpeculae 1910–15," *Astronomische Nachrichten*, **208**, 53, 1919, which appeared after the foregoing was in type. He finds a gradation 3 per cent steeper for the red star B.D.+27°3911 than for the neighboring white star +27°3868. The color-indices of these stars are $2^{\mathrm{M}}.34$ and $0^{\mathrm{M}}.22$ respectively. The plates used were Schleussner "blaue Etikette."

Summary.—A property analogous to the "Purkinje Phenomenon" has been found by Abney and other investigators to be present in the photographic plate, but the amount of the error introduced by neglecting it on Seed 27 plates and with extra-focal images is of the same order as the accidental errors.

J. A. PARKHURST

YERKES OBSERVATORY
February 19, 1919

NOTICE

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention is given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

Articles written in any language may be accepted for publication, but they usually will be translated into English. It is the practice of this JOURNAL to have tables of wave-lengths printed with the short wave-lengths at the top, and maps of spectra with the red end on the right.

Accuracy in the proof is gained by having manuscripts typewritten, provided the author carefully examines the sheets and eliminates any errors introduced by the stenographer. It is suggested that the author should retain a carbon or tissue copy of the manuscript, as it is generally necessary to keep the original manuscript at the editorial office until the article is printed.

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THE STELLAR SPECTROGRAPH OF THE 72-INCH REFLECTING TELESCOPE

By J. S. PLASKETT

The telescope has been described in two issues of the *Journal of the Royal Astronomical Society of Canada* and in the *Publications of the Astronomical Society of the Pacific* for October, 1918, but it seems advisable to summarize the principal optical and operating details of this light-gatherer for the spectrograph.

The aperture of the principal mirror is 184.5 cm (72.6 in.) and its focal length 917.9 cm (361.38 in.). It has a central hole 25.7 cm (10.13 in.) in diameter which allows the spectrograph, which is used with the Cassegrain form of the telescope, to be placed along the axis of the mirror and attached to the bottom of the tube, a much more convenient and symmetrical position than if a third reflection were used and the spectrograph were attached to the side of the tube. The Cassegrain mirror is 50.8 cm (20 in.) in diameter, has a radius of curvature of 605.6 cm (19.87 ft.), and is placed about 218 cm (7.15 ft.) inside the principal focus of the main mirror. The equivalent focal length of the combination is approximately 32.92 meters (108 ft.), and the aperture ratio is reduced from 1:5 to 1:18.

Tests of the figure of the principal mirror in the optical shop under constant-temperature conditions showed a maximum longitudinal aberration of 0.25 mm (0.01 in.) and this, being for a median zone, is equivalent to lateral aberration 0.025 mm to a circle of confusion of less than the thousandth of an inch in diameter. Under changing conditions of temperature in the dome this is much increased, and after a daytime rise of 5° C. the aberration, under-correction, became about 3 mm. The application of heat-insulating covers to the lower end of the tube completely inclosing the mirror reduced the temperature change to less than one-third that in the dome, with the result that, as the nocturnal change of temperature is very small and slow, the working figure remained practically perfect, and the longitudinal aberration, always under-correction, was of the order of 0.5 mm.

The figure of the Cassegrain combination has recently been tested by the Hartmann method. The figure is also very good, the effective aberration under average observing conditions being about 1.5 mm, equivalent to a circle of confusion 0.05 mm in diameter. This is an example of the skill of Mr. J. B. McDowell, of the John A. Brashear Co., Ltd., as this mirror was corrected in combination with the 72-inch while only a 33-inch plane was available for producing a parallel pencil. Hence less than one-half the diameter, one-fourth the area, of the secondary could be tested at a time, and the difficulty of figuring was thereby much increased.

The general form of the mounting and the appearance and position of the spectrograph can be seen in Plate XII. The mounting is beautifully designed and constructed and is undoubtedly the most perfect and complete of any hitherto made by the Warner & Swasey Co. The driving is perfect, without a trace of periodic or other error, which is so frequent a source of trouble and annoyance in telescopes. The following is so close, the figure of the mirrors and generally the seeing so good, that it is necessary to set the clock driving slightly fast or slow, else the image remains so nearly in one position on the slit, which subtends an angle of 0.″3 by 3″, as to produce too narrow or unevenly exposed spectra.

The arrangements for quick and fine setting of the telescope are very convenient, so that little time is lost in changing from star to star in spectroscopic observation. There are three speeds in each co-ordinate, all electrically driven, a quick motion of 45° to the minute, a fine setting speed of 10′ to the minute, and a guiding speed of 30″ to the minute. The quick setting, clamping, and dome operating are performed at the more convenient of duplicate operating boards, one of which may be seen in Plate XII, on the east and west sides of the south pier. The fine setting and guiding are effected by means of a small portable board carried by the observer. There are no fine circles, but the coarse circles are graduated to single minutes of time in right ascension and to five minutes of arc in declination. The telescope can always be certainly set to within two minutes of arc of the catalogue position, and charting and uncertainty of identification for stars brighter than about the seventh magnitude are generally avoided. A further convenience is the fact that the settings in right ascensions are made on a large circle moving with the worm wheel, set to the sidereal time at the beginning of a night's work, and all mental arithmetic, liability to mistakes, and difficulty of setting to constantly changing hour angle are avoided.

In consequence of the perfection of the design and construction of this operating mechanism, the average time required to change from star to star, the time from the end of one spectrum exposure to the beginning of the next is only about three minutes and, if the stars are not far apart, is frequently only two minutes. Even though the moving parts weigh 45 tons, the telescope can be handled as easily and quickly as one of less than a fourth the aperture. In consequence of this, of the quality of the star images, and of the efficiency of the spectrograph, spectra can be accumulated rapidly, and the equipment has proved most efficient in this its principal work.

THE SPECTROGRAPH

As the stellar spectrograph to be described is in one sense a reversion to an early form, it will be of some interest to trace briefly the method of evolution. The universal spectroscope was probably the earliest type to be attached to a telescope and was probably chosen, as observing methods had not become standardized, because a choice of different dispersions and spectral regions would prove useful in experimental research. The best examples of this type were those supplied by the Brashear Co. to the Allegheny, Yerkes, Lowell, Ottawa, and other observatories, which were undoubtedly beautiful instruments for general work but unsuited for the particular line of work—the determination of stellar radial velocities—into which stellar spectroscopy was soon narrowed. The adjustable features were not needed and were indeed a source of weakness in rendering the spectroscope more subject to differential flexure. Consequently the universal type was soon abandoned, and a rigid fixed form was developed, of which the Mills at Lick, the Bruce at Yerkes, and the Hartmann at Potsdam were all successful examples, and with which very accurate observations were obtained.

Both the universal and these later spectrographs had the drawback that the spectrograph proper and the truss which attached it to the telescope were one and the same construction, and the flexure, just as in a beam supported at one end, was a maximum. A great improvement in design, introduced by Campbell and Wright at the Lick Observatory, was to make the spectrograph proper self-contained in box form and attach it to the telescope by an external truss frame in which the spectrograph box was held flexibly at two suitably chosen supporting points. The flexure in such a form is evidently only about one-fourth that of the earlier types, and this form has been adopted in nearly all recent spectrographs.

In the spectrograph for the 72-inch telescope I have attempted to combine the advantages of the self-contained spectrograph box carried in an external attaching-frame by a two-point flexible support-system with the flexibility and general usefulness of a universal instrument. In more recent years the narrow field of radial-velocity observation of stars in the spectral region near Hγ, with a three-prism spectrograph of linear dispersion about 10 A to the millimeter, has been broadened to include observations with one and two prisms, with various lengths of camera, and at various regions of the spectrum. No single spectrograph, indeed no two or three spectrographs, would be likely to meet the varied demands that might arise in spectroscopic investigation with such a powerful instrument as the 72-inch reflector. Hence, owing to the expense that would have to be incurred for such a battery of spectrographs, it seemed wise to try to devise an instrument in which the change from one type to another could be made as quickly or even more quickly than spectrographs could be changed on the reflector, while any desired or used adjustment could be rapidly and certainly obtained. At the same time the spectrograph must be as rigid and as little subject to flexure as any of the fixed-form types.

These conditions have, I am convinced, been successfully met, and the methods employed will be developed as the description proceeds. The optical parts of the spectrograph were made by the Brashear Co. and give exquisite defining power, while the mechanical parts were made by the Warner & Swasey Co. The material, size, and form of the optical parts were determined by the writer, and the principles and general form of the mounting, of the minimum-deviation link-work, of the slit-head, comparison, and guiding apparatus, and of the constant-temperature arrangements were given to the Warner & Swasey Co. The latter is, however, responsible for the working out of the mechanical details, the form and material of the spectrograph box and attaching frame, and the style of construction and finish of the instrument. I need say no more than to state that in harmony of design and in character of workmanship and finish it is in keeping with the work on the telescope mounting, all of the very highest order.

THE OPTICAL PARTS

Practically all spectroscopists are agreed as to the advantage of a long collimator, the practical limit being the possibility of obtaining homogeneous material for the correspondingly large prisms. I had successfully used a prism of 51 mm (2 in.) aperture in the Ottawa single-prism spectrograph, and Adams has since used prisms of 63 mm (2.5 in.) aperture without difficulty. Hence it was decided to use prisms of the latter aperture, and, with the aperture-ratio of 1 to 18, this fixes the focal length at 114.3 cm (45 in.).

The most suitable material for the prisms was the subject of considerable experiment and calculation. I have previously said,[1]

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[1] *Pub. Dom. Obs.*, Ottawa, 1, 188.

"I have long been of the opinion that the dense flint, O 102, used in most recent spectrographs is too absorbing for the best results." This opinion has been borne out by experiments with a prism of baryta light flint and more recently by the results obtained with a prism of ordinary flint glass in the spectrograph here described. Measurements of the absorptions of various Jena glasses were obtained, and calculations of the losses due to reflection and absorption in prisms of 63 mm (2.5 in.) aperture of these glasses and of their relative efficiencies for given dispersion and resolving power as given in the accompanying table, showed that one of the ordinary flints, O 118 of the Jena works, a glass very generally used for the flint component of telescope objectives, gave the most favorable showing. Schlesinger's measures of the absorption of the flint element of the 30-inch Allegheny objective, made of O 118 glass, confirmed these calculations, and, as this type is made in large quantities and is more likely to be homogeneous

TABLE OF TRANSMISSION, DISPERSION, AND RESOLVING POWER

(Prisms of 63 mm aperture of various glasses)

Material Jena Numbers	Number of Prisms	Deviation at λ 4200	Angle of Prisms	Transmission of Prisms	Angular of Dispersion at Hγ	Linear Dispersion A per mm at Hγ Cameras of Focal Lengths			Resolving Power at at λ 4200
						381 mm	711 mm	965 mm	
U.V. 3248..........	1	48°31′	65 45	.822	5″.24	103.3	55.3	40.8	16,160
Ultra-violet flint....	2	97 02	....	.703	10.48	51.6	27.7	20.4	32,320
	3	145 33	....	.615	15.72	34.4	18.4	13.6	48,480
	4	194 04	....	.556	20.96	25.8	13.8	10.2	64,640
O 722.............	1	51 41	64 15	.657	5.90	91.8	49.2	36.2	18,260
Baryta light flint....	2	103 22	....	.451	11.80	45.9	24.6	18.1	36,520
	3	155 03	....	.321	17.70	30.6	16.4	12.1	54,780
O 578.............	1	52 07	64 0	.636	7.12	76.0	40.7	30.0	22,220
Baryta flint.........	2	104 14	....	.422	14.24	38.0	20.4	15.0	44,440
	3	156 21	....	.291	21.36	25.3	13.6	10.0	66,660
O 118.............	1	50 0	60 0	.768	8.51	63.6	34.1	25.1	....
Ordinary flint.......	1	54 40	63 0	.756	9.86	54.9	29.4	21.7	30,450
	2	109 30	....	.603	19.72	27.4	14.7	10.8	60,900
	3	164 00	....	.503	29.58	18.3	9.8	7.2	91,350
O 102............	1	60 0	64 0	.467	12.89	42.0	22.5	16.6	52,700
Dense flint.........	2	120 0	....	.235	25.78	21.0	11.2	8.3	105,400
	3	180 0	....	.126	38.67	14.0	7.5	5.5	158,100

NOTE.—It is evident, from a comparison of dispersion, resolving power, and transmission, that, with slit-widths giving the same purity of spectrum, the ultra-violet flint and the ordinary flint are much superior to the others and approximately equal themselves. The greater dispersion given by the ordinary flint, the higher resolving power obtainable, its lower cost, and the greater likelihood, owing to its being produced in large quantities, of homogeneity, led to its choice.

than some of the less-used varieties, I had no hesitation in choosing it for the prisms of the spectrograph. Material for the three prisms was ordered from Jena by the Brashear Co. in August, 1914, but of course could not be supplied, owing to the war. Until these or some similar prisms are available, we are fortunate in being able to use a 60° prism of the same aperture and the same material made by the Hilger Co. for a Littrow spectrograph for Toronto University and kindly lent temporarily to me by Professor Chant. Use of this prism has shown it to be of excellent quality and of remarkable transparency in the violet, considering its density and dispersing power. Although its dispersion is only about 25 per cent less than O 102, its absorption at λ 4000 is certainly less than half, and in spectra of Nova Aquilae measures of 19 displaced hydrogen lines, extending to λ 3655, were made.

Owing to this transparency in the violet of the prism and to the absence of absorbing material in the telescope objective, the central ray could be advantageously placed considerably farther to the violet than has been usual hitherto. The position chosen was at λ 4200 in order to take advantage of the larger number of lines in early-type stars around this region than to the red of λ 4500. However, the adjustable feature of the spectrograph will readily admit of shifting the camera and prisms in a few moments, so that any other region is central, and any part of the photographic spectrum can be reached with the same objectives without the color-curve becoming unmanageably steep.

The collimator and the two camera objectives supplied are all of the Hastings-Brashear triplet type, and as the former is 63 mm and the latter 76 mm aperture the use of Canada balsam as a cementing agent was not deemed allowable on account of the strain almost certain to be introduced. However, as the internal reflections in such triplets when uncemented cause a loss of about 20 per cent in each, these objectives had at my suggestion a film of watch oil placed between the components, which has acted perfectly and shown no signs of deteriorating or drying out after a year's use. The aperture of the collimator objective is 63.5 mm (2.5 in.) and its focal length 1143 mm (45 in.). The apertures of the camera objectives are 76.2 mm (3 in.), and their focal lengths are 711 mm

(28 in.) and 965 mm (38 in.) respectively. These objectives and the prism are so well corrected that the spectra given by the instrument are of exquisite definition and leave nothing to be desired. A third camera objective of the Cooke triplet type of 76 mm aperture and 380 mm focus, ordered from the Brashear Co., has not yet been received.

The linear dispersions of the spectrograph with these three cameras and with the one, two, and three prisms destined for the instrument, and also with the 60° prism now in use, are given in the table on p. 214.

SLIT AND ACCESSORIES

The slit-jaws are of polished nickel inclined at an angle of 3°5 to the normal to the optical axis. One of the jaws is fixed, and the other is separated from it by a micrometer screw reading to thousandths of an inch. The guiding is effected in the usual way by the star's light reflected from the jaws into a right-angled reflecting prism at the end of a broken guiding telescope. The bend of 45° allows the guiding ocular to be rotated into the most convenient observing position.

The method of applying star and comparison spectrum is new, offers some advantages over other devices, and hence merits a more detailed description. Directly above, and with their edges parallel to the slit, are two small right-angled reflecting prisms, one for each set of comparison lines, which reflect the comparison light from its original direction, perpendicular both to the slit and the optical axis, down through the slit. These prisms can be brought into contact or separated symmetrically along the direction of the slit, so as to change the distance apart of the comparison spectra, by means of a right-and-left screw, and can also be moved, transversely to the slit, toward or from the comparison source, by rack and pinion. The prisms are held in their cells by thin metal plates shaped like the diagram, Fig. 1, which just clear the slit-jaws and serve as limiting diaphragms for star and comparison light. The star's light passes through the wedge-shaped opening *a*, and the comparison light through the rectangular openings *b*, *b*. The wedge-shaped opening will give any width of spectrum between 0 and 0.8 mm, depending upon its position when the prisms are in contact, a notch being cut, as shown, in the front of each above the wedge to allow the light unobstructed passage. Three adjustable abutting screws, against which the rack and pinion can move the prisms, allow the usual widths to be reproduced at will. At the same time the rectangular openings give comparison spectra of the same length and separation, whatever the width of star spectrum within the preceding limits. If nebular or planetary spectra are desired, all that is necessary is to separate the prisms by the right-and-left-hand screw the required distance, and the comparison spectra remain automatically at the same separation from the edges of this widened spectrum. The rectangular notches *c* are for the purpose of placing a central strip of comparison spectrum between the outer strips, which, in conjunction with a semicircular diaphragm inserted at the collimator objective, enables the Hartmann method of focusing the camera to be readily applied.

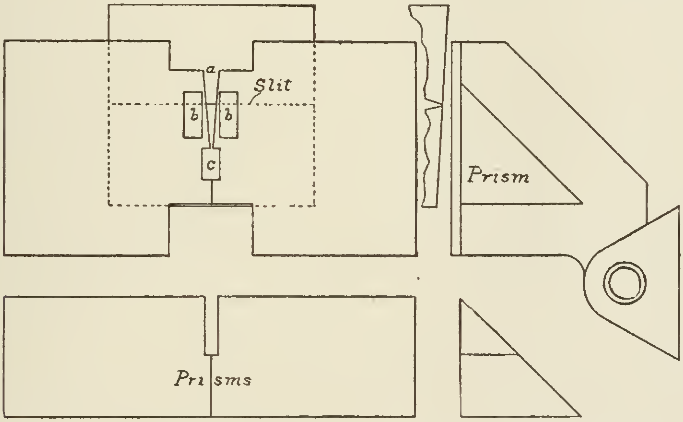

FIG. 1.—Slit diaphragm and comparison prisms, 1.7 times natural size

Evidently the comparison spectrum can be applied as often as desired without stopping or affecting the exposure on the star spectrum or touching the slit-head. All that is necessary is to snap on the switch that starts the iron arc used as comparison source, which is fed by the 220-volt direct current used in operating the telescope. This device works admirably, gives exactly similar and uniformly exposed comparison spectra, always remains in adjustment, and has the additional advantage over previous devices of readily giving adjacent comparison spectra for determining camera focus by the Hartmann method. Both arc and guiding telescope are adjustably attached to the supporting frame, do not touch spectrograph box or slit-head, and no pressure on them can possibly affect the position of the lines.

THE SPECTROGRAPH BOX

As previously stated, the box carrying all the optical parts is a self-contained unit and is carried and supported without stress in collimation with the telescope by a surrounding frame, to which it is attached at two points. The general shape of this box, which is a single aluminum casting, is well shown in Plate XIII, which illustrates the spectrograph proper in its supporting frame attached to the telescope. The circular opening in the side directly over the prisms and link-work enables them to be readily reached for changes and is normally covered by the three plates on the floor. The three projections to the right are for inserting the cameras for use with one, two, or three prisms. Another projection for carrying the collimator tube extends centrally up within the supporting frame, and this is united to the third camera projection by a box girder, seen in the photograph, cast integral with the spectrograph box. This box, as it is in one casting with only the circular opening at one side, is exceedingly stiff and, being of aluminum, comparatively light.

The minimum-deviation link-work, which carries and maintains prism cells and prisms always at minimum deviation, whatever part of the spectrum is central, is similar in design to that used by the Brashear Co. in their universal spectroscopes, with the addition of guiding links to always align the cameras, when used with one, two, or three prisms, along the optical axes. It is exceptionally well made, without lost motion or backlash of any kind, and one can be certain that after the prisms are adjusted in their cells they are always at minimum deviation and the axes of the cameras parallel to the central emergent ray, whether one, two, or three prisms are used, and whatever the region of the spectrum. A scale and magnifying index enables any region to be accurately set to or recovered when desired. This link-work, however, is not depended on for rigidly holding prisms and camera in position but only serves to fix their relative positions. Bolts through each corner of the triangular-shaped prism tables, and two at each end of the cameras, passing through slotted holes in the 16-mm thick base of the box, enable prisms and camera, when adjusted by the link-work, to be as rigidly fixed to the spectrograph box as if it had no adjustable features whatever, or was specially made for one dispersion and wave-length only. The prisms are held in substantial cast-iron cells, these latter being attached to the movable prism tables by screws and dowel pins, so that if, for example, the second prism has to be removed to use the one-prism form, this is done by unscrewing four screws and taking out cell and prism as a unit. The dowel pins insure its return to exactly the same position and adjustment.

The collimator tube is of steel, and the cameras are of brass. Previous experience has shown that with the triplet type of objectives such construction made change of focus with change of temperature compensatory, and no change of camera setting is necessary between winter and summer temperatures, which is a great convenience. The collimator tube is placed in the most convenient position and firmly clamped there, as the focusing of the star image on the slit is effected by moving the secondary mirror in and out with a slow motion operated by a hand wheel near the guiding eyepiece. This method is very accurate, sensitive, and convenient, and the focus can be readily and closely determined visually by the appearance of the image on the slit-jaws.

The medium-focus camera is in position for using one prism, in the photograph of the spectrograph, Plate XIII, while the short-focus camera is on the floor. They are of exceptionally rigid construction to avoid flexure; the objectives are permanently screwed in the inner ends of the tubes, while focusing is effected by rack and pinion on the plate-holder end, read by scale and vernier to tenths of millimeters. The plate-holders are of metal, book-form style, opening in the middle, the plate 2×4 in. in size being held down by a spring against a raised surface on the edge of an opening 1 by 35 in. in the front half of the holder. The plates are hence supported parallel to the spectrum all along its length, only half an inch away from it, and any curvature of the plate will hence have negligible influence on the focus or measures. The plate-holder and carrier move transversely in ways so that, if desired, several spectra may be made adjacent to one another on the same plate. In addition a tilt of about 15° each way can be given to the plate to enable it to be adjusted tangent to the focal plane. The medium- and long-focus cameras can be operated through doors in the temperature case, but, as the plate comes within the spectrograph box in the short-focus camera, suitable openings covered by hinged doors are made in the camera projections of the box and corresponding openings in the temperature case to enable the plate-holders to be inserted and changed.

THE SUPPORTING FRAME

This, as well as the spectrograph box, is of cast aluminum, both being exceptionally fine castings. It is of hollow, rectangular, prism-shaped form, with a substantial circular flange at the top, which is held by 8 cap screws firmly to the revolving cast-iron ring at the lower end of the tube. The spectrograph box is held in the frame by two shafts, of which the lower one, which can be seen in the figure, Plate XIII, at the bottom of the frame, forms the principal support. The upper shaft is within the small circular cover plate on the side of the frame and serves to keep the instrument from rotating on the lower shaft. Both of these shafts are so attached to the frame and pivoted in the box that no possible flexure of the former can produce any stress in the latter, which is carried, as it were, in a flexible cradle. Collimation of the spectrograph with the optical axis of the telescope is effected by adjustments on these two shafts. This collimation was performed before the optical parts were placed in the spectrograph by turning the telescope tube vertical and hanging a steel plumb line down its exact center and through the collimator tube. The spectrograph box was then adjusted on the two shafts until the wire was exactly central at the upper and lower ends of the collimator tube. Flexure in other positions of the telescope will change the collimation slightly, but this will be of the second order, unavoidable, and cannot cause appreciable error in the observations. The inside of the frame where it surrounds the collimator projection and tube is lined with felt, has part of the heating wires on the two sides, and really forms part of the temperature case.

THE TEMPERATURE CASE

The temperature case is attached to, and indeed forms a part of, the supporting frame and does not touch the spectrograph box at any point. The very workmanlike and convenient combination of box, frame, and case, due to the Warner & Swasey Co., forms a new type of design of simple arrangement and fine proportions, and one in which the necessary adjustments or changes can be made with the greatest convenience. The temperature case proper can be seen detached in Plate XIII and attached in Plate XII. It has sheet-aluminum sides and cast-aluminum edges, is firmly screwed to the supporting frame, and is lined throughout, and the entire spectrograph box is hence inclosed with felt one-half inch thick, forming a very efficient heat insulation. Large circular openings at each side and covers for the cameras, all quickly detachable, enable any changes of adjustment to be made without removing the case, while plate-holders are changed through hinged doors.

The inside of the case is heated by passing the 110-volt lighting current through wires uniformly distributed over and sewed to the felt, and, as the difference between inside and outside temperatures is generally not more than three or four degrees Centigrade and the case is well heat-insulated, very little current is required. Exploring thermometers have shown very nearly uniform temperature all over the inside of the case, even when the air-stirring fan within is not operating. The heating current is turned on and off automatically by a mercury-contact thermometer operating a relay, and this works satisfactorily, there rarely being a change greater than 0°.1 C. during a night's work. It is proposed, as soon as it can be obtained, to instal a Callendar recorder to regulate and record the temperature within the case. As the regulating resistance can be divided between the inside of the case and the inside of the prism box, it should be possible by suitably proportioning the amount of resistance in each to keep the spectrograph constant to within 0°01 C.

PERFORMANCE OF THE SPECTROGRAPH

The collimator focus was adjusted by Schuster's method, and the camera is focused by the Hartmann method, the very convenient means for applying this test having already been described. The field of the medium-focus camera, focal length 711 mm, which is the one now in use, is slightly convex toward the lens, the deviation at λ 4600 at one end and at λ 3900 at the other from the tangent plane at λ 4200 being less than 0.1 mm. Evidently, when the dispersion is greater, with two prisms, say, the field will probably be practically flat. As it is, by accommodating the focus slightly, no part of the spectrum between λ 3900 and λ 4600 will be out of focus to a greater extent than 0.05 mm, too small, when even only approximately uniform illumination by star and spark light is secured, possibly to affect the measures. As previously stated, no change of camera setting with change of temperature is required, but as a precaution the setting is periodically tested.

Tests for flexure with the extended one-prism form, where it would be the maximum, showed quite inappreciable displacements, even under the most unfavorable conditions and abnormal hour-angle changes.

However, the best test of the performance of the spectrograph is actually making and measuring spectrograms, and under this final exacting test the instrument has exceeded expectations. It has only been used with the single prism and medium-focus camera giving a linear dispersion at Hγ of about 35 A to the millimeter. About 1300 spectra of stars, mostly fainter than 6.0 photographic magnitude, have been obtained, and half of them measured. Under average conditions of seeing and of the silver surfaces, a well-exposed spectrum of a star of photographic magnitude 7.0 can be obtained in twenty to twenty-five minutes. I am convinced that with a good silver coat and good seeing this could easily be reduced to fifteen minutes.

The accuracy of the resulting spectrograms is also very gratifying. It is generally considered that a three-prism spectrograph should give radial velocities with a probable error of half a kilometer. It would naturally be expected that with a dispersion less than one-third the probable error would be trebled, but this is by no means the case. The resulting probable error for stars with good lines, F to M types, is well under 1 km, and for G, K, and M types seems to average about ±0.85 km per second. On stars with few or poor lines this will of course be increased, but if the lines are poor better values can probably be obtained with low than with high dispersion.

In view of the accuracy obtainable with this dispersion, and in consideration of the fact that three-prism spectra require at least five times the exposure, it seems to me questionable, for radial velocities of stars fainter than sixth magnitude, whether the doubled accuracy of determination justifies the additional time consumed at the telescope. Some special tests have shown that as far as can be determined the systematic displacements are negligible in comparison with the errors of measurement of the spectra, and that by remeasurement the range in values and the probable errors are generally reduced. Double the number of single-prism plates could be obtained in 40 per cent of the time of three-prism plates, and, with care in the measurement, the mean values should not greatly differ in accuracy in the two cases.

The test of actual use has convincingly shown that both telescope and spectrograph are most efficient and convenient to operate, and we have not been able yet to discover any features that could be improved. A great part of this success is undoubtedly due to the ever-present determination of the makers, of both optical and mechanical parts, to produce the best possible instrument regardless of cost, and it is only my duty as well as my pleasure to express here my gratitude to the John A. Brashear Co., Ltd., and the Warner & Swasey Co., for the spirit with which they attacked the various problems that arose, and for the success that attended their efforts to make an equipment for astronomical research capable, by its accuracy and efficiency as well as by its size, of so greatly extending our knowledge of the universe.

Dominion Astrophysical Observatory, Victoria, B.C.
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A FURTHER STUDY OF METALLIC SPECTRA PRODUCED IN HIGH VACUA[1]

By EDNA CARTER and ARTHUR S. KING

Using the methods employed in a former investigation,[2] we have made a study of the spectra of manganese, titanium, magnesium, and cadmium, as produced by vaporizing the substance through the heating effect of a stream of cathode rays and exciting the vapor to luminescence by the bombardment of the cathode particles. The spectra thus obtained seem to be quite definite and characteristic of this mode of excitation. Additional observations of the calcium spectrum have confirmed the previous results, and for the iron spectrum a considerable number of lines has been obtained with a slightly different arrangement of circuit.

The form of the discharge-chamber is similar to that used in the previous work. In order to withstand the heat developed by the heavier discharge required to vaporize the more refractory metals, the parts formerly of glass were replaced by fused quartz or silica-ware. The glass bell-jar was superseded by an inverted crucible of silica-ware eight inches in diameter. Through the top a hole was bored into which a tube, *T*, Fig. 1, of opaque quartz was fitted by tapering it slightly at the end. Through this was passed another similar tube, which was fitted into a hole bored into a small quartz crucible, *B*, about two inches in maximum diameter. This flared around the cathode, *C*, and helped to concentrate the discharge. The aluminum cathode was melted by the heat, so it was replaced by a more massive one of copper faced with iron. This was connected by a copper rod with a copper receptacle, *E*, in which ice was placed. The joints were cemented with sealing-wax. In the large crucible two holes were bored opposite each other, into which tubes bearing quartz windows were set for viewing and photographing the luminescence spectrum just above the anticathode.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 166.

[2] *Mt. Wilson Contr.* No. 125; *Astrophysical Journal*, 54, 303, 1916.

The base-plate of $\frac{3}{4}$-inch glass was retained, protected from the heat by a $\frac{1}{4}$-inch aluminium plate, *D*, supported a short distance above it. An opening in the aluminum plate, covered by a quartz window, permitted a view of the discharge from below. The anode, *A*, was introduced through a quartz tube, which projected through the base-plate and the aluminum plate. The metal to be examined, *M*, was contained in a quartz tube supported in a base-plate, *F*, of the same material. The tube was slotted vertically at the upper end to expose the luminescent vapor, while also confining it. The metal was placed just below the slot, the quantity used depending upon the ease with which it could be vaporized.

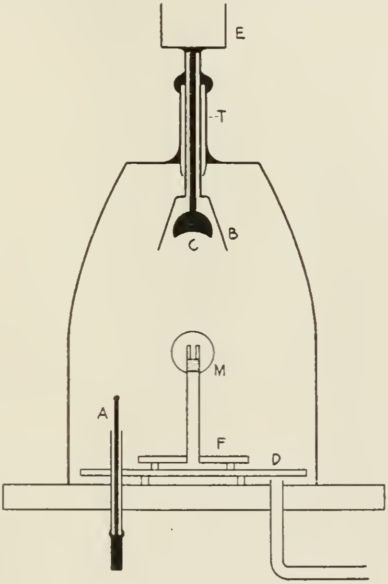

FIG. 1

The discharge was produced by a transformer with a rotary spark-rectifier. The evacuation was effected by two rotary oil pumps in series. In general, the metal, as soon as it began to vaporize, aided the evacuation by absorbing the gases in the chamber. Sometimes the absorption was so complete that it was necessary to leave a fine capillary tube open to the outside air in order to keep up the discharge. The iron vapor did not seem to have this absorbing power, so that it was difficult to obtain a satisfactory spectrogram of iron except by making the metal the anode. When a Fe-Mn alloy was used there was no trace of iron in the spectrum of manganese thus obtained, although a hole several millimeters deep was made in the metal at the focus of the rays.

The spectra were photographed for the most part in the first order of a 1-meter concave grating, the scale being 1 mm = 17 A. In order to detect any differences in the spectrum at different distances from the hot metal, photographs of the spectra of titanium, magnesium, and calcium were also made with a quartz spectrograph, but no notable difference was found. The exposure time varied from fifteen minutes in the case of cadmium, with a discharge current of about 5 milliamperes and a parallel spark of 20–30 mm, to seventy minutes for titanium, the current being 9–12 milliamperes and the parallel spark 60–120 mm in length. It was imprudent to run this latter discharge more than ten minutes at a time on account of the warming of the sealing-wax joints.

MANGANESE

The number of manganese lines identified is the largest thus far photographed in luminescence spectra. The best spectrogram was obtained with a current of 8 milliamperes, the parallel spark-gap being about 5 cm. The lines which appear in the luminescence spectrum are listed in Table I, with their intensities in this source and for the furnace (at about 2200° C), arc, and spark. The intensities for the arc and spark are taken from the tables of Exner and Haschek, and, while on a different scale from the other estimates, they serve to show in which of these two sources a line is relatively strong.

A comparison of the luminescence lines with those of the three other sources serves to show the distinctive character of the luminescence emission. Among the lines relatively strong in the spark, only λ 4235 is notably strong in the luminescence. Of a group of nine enhanced lines from λ 3442 to λ 3498 only a trace of the strongest appears.

An inspection of Table I shows also many divergences of the luminescence spectrum from that of the furnace. A notable contrast is given by the moderately strong luminescence lines λλ 4058, 4060, and 4062, which do not appear in the furnace. The resemblance to the arc spectrum is closer, but this is found to result from the fact that the luminescence emission picks out certain arc lines and gives them with high intensity. That most of the luminescence lines appear also in the furnace is a consequence of the usual condition that most of the arc lines are given with greater or less intensity by the furnace.

TABLE I

MANGANESE

λ (Exner and Haschek)	Luminescence	Furnace	Arc	Spark
2576.20	6	....	4R	30R
2593.82	5	....	4R	15R
2605.78	3	....	4	10R
2704.00	2	....	5	....
2713.42	2	....	5	....
2726.22	2	....	5	....
2761.03	3	....	3	3
2776.25	1	....	2	....
2778.68	1	....	2	....
2782.85	1	....	2	....
2794.92	150	200R	50R	5R
2798.37	125	150R	50R	5R
2799.99	1	2	2	1
2801.20	100	100R	50R	5R
2802.90	1	1	2	1
2818.09	1	3	3	1
2858.85	1	1	2	....
2872.68	1	1	2	....
2914.71	3	5	8	1
2925.67	5	10	8	1
2930.35	1	1	2	....
2933.19	2	1	3	15
2939.40	3	1	3	20
2940.50	6	4	8	1
2949.31	5	1	3	30
3044.69	25	5	5	2
3054.53	8	8	4	2
3062.30	4	5	4	1
3066.19	5	5	4	1
3070.46	6	5	4	1
3073.33	5	5	4	1
3079.80	3	5	3	1
3081.52	1	4	3	1
3148.36	3	5	3	1
3161.19	4	6	3	1
3178.61	5	7	3	1
3212.98	1	10	2	2
3217.04	10	30R	1	1
3224.90	25	30R	2	1
3228.20	15	20r	3	3
3230.81	2	8	2	2
3234.0	2	1	2	....
3235.1	3n	1	2	....
3236.90	8	8	2	3
3237.4	4	3	2	....
3240.53	1	10	2	1
3240.75	1	7	2	1
3243.93	3	6	3	2
3247.80	1	....	1	....
3248.64	6	6	3	3
3253.09	2	7	3	2
3254.14	1	3	2	1
3256.25	3	6	3	2
3258.52	2	6	3	2
3260.40	1	6	3	2
3264.83	4	8	3	2
3278.65	1	4	2	1
3280.90	3	4	2	1
3297.01	2	4	2	1
3312.05	3	2	2	....
3313.41	5	1	2	....
3313.70	5	1	2	....
3314.59	5	2	2	....
3315.07	8	2	2	....
3316.61	4	1	1	....
3317.47	20	4	3	1
3320.82	4	4	2	1
3330.80	4	6	2	....
3442.13	1	....	2	30
3532.02	30	10	20	2
3532.15		15		3
3532.26		15		3
3547.93	40	15	10	4
3548.16		15	10	3
3548.30		10	3	3
3569.65	40	20	15	5
3569.96		15	10	4
3570.23		12	4	3
3576.25	5n	....	1	....
3577.99	20	15	10	5
3580.3	4n	....	1	....
3586.69	12	10	5	4
3595.29	3	7	4	2
3607.69	6	10	5	3
3608.66	8	10	5	3
3610.49	5	8	5	3
3619.42	5	6	4	3
3623.96	3	6	4	2
3629.89	1	4	3	1
3790.38	1	6	3	4
3800.70	1	....	2	2
3806.90	15	12	10	8
3809.75	2	8	5	6
3823.64	10	10	5	6
3824.03	1	6	4	4
3829.80	3	3	3	3
3834.01	2	6	4	4
3834.50	7	10	8	8
3839.90	1	5	4	5
3841.23	3	7	5	6
3844.12	1	5	3	4
3926.63	2	....	3	4
3943.05	1	....	2	2
3982.73	1	5	2	3
3985.40	1	1	2	3
3986.98	4	1	2	4
3987.26		1	2	4
4018.28	3	20	10	8
4030.92	150	200R	100R	20R
4033.21	100	150R	100R	20R
4034.62	100	150R	100R	10R
4035.88	1	15	5	8
4041.53	10	40	20	10
4048.90	3	15	8	8
4055 70	4	20	4	8
4058.10	6	....	3	2
4059.54	8	....	3	2
4061.90	10	....	4	3
4063.70	1	20	5	6
4079.38	3	10	3	5
4079.61		10	3	5
4083.11	2	10	4	6
4083.82	2	12	3	6
4147.71	1	....	2	2
4235.41	10	4	10	20
4239.90	1	1	3	5
4257.83	1	1	3	4
4266.10	2	1	3	5
4281.30	2	2	3	5
4436.52	1	1	8	5
4451.78	2	8	8	10

TABLE I—*Continued*

λ (Exner and Haschek)	Luminescence	Furnace	Arc	Spark	λ (Exner and Haschek)	Luminescence	Furnace	Arc	Spark
4455.20	15	2	5	3	4766.63....	1	6	8	3
4455.51			5	3	4783.62....	15	50	30	8
4456.02.....	5	1	5	3	4823.71....	15	50	30	10
4457.76.....	8	1	5	4	5341.25....	15	50	15	8
4458.48.....	15	2	6	5	5394.89....	1	40	8	1
4461.30.....	8	1	5	4	5420.61....	6	25	10	3
4462.20.....	40	3	10	8	5470.88....	5	15	10	2
4464.88.....	1	4	8	5	5481.61....	2	8	8	1
4754.24.....	10	50	30	8	5517.00....	2	10	10	2
4762.60.....	2	10	10	4	5537.99....	2	6	10	1
4766.08.....	1	4	6	3					

Two sections of the manganese luminescence spectrum, with that of the arc for comparison, are reproduced in Plate XIV. While many of the luminescence lines are too faint to appear in the copy, the general effect of certain arc lines being brought out while other strong ones are absent may be clearly seen.

TITANIUM

The titanium luminescence lines are given in Table II, with the arc and spark intensities of Exner and Haschek for comparison. The results, considering the high melting-point of this element, show the possibility of studying the more refractory substances by this method. It will perhaps be sufficient to note briefly the points of difference from the furnace, vacuum arc, and the arc and spark in air, photographs of which were available for comparison. As for manganese, a decided selective action by the luminescence is apparent, many strong arc lines being absent or very faint. A comparison with the furnace spectrum, however, shows many resemblances. The prominent luminescence lines are strong also in the furnace and reverse at high temperature, their relative strength in a given region often being not greatly different in the two sources. There are exceptions, however, to this rule, and strong furnace lines are frequently lacking in the luminescence spectrum. There is an absence of even a slight approach to the spark spectrum, since the enhanced lines of titanium, many of which appear with considerable strength in the arc and can be obtained faintly even in the furnace, are either absent in the luminescence or among the fainter lines of this spectrum. The

TABLE II

TITANIUM

λ (Exner and Haschek)	Luminescence	Arc	Spark	λ (Exner and Haschek)	Luminescence	Arc	Spark
2641.20......	1	2	2	3753.77......	3	3	3
2644.35......	1	2	3	3759.48......	1	10	20
2646.71......	1	2	2	3761.50......	1	10	10
2942.12......	4	3	4	3771.84......	10	4	3
2948.37......	4	3	2	3786.20......	1	3	3
2956.27......	4	2	2	3789.43......	tr	2	2
2983.41......	tr	2	2	3796.02......	tr	2	2
3000.99......	1	2	2	3798.44......	tr	2	2
3186.58......	10	8	3	3818.35......	tr	2	1
3192.14......	12	8	1	3836.90......	1	2	....
3200.08......	20	8	3	3846.54......	1	2	2
3203.55......	1	2	2	3921.61......	2	5	2
3214.38......	1	2	1	3924.71......	10	8	3
3234.68......	1	8	15R	3930.04......	8	8	3
3236.72......	1	5	6R	3947.98......	10	10	3
3342.01 }	50	4 }	10R	3948.87......	40	12	4
3342.30 }		3 }		3956.50......	40	15	4
3349.17 }	6	3 }	8R	3958.39......	60	15	5
3349.56 }		8 }	10R	3963.05......	6	8	3
3354.80......	60	8	3	3964.48......	6	8	3
3358.56......	1	2	....	3981.95......	50	15	3
3361.41......	8	10	30R	3982.63......	2	8	3
3370.61......	40	5	2	3989.94......	60	10	6
3371.62......	70	10	2	3998.80......	70	20	6
3373.03......	1	3	20R	4009.12......	15	8	4
3377.70......	15	10	3	4024.76......	20	10	3
3379.35......	10	3	1	4078.62......	1	8	4
3382.47......	3	3	1	4112.87......	6	5	2
3383.90......	3	3	20R	4122.30......	1	3	2
3385.80 }	20	2 }	....	4123.70......	1	3	2
3386.10 }		5 }	....	4127.67......	2	3	3
3390.84......	tr	2	2	4137.46......	tr	3	2
3394.74......	tr	2	10	4151.12......	1	3	3
3444.49......	tr	4	10	4171.20......	tr	3	2
3505.06......	1	3	30	4200.90......	tr	2	2
3610.35......	1	4	2	4282.87......	2	5	3
3635.61......	50	15	3	4286.19......	1	10	4
3642.81......	70	15	3	4289.26......	1	15	4
3653.66......	100	15	4	4291.19......	1	10	2
3654.74......	2	3	2	4295.93......	1	10	4
3658.26......	12	4	3	4298.89......	1	12	4
3660.80......	2	3	2	4300.73......	4	15	2
3669.15......	3	3	2	4301.24......	5	15	3
3671.85......	10	4	3	4306.09......	6	20	8
3685.37......	2	8	100	4314.96......	10	5	3
3690.09......	2	3	2	4318.85......	1	10	3
3717.52......	5	5	2	4321.89......	tr	8	3
3722.73......	7	3	3	4325.36......	tr	8	3
3729.97......	50	8	4	4326.54......	tr	4	2
3741.25......	70	15	2	4427.28......	tr	10	4
3748.22......	1	2	6	4430.52......	tr	3	2
3753.00......	100	15	5	4434.16......	tr	5	3

TABLE II—*Continued*

λ (Exner and Haschek)	Luminescence	Arc	Spark	λ (Exner and Haschek)	Luminescence	Arc	Spark
4449.35	1	10	5	4599.40	tr	3	1
4451.13	1	8	3	4617.40	tr	10	8
4453.52	12	8	3	4656.63	25	8	3
4455.50	12	12	4	4667.77	25	10	5
4457.61	15	15	5	4682.10	30	10	6
4496.35	tr	10	3	4981.93	6	20	10
4512.90	1	15	4	4991.24	5	20	10
4518.19	2	15	4	4999.68	4	20	10
4522.98	2	15	4	5007.35	2	20	10
4527.47	1	15	4	5014.39	8	20	8
4533.40	20	20	5	5036.08	2	10	8
4534.95	10	15	4	5036.65		10	8
4535.71	40	8	3	5038.59	1	10	8
4536.12		10	2	5040.14	3	10	3
4536.25		10	2	5064.79	3	10	5
4544.88	1	10	3	5173.92	1	15	5
4548.93	1	8	3	5193.12	1	20	8
4552.70	1	10	4	5210.59	2	20	10
4555.70	1	10	3				

titanium vacuum arc also does not show differences from the ordinary arc in the direction of a resemblance to the luminescence spectrum. The effects point to a distinctive character in the emission resulting from the cathode bombardment.

MAGNESIUM

Magnesium metal was used as the anticathode, and spectra were obtained both for the usual arrangement with an insulated target and when it was connected as anode. The intensities of the lines under these conditions are given in Table III. The difference found was chiefly in the general intensity of the light, the anode connection giving a much stronger spectrum when the same length of parallel spark was used.

An interesting mixture of arc and spark lines is presented by these high-vacuum spectra. Six ultra-violet enhanced lines, λλ 2791, 2796, 2798, 2803, 2929, and 2937, are strong for both arrangements of the anticathode. These are placed by Fowler[1] in a series of "wide doublets." The much-studied enhanced line λ 4481, belonging to a different series, is just visible in the luminescence spectrum, and in the anode spectrum is much weaker than the adjacent arc lines. The ultra-violet doublets appear in the ordinary arc in air, while λ 4481 does not. It appears that in the high-vacuum spectra for magnesium we have a slight approach to the spark spectrum, but not as pronounced as can be obtained in the arc under special conditions, either in air at a few millimeters pressure, in hydrogen, or in the "tube-arc" formed when the furnace tube is burned apart at a low voltage. In these sources λ 4481 appears with great intensity. Without further data on the state of the electric field producing the high-vacuum spectrum, it is perhaps useless to speculate on what features may bring about this difference.

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[1] *Philosophical Transactions of the Royal Society*, A, 214, 225, 1914.

TABLE III

MAGNESIUM

λ (Exner and Haschek)	Luminescence	Anticathode as Anode	λ (Exner and Haschek)	Luminescence	Anticathode as Anode
2630.52	....	1	2936.99	15	30
2633.13	....	2	2938.67	4	8
2646.61	....	tr	2942.21	6	15
2649.30	....	tr	3091.18	8	30
2659.5	....	1	3093.14	20	50
2668.26	....	2	3097.06	30	60
2669.84	....	6	3106.5	....	3
2672.90	....	8	3330.08	5	10
2695.53	....	1	3332.28	10	20
2698.44	....	2	3336.83	15	40
2732.35	1	5	3829.51	15	50
2733.80	5	10	3832.46	40	80
2736.84	8	12	3838.44	60	100
2765.47	....	2	3904.0	1	2
2768.57	1	5	3938.6	2	3
2776.80	3	12	3987.08	3	5
2778.36	2	10	4058.45	5	8
2779.94	10	15	4167.81	8	12
2781.53	3	10	4352.18	12	20
2783.08	2	10	4481.3	1	6
2790.88	10	20	4571.33	10	25
2795.63	80	80	4703.33	10	35
2798.07	15	20	4730.42	1	4
2802.80	60	60	5167.55	2	15
2846.91	1	8	5172.87		15
2848.53	2	10	5183.84	2	30
2852.22	500	500	5528.75	....	25
2915.57	....	tr	5711.38	....	2
2928.74	8	20			

The line λ 4571, faint in both arc and spark, is strong in the luminescence and in the anode spectra. This furnished the only point of similarity to the furnace spectrum, in which at low temperature λ 4571 dominates this region of the spectrum.

A high intensity of the Rydberg series of single lines is a notable feature of both of the high-vacuum spectra. The members in this list are λλ 5529, 4703, and λλ 4352–3987. Two other lines of this series appear, which have been observed by Fowler and Payn[1] in the vacuum arc. We have obtained these two lines also by prolonged exposure of the magnesium arc in air, the spectrum being formed by the 15-foot concave grating. Their wave-lengths were then measured as 3938.6 and 3904.0, close to the calculated values of the tenth and eleventh members of the Rydberg series.

Janicki and Seeliger[2] obtained several metallic spectra in a discharge at low pressure and observed the lines occurring near the anode and cathode respectively. Those in the positive column were the stronger arc lines, while in the cathode glow the spectrum contained also enhanced lines. Comparing their results for magnesium with ours, it is evident that the presence of enhanced lines in the luminescence spectrum and in that obtained with the anticathode connected as anode corresponds most nearly with conditions in the cathode glow spectrum of Janicki and Seeliger, even when we observed the vapor close to the anode. The vacuum employed by us was probably much higher than the vacuum they used, and the similarity to the cathode glow spectrum indicates that, with a high vacuum and the concentration of cathode particles which we have employed, the cathode emission is transferred to the focus of the stream at the anticathode. With magnesium the effect of the cathode bombardment at this point appears to be practically the same when the anticathode is also the anode.

CADMIUM

The cadmium luminescence spectrum shows a much closer similarity to that of the spark than was found for magnesium. The lines obtained are listed in Table IV, with their intensities, and also the arc and spark intensities given by Exner and Haschek. Three enhanced lines, λλ 3250.51, 3535.83, and 4415.89, are among the stronger lines of the luminescence spectrum. Certain strong arc lines are seen by the table to be greatly weakened in the luminescence.

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[1] *Proceedings of the Royal Society*, 72, 253, 1904.

[2] *Annalen der Physik* (4), 44, 1151, 1914.

The vacuum arc spectrum of cadmium was photographed for comparison, but no large difference from that of the arc in air was observed, the effects offering a decided contrast with those for magnesium, for which the spectrum of the arc in vacuum is more like that of the spark than is the luminescence spectrum.

TABLE IV

CADMIUM

λ (Exner and Haschek)	Luminescence	Arc	Spark	λ (Exner and Haschek)	Luminescence	Arc	Spark
2265.11	6	2	10	3467.76	3	50	15
2288.09*	50	500	10	3535.83	4	1	4
2880.89	2	10	3	3610.72	40	500	100
2980.80	3	30	3	3613.04	5	50	15
3250.51	30	......	10	4415.89	100	1	20
3252.86	1	20	3	4662.8	4	5	2
3261.23	100	20	5	4678.38	6	50	50
3403.86	5	100	30	4800.14	10	100	100
3466.33	15	100	30	5086.06	5	100	50

* Reverses widely in arc, with low photographic density. Not given in arc tables of Exner and Haschek.

A comparison with the results of Janicki and Seeliger[1] shows, as with magnesium, a qualitative correspondence with the spectrum of the negative glow, the lines strengthened in the luminescence spectrum being absent in that of the positive column.

CALCIUM

Two successful spectrograms for calcium with the quartz spectrograph confirmed the effects described in the former paper, the distinctive features being great intensity of λ 4227 and relatively high strength of the spark line-pairs and members of the single-line series. The position of the luminescence spectrum of calcium with reference to the arc and spark spectra is about the same as in the case of magnesium.

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[1] *Loc. cit.*

ANODE SPECTRUM OF IRON

Several attempts to obtain the luminescence spectrum of iron yielded no better results than those reported in the former paper. An experiment in which the anticathode was made the anode, however, gave a spectrum of such distinctive character that it seems worth while to print a list of the lines which appeared and the results of a brief comparison with other sources. The vapor a few millimeters above the anticathode was photographed in the same way as when the anticathode was without anode connection. A current of 15 milliamperes was used, with a parallel spark-gap in air varying from 22 to 38 mm.

The lines of the spectrum thus obtained, with their intensities, are given in Table V. The effect is that of a picking out of certain arc lines, but this selection results in a spectrum apparently unique. Among its many differences from the ordinary arc or spark spectrum may be mentioned the extreme faintness of the triplets, whose strongest lines are at λ 4045 and λ 4383, and the absence of enhanced lines. Certain low-temperature lines are strong, λ 4376 being twice as strong as λ 4383, while λλ 4427, 4461, and 4482 are relatively prominent. This feature can be reproduced at low temperatures of the electric furnace, from 1500° to 1600° C., and in the low-temperature flames. In the furnace at this temperature, however, λ 4260 does not appear, and the very strong anode line λ 4227.45 belongs also to higher furnace temperatures. The vacuum arc spectrum of iron shows no approach to this high-vacuum spectrum. In the ultra-violet, while many of the strong lines are prominent also in arc, spark, and furnace, others are abnormally strong in the anode spectrum, and many arc lines of high intensity do not appear. The extension toward shorter waves is much beyond that reached by the high-temperature furnace, though the spectrum of the latter is richer in the region which it covers.

Plate XV illustrates the strongly selective action of the anode discharge as compared with the arc in air. The lines not belonging to iron are the manganese triplet at λ 4030 and several nitrogen bands. The latter occur in both the negative and the positive pole groups. The presence of the negative bands is another indication of cathode conditions due to the pencil of rays focused here. Both kinds of bands were found also when the anticathode was insulated.

TABLE V

ANODE SPECTRUM OF IRON

λ (I A)	I	λ (I A)	I	λ (I A)	I
2462.65	1	3465.86	3	3859.91	100
2472.91	1	3475.45	8	3865.53	1
2479.78	1	3476.71	1	3872.51	2
2483.28	10	3490.58	15	3878.02	2
2488.15	7	3497.11	tr	3878.58	5
2490.66	5	3497.84	1	3886.29	30
2491.16	2	3581.20	8	3895.66	2
2522.86	6	3608.86	1	3899.71	3
2527.44	3	3610.15	1	3902.95	2
2545.98	1	3618.77	1	3920.26	1
2549.62	2	3631.46	2	3922.92	6
2719.04	5	3647.85	1	3927.93	5
2720.91	2	3679.92	7	3930.30	5
2741.83	2	3687.46	2	4045.82	2
2749.49	4	3705.57	12	4062.45	1
2966.90	2	3709.25	3	4063.60	2
2981.45	tr	3719.94	150	4066.98	2
2983.57	3	3722.57	10	4067.99	1
2994.43	4	3727.62	1	4071.75	1
2999.52	35	3733.32	4	4084.51	1
3000.95	2	3734.87	100	4198.31	1
3008.14	1	3737.14	100	4216.19	2
3009.58	4	3745.56	80	4227.45	15
3018.99	1	3748.26	20	4238.83	1
3020.50	40	3749.49	40	4247.44	tr
3020.64		3758.23	15	4258.39	1
3021.08		3763.79	5	4260.49	2
3024.04	1	3767.19	2	4271.76	3
3025.85	1	3795.00	2	4282.41	1
3037.39	3	3812.97	2	4307.91	tr
3040.43	2	3815.84	2	4325.77	tr
3047.61	4	3820.43	100	4375.93	4
3057.45	20	3824.44	10	4383.55	2
3059.09	5	3825.89	50	4404.75	1
3067.25	5	3827.83	tr	4427.31	3
3286.76	1	3834.23	15	4461.66	2
3440.61	100	3840.44	4	4482.26	1
3440.99	50	3849.97	1	4528.62	2
3443.88	1	3856.37	10		

SUMMARY OF RESULTS

Fairly rich spectra of manganese, titanium, iron, magnesium, calcium, and cadmium have been obtained by vaporizing the metal at the focus of a beam of cathode rays in a high vacuum and photographing the spectrum of the vapor in the path of the rays. The spectrum of iron was obtained by making the anticathode the anode, but photographs of the magnesium spectrum made in both ways indicated that, although the spectrum was more intense when the metal acted as anode, it did not differ in character from that produced with a separate anode so placed that the anticathode was out of the path of the current.

Comparison of these luminescence spectra with the arc, spark, and furnace spectra of the same elements shows that, although in general the majority of the lines are those easily excited in the arc and furnace, the following peculiarities mark this as a source having special characteristics:

1. Certain arc and furnace lines are intensified, while other lines, strong in these sources, are greatly weakened or are absent.

2. The resemblance to the spark spectrum, as measured by the tendency to give enhanced lines, differs greatly for different elements. The production of enhanced lines is very marked in the case of cadmium; they are present also to a considerable extent with magnesium and calcium, but to a very slight degree for titanium, manganese, and iron.

3. Where series relations are known, "single-line" series are likely to show a relatively greater intensity than other series lines.

4. There is a relatively high intensity of lines in the ultra-violet as compared with the arc and furnace spectra.

MOUNT WILSON SOLAR OBSERVATORY
March 1919

MONOCHROMATIC AND NEUTRAL-TINT SCREENS IN OPTICAL PYROMETRY

By W. E. FORSYTHE

It is very difficult to find a monochromatic or a neutral-tint glass screen. Very fortunately for optical pyrometry absolutely monochromatic or absolutely neutral-tint screens are not necessary. In general, for the so-called monochromatic screen, what is wanted is a screen that is sufficiently monochromatic to enable different observers to obtain very closely the same comparisons in brightness between the comparison source (or pyrometer filament for the Morse pyrometer) and the source that is being investigated.

In Fig. 1 is shown the spectral transmission of several red glasses. It has been found that two thicknesses of any one of the glasses whose transmission is shown by the curves *B*, *C*, and *D* give a transmission band sufficiently monochromatic to permit comparisons of brightness to be made between a pyrometer filament and a source, even where the color-difference corresponds to that of two black bodies operated at 1800° and 3100° K respectively. At the same time two thicknesses of each of the glasses whose transmissions are given by curves *B* and *C* transmit enough light to permit comparisons to be made for sources at very low temperatures.

The spectral transmission of the different pieces of glass was measured with the spectral pyrometer shown diagrammatically in Fig. 2. This instrument is quite similar to the one used by Henning.[1] The pyrometer filament was made of two-mil tungsten wire and was so mounted that the image formed by the lens at *C* was horizontal, while the slit was vertical. The background lamp consisted of a large, gas-filled tungsten lamp having a ribbon filament of about two millimeters' width. This filament was so mounted that its image at the slit was vertical, and of such width as to cover much more than the slit. To match the brightness of the image of this pyrometer lamp with the background it was found

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[1] *Zeitschrift für Instrumentenkunde*, 30, 61, 1910.

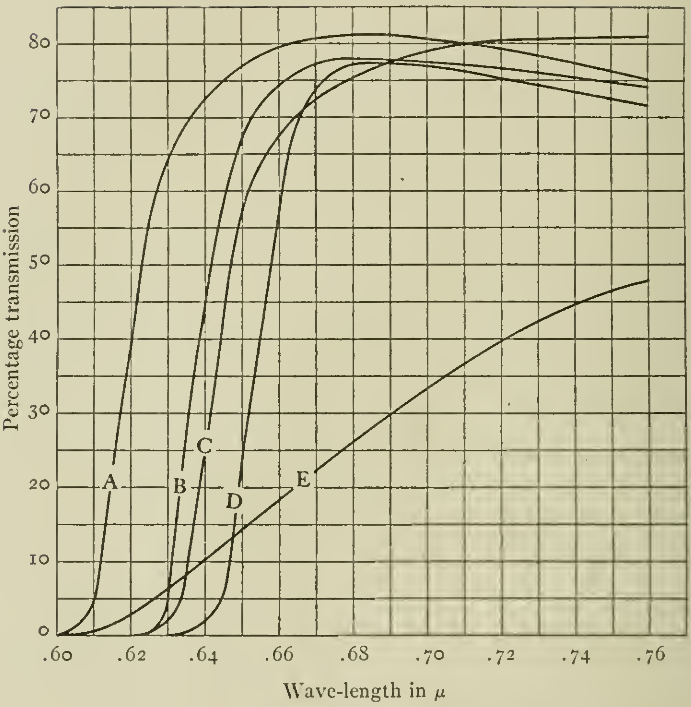

Fig. 1.—Spectral transmission of various red glasses

Curve C for Jena red 4512, 2.93 mm thick.
Curve E for Jena red 2745, 3.2 mm thick.
Curve A for Corning high-transmission red, marked 150 per cent, 5 mm thick.
Curve B for Corning high-transmission red, marked 50 per cent, 5 mm thick.
Curve D for Corning high-transmission red, marked 28 per cent, 6 mm thick.

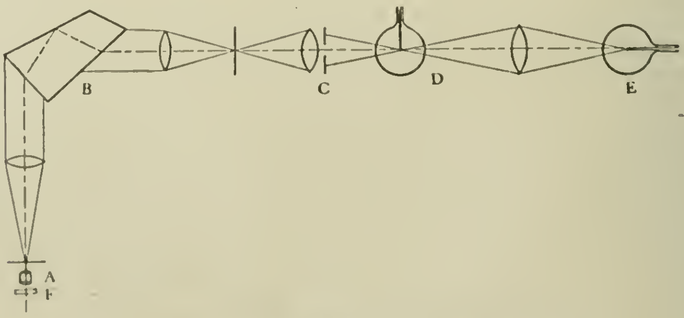

Fig. 2—Diagrammatic sketch of spectral pyrometer

necessary to have all the different parts well lined up. It was also necessary to use a diaphragm with a small opening at *C* for the more luminous parts of the spectrum. The opening in this diaphragm could be made larger for work at the ends of the spectrum, where the amount of light was much less. The image of the pyrometer filament, as seen through the eyepiece, had to be carefully focused for accurate work. As this depended somewhat upon the wave-length used, a scale was attached to the lens at *C*, so that it was possible to reset this at the proper place for the different parts of the spectrum after it had once been calibrated.

The ribbon filament background gave the very high brightness necessary for measuring transmissions that were as small as a fraction of 1 per cent. The glass whose spectral transmission was to be measured was mounted just in front of the pyrometer lamp. By making readings of the current through the pyrometer filament for an apparent match of brightness with the background, as seen first through the glass and then through sectors having very nearly the same transmission as the glass, it was possible to determine the transmission of the glass. Interpolated values were obtained by using the logarithms of the transmission of the sectors and of the current through the pyrometer lamp instead of the direct values, because the logarithms gave linear relations.

If the transmission of a piece of glass was measured over a range of wave-lengths and the currents through the pyrometer lamp plotted against wave-lengths, errors of measurements or sudden changes in the transmission of the glass were easily detected. To eliminate stray light a piece of colored glass was used at *F*. This was necessary when working in the extreme red or blue end of the spectrum. With this arrangement the transmission of different pieces of glass has been measured out to wave-length 0.76 μ. It was found that if the collimator and eyepiece slits were 0.3 mm, corresponding to a range of 0.005 μ in the transmitted interval, the slit-width corrections would be negligible.

When it is necessary to use glass absorbing screens to reduce the apparent brightness of the source studied, the main requirement is to have a screen that approximates a neutral tint sufficiently well to enable comparisons in brightness to be made by different observers with very approximately the same results. The degree to which it is necessary for the absorbing screen to have a spectral transmission independent of the wave-length depends upon the so-called monochromatic glass used in the eyepiece. It is quite evident that if this eyepiece glass is absolutely monochromatic any absorbing glass will answer.

In Fig. 3 are shown the spectral transmissions of a piece of noviweld (curve *C*) and of a piece of Jena absorbing glass (curve *B*). Either one of these glasses is nearly enough of neutral tint for use with the red glasses whose transmission curves are shown by curves *B*, *C*, and *D* in Fig. 1. The noviweld absorbing glass and also the samples of Corning high-transmission red glass were obtained from Mr. F. P. Gage, of the Corning Glass Works. This absorbing glass is made in different shades, with transmissions, when used in connection with red glass, ranging from less than 1 per cent to several per cent.

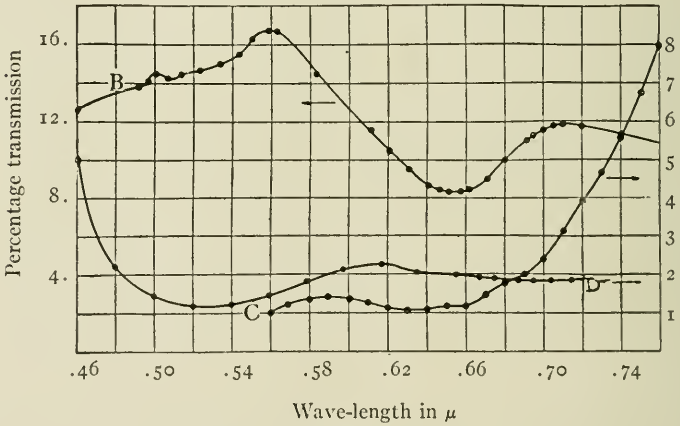

FIG. 3.—Spectral transmission of various absorbing glasses

Curve B—Jena absorbing glass 1.5 mm thick.
Curve C—Noviweld obtained from Corning Glass Works. Shade about 6.
Curve D—Leeds & Northrup absorbing glass made of purple and green glass.

If a red glass is used in the eyepiece, by total transmission for a particular temperature is meant the ratio of the brightness of the source observed through both the red glass and the absorbing glass to the brightness of the same source observed through the red glass alone. Without a red glass, using the entire visible spectrum, it is generally very difficult to make such measurements, owing to the color-differences introduced by even the best absorbing glasses, but with a good red glass in the eyepiece measurements of transmission can be made easily.

The transmission of the absorbing glass when used with a red glass can be calculated for any black-body distribution by the following formula, taken from Preston's *Theory of Light:*

$$T_B = \frac{\int_0^\infty J_\lambda V_\lambda T'_R T'_B d\lambda}{\int_0^\infty J_\lambda V_\lambda T'_R d\lambda} \qquad (1)$$

$J_\lambda d\lambda$ = black-body energy for interval λ to $\lambda + d\lambda$, V_λ = visibility, T_R and T'_B = spectral transmissions of the red glass and absorbing glass respectively. It is very evident that if the spectral transmission of the absorbing glass is different for different wave-lengths, the total transmission will be a function of the temperature of the source under investigation.

Experimental results.—First with two pieces of Jena red glass No. 4512 (spectral transmission shown by curve *C*, Fig. 1) and secondly with two pieces of Corning red 50 per cent (spectral transmission shown by curve *B*, Fig. 1) in the eyepiece of the pyrometer, readings were made on the apparent brightness of a particular source as observed through (1) a rotating sector with two one-degree openings; (2) the noviweld absorbing glass, whose spectral transmission is shown by curve *C*, Fig. 3, and (3) two pieces of the Jena absorbing glass, whose spectral transmission is shown by curve *B*, Fig. 3. The source used was a fifteen-mil tungsten lamp operated at a color temperature of 2610° K. The brightnesses were measured in terms of the current through the pyrometer filament for an apparent match of brightness. Four different observers made the measurements, three of them having had considerable experience with that kind of work, and the fourth (K.H.M.) having had much less experience. Values thus obtained are given in Table I. The maximum range with the two glasses occurs for K.H.M. and W.E.F. for the noviweld glass when the Jena red No. 4512 was used. This amounted to about 1 per cent in brightness and to less than five degrees in temperature at about 2500° K.

TABLE I

Readings by Different Observers Using Different Red Glasses and Different Absorbing Glasses

Observer	Red Glass Used	Current in Amperes through Pyrometer Filament, for Apparent Match of Brightness with:		
		2° Sector	Noviweld Absorbing Glass	Two Jena Absorbing Glasses
I.A.V.	Jena 4512	0.3358	0.3804	0.3547
K.H.M.	Jena 4512	.3361	.3807	.3546
W.E.F.	Jena 4512	.3361	.3803	.3546
A.G.W.	Jena 4512	.3358	.3805	.3547
I.A.V.	Corning red	.3380	.3784	......
K.H.M.	Corning red	.3380	.3785	......
W.E.F.	Corning red	.3380	.3783	......
A.G.W.	Corning red	.3378	.3784	......

For the currents given above, a change of 0.0005 ampere corresponds to a change of about 1 per cent in the brightness of the background. This same change in current through this pyrometer filament corresponds to about 4° K in temperature at about 2000° K.

The visibility-curves of the four observers are quite different, as can be seen by referring to a paper on "Visibility of Radiation" in the *Astrophysical Journal*, **48**, 65, 1918. Two of the observers (I.A.V. and K.H.M.) are quite blue-sensitive, one (W.E.F.) is somewhat red-sensitive, and the other (A.G.W.) is very much red-sensitive. These values extend toward the red end to wave-length 0.66 μ. In this work the visibility much beyond this point must be taken into consideration. It is not the visibility in the blue end of the spectrum that is important but rather the relative shapes of the different visibility-curves in the red end of the spectrum. In some other work[1] it was shown that, though there was a great variation in the values given by the individual observers to the brightness in the extreme red, the relative values do not vary so widely. From this it is to be expected that different observers will get very closely the same values of brightness if they are limited to the extreme red.

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[1] *Astrophysical Journal*, **42**, 285, 1915.

In Fig. 4 is shown, as a function of the temperature of the source, the transmissions for red light of the absorbing glasses whose spectral transmission is given by curves *B* and *C*, Fig. 3. The measured points were determined by the author, and the calculated values were obtained by means of equation (1) by making use of an average visibility-curve[1] for this spectral region. Values of transmission were also calculated, using the observer's visibility-curve. Values thus obtained, using the two different visibility-curves, differ from each other by only a small fraction of 1 per cent.

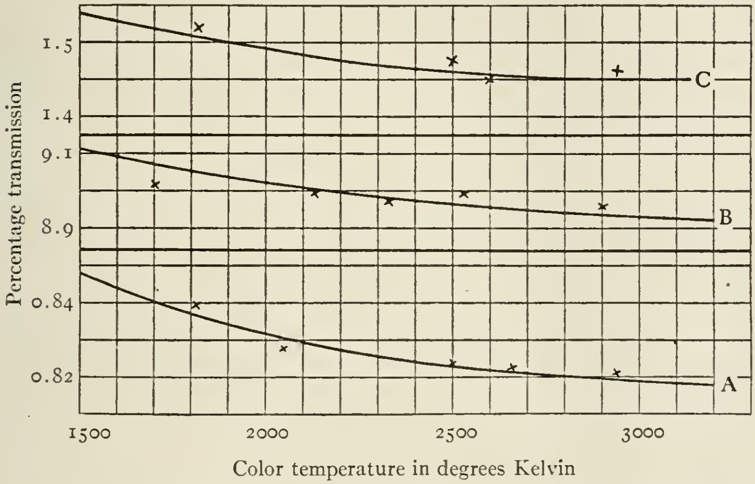

FIG. 4.—Transmission of absorbing glasses as a function of temperature when used with red glass No. 4512, 5.8 mm thick.

Curve A.—Two pieces Jena absorbing glass.

Curve B.—One piece Jena absorbing glass.

Curve C.—Noviweld glass from Corning Glass Works.

Curves drawn through points calculated from equation $T_B = \frac{\int_0^a J_\lambda V_\lambda T'_R T'_B d\lambda}{\int_0^a J_\lambda V_\lambda T'_R d\lambda}$.

Crosses represent values of transmission obtained with optical pyrometer.

Inasmuch as a question has been raised[2] concerning the accuracy of the results thus obtained, a word might be said concerning this method of calculating values of transmission. In using equation

[1] *Astrophysical Journal*, **48**, 87, 1918.

[2] *Bulletin of the Bureau of Standards*, **12**, 485, 1916.

(1) in this work, as well as in determining the effective wave-length, the integral was found by the step-by-step method. Results can be obtained that are as accurate as desired by making the steps small enough. If the two-thirds rule is used very accurate results can be obtained with fewer steps. Transmission values can thus be obtained that are as accurate as the data warrant. If the computation be carried too far, and if such values of transmission are used in determining the effective wave-lengths, the latter will, of course, appear to be extremely accurate, more so than the experimental data warrant.

An absorbing glass that is not strictly of neutral tint is frequently used to cut down the apparent brightness of a source when measuring its temperature with an optical pyrometer having a red glass in the eyepiece. It has been stated[1] that a different value of the effective wave-length of the red glass must be used in connection with the so-called neutral-tint glass from that used in connection with a rotating sector of the same transmission. In what follows it is shown that such is not the case, but that the same effective wave-length is to be used for both.

Suppose that, using the same red glass in both cases, a sector with a transmission T_S were found such that the brightness observed through the black glass would equal that observed through the sector (i.e., sector and glass have same transmission). Then

$$\int J_\lambda V_\lambda T'_B T'_R d\lambda = \int J_\lambda V_\lambda T'_R T_S d\lambda = T_S \int J_\lambda V_\lambda T'_R d\lambda \,.$$

Since the brightness is measured in terms of the current through the pyrometer filament, this current will be the same in the two cases. This means that in both cases the temperature T_2 that is being determined must be calculated from the same initial temperature.

The question to be considered is what effective wave-length is to be used in calculating the temperature of the source whose brightness is thus measured. When the brightness is measured with the use of the rotating sector the temperature T_2 is calculated from the transmission of the sector and T_1, the temperature corresponding to the pyrometer reading when no sector is used. For this calculation the following formula derived from Wien's equation is used:

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[1] *Bulletin of the Bureau of Standards*, 12, 483, 1916.

$$1/T_2 - 1/T_1 = \frac{\lambda_e \log T_S}{C_2 \log e}. \tag{2}$$

In this expression λ_e is the ordinary effective wave-length between T_1 and T_2 and is defined as the wave-length such that the ratio of the intensities of radiation for the temperature-interval for this wave-length shall equal the ratio of the integral luminosities through the screen used, or, stated in the form of an equation,

$$\left(\frac{J(\lambda T_1)}{J(\lambda T_2)}\right)_{\lambda_e} = \frac{\int J(\lambda T_1) V_\lambda T'_R d\lambda}{\int J(\lambda T_2) V_\lambda T'_R T_S d\lambda}. \tag{3}$$

When the brightness is measured with the use of an absorbing glass, the temperature T_2 must be calculated from the transmission of the absorbing glass and T_1, the temperature corresponding to the pyrometer current when no absorbing glass is used. Inasmuch as we know the ratio of the brightness of the black body at the unknown temperature T_2 to that of the black body at temperature T_1, the temperature T_2 is to be calculated by an equation similar to equation (2), that is, by

$$1/T_2 - 1/T_1 = \frac{\lambda'_e \log T_B}{C_2 \log e}, \tag{4}$$

where λ'_e is an effective wave-length. As T_1 and T_2 are the same in equations (2) and (4),

$$\lambda_e \log T_S = \lambda'_e \log T_B,$$

whence

$$\lambda_e = \lambda'_e.$$

That is, since the transmission of the absorbing glass given by equation (1) is the same as that obtained experimentally by comparing its transmission with that of a sector, the same effective wave-length of the red glass is to be used with both the absorbing glass and a sector having the same transmission.

Thus to calculate the extrapolated brightness-temperature of a source whose brightness is measured through a black glass, it is necessary to know the transmission of the glass as a function of the temperature of the source studied, and also the ordinary effective wave-length, defined by equation (3), for the red glass used. In calculating the extrapolated temperature when using a sector disk, it is necessary to know this temperature very approximately in order to find the effective wave-length for the interval. The calculation is therefore one of successive approximations. When a so-called neutral-tint glass is used, additional care is required, because both the effective wave-length and the transmission depend upon the temperature reached. When the effective wave-length of the red glass and the transmission of the absorbing glass are known, the extrapolated temperature is calculated by means of equation (4).

In Fig. 5 are shown the extrapolated temperatures that are obtained with the absorbing glasses whose transmissions are shown in Figs. 3 and 4. These values were obtained from equation (4) and the transmissions of the different glasses as shown in Fig. 4. The dotted line under each curve shows what the resulting temperature would be if the transmission of the glass did not vary with the temperature of the source but remained the same at high temperatures as was found for the lower temperatures. For the Jena absorbing glass this change amounted to only 3° K for a range from 1900° K to 2400° K, and for the noviweld glass the change amounted to 8° K for a range from 1900° K to 3000°K. These two glasses were the best ones found among those investigated. The spectral transmission of the glass obtained from Leeds & Northrup, given in Fig. 3, curve *D*, shows that this glass would give almost the same results as a sector. Lack of time prevented a complete test of this glass.

Only one absorbing glass was tested that gave any trouble due to lack of color-match of the pyrometer-lamp filament at a temperature of 1800° K and a background at 3000° K. This glass had an almost uniform spectral transmission of about one-half of 1 per cent from 0.60 μ to 0.65 μ, and from there on to longer wave-lengths increased very rapidly, being about 5 per cent at 0.68 μ, 20 per cent at 0.70 μ, and 40 per cent at 0.72 μ. Thus, in general, if a good red glass is used in the eyepiece of the pyrometer but little trouble is experienced in obtaining an absorbing glass that is satisfactory for extrapolation of the temperature-scale.

Attempts have been made to obtain an absorbing glass that is of strictly neutral tint, or even one that has such a transmission as to correct for the change in effective wave-length of the red glass used.

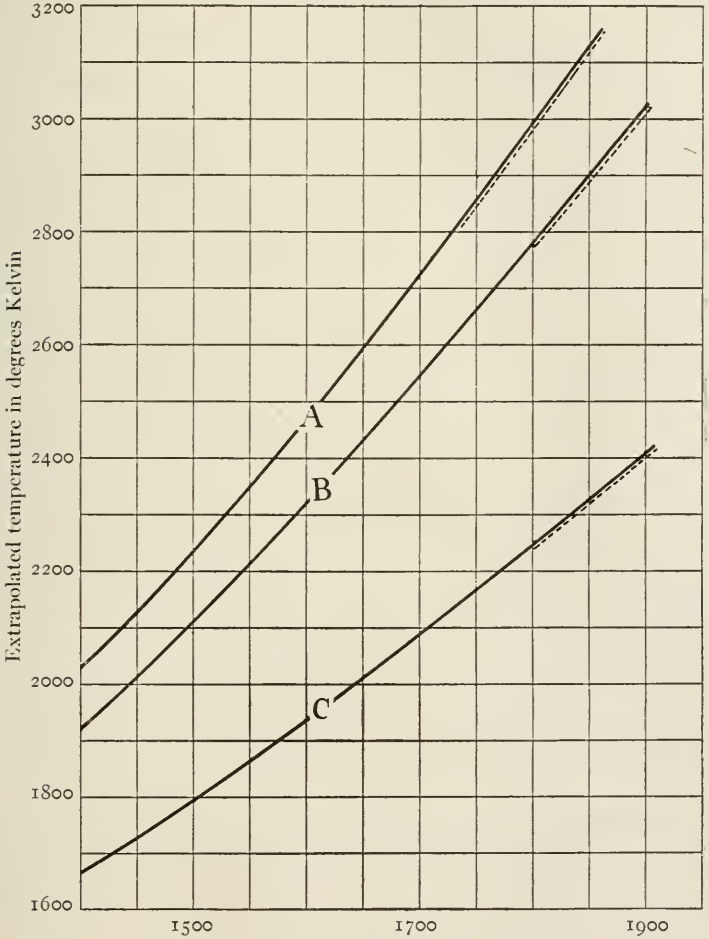

Fig. 5.—Curves showing extrapolated temperature given by various absorbing glasses.

Curve A.—Two Jena absorbing glasses.

Curve B.—One Jena absorbing glass.

Curve C.—Corning noviweld—shade about 2.4.

If obtained, this would probably be very good, but it is not at all necessary. What is wanted is a glass that will permit comparisons of brightnesses to be made by different observers with very nearly the same result. It was shown above that if a good red glass is used absorbing glasses can easily be found that are suitable. The same thing might be said about the red glasses. Many attempts have been made to obtain absolutely monochromatic screens for optical pyrometry. This is very good for some purposes, but is not necessary in general, and such glasses have the disadvantage of not transmitting enough light to permit of accurate comparisons of brightness at low temperatures. A good red glass can easily be obtained that transmits enough light to permit comparisons of brightness at low temperatures and at the same time is sufficiently monochromatic to enable different observers to obtain the same results, even under the unfavorable conditions existing when the comparison source and the source studied are quite different in temperature. In addition to this, if the effective wave-length of the red glass is known, all results can, in general, be reduced to the condition for a common wave-length.

Summary.—In this paper it has been shown that with a good red glass in the eyepiece of pyrometers absorbing glasses can be found that are near enough to being of neutral tint so that different observers obtain the same values of measured temperature. It has also been shown that the same effective wave-length of the red glass is to be used both with the rotating sector and with an absorbing glass of the same total transmission that is not of neutral tint.

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National Lamp Works of General Electric Co.
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January 1919

STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS

ELEVENTH PAPER: A COMPARISON OF THE DISTANCES OF VARIOUS CELESTIAL OBJECTS

By HARLOW SHAPLEY

The bearing that parallaxes of clusters and variable stars may have on the general structural problems of the sidereal system is well illustrated through a tabulation and diagrammatic representation of the distances of various remote celestial objects. Part of the data compared has been obtained directly or indirectly from the statistical investigations of Kapteyn, Charlier, and Dyson, and from recent trigonometric parallaxes, mainly by van Maanen, but the greater part is from the observations discussed or summarized in the preceding contributions or from similar work that will be reported more fully at another time.

Table I summarizes the distances illustrated in Fig. 1. On the scale of the diagram objects as near as the Hyades or the brighter naked-eye stars can be shown only with difficulty, even when the lines representing distances of clusters are folded many times. The accuracy of the values in the second column of the table differs considerably, and for only a few, which are averages for many objects, is there any meaning in the third significant figure. For some of the results the relative certainty is indicated in the last column, but for most of them reference must be made to the brief discussion of the individual entries to which the remainder of this paper is devoted. Special attention may be called to Section J, which contains a discussion of pre-giant stars and the variables of the Orion nebula.

A, 1. *The most distant globular cluster.*—The four most distant clusters now known are N.G.C. 7006, 4147, 6266, and 6316; the adopted parallaxes in millionths of a second are 15, 19, 19, and 20, respectively, corresponding in the mean to a distance of 55,000 parsecs (180,000 light-years). For N.G.C. 4147 the adopted value

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 156.

depends upon measures of both magnitudes and diameter, for the others on diameter alone; but, subsequent to the preparation of the seventh paper, the parallax of N.G.C. 7006 has been studied by means of the magnitudes of the brightest stars (p. 14, n. 1, of seventh paper), and for the present, at least, its distance seems

TABLE I

Comparison of the Distances of Various Stars, Clusters, and Nebulae

Object	Distance (Unit is 100 parsecs)	Remarks
A. Globular clusters:		
1. Most distant cluster........	670	N.G.C. 7006
2. Diameter of typical system..	1.5	Messier 3
3. Nearest cluster............	65	ω Centauri
4. Mean distance.............	230	69 clusters
5. Maximum $R \sin \beta$.........	500	N.G.C. 4147
6. Minimum $R \sin \beta$..........	13	N.G.C. 6656 (M 22)
B. Distance covered in a million years by constant velocity of 1180 km/sec..............	12	
C. Cepheid variables:		
1. Most distant..............	60	Three stars
2. Maximum $R \sin \beta$, periods less than a day............	29.3	Mean of five
3. Mean $R \sin \beta$, periods less than a day...............	9.63	45 variables
4. Mean $R \sin \beta$, periods greater than a day...............	1.48	94 variables
D. Eclipsing binaries:		
1. Most distant..............	31	Mean of five
2. Mean distance.............	7.6	90 variables
E. Galactic clouds near Messier 11	50	Four fields
F. Messier 37...................	40	Preliminary estimate
G. Small Magellanic Cloud:		Provisional values from variable stars
1. Radial distance...........	190	
2. Approximate diameter......	15	
3. $R \sin \beta$...................	130	
H. Three galactic novae..........	1.1	Direct measures
I. Nebulae (van Maanen):		Direct measures with 60-inch reflector
1. Two spirals...............	2	
2. Three planetaries..........	0.7	
J. Orion nebula (Kapteyn).......	1.9	*Mt. Wilson Contr.* No. 147
K. Hyades (Boss, Kapteyn)......	0.4	
L. Most distant naked-eye star...	10	
M. B-type stars (Charlier):		
1. Most distant..............	8.5	Mean of five
2. Dispersion in galactic plane.	1.8	751 stars
3. Diameter of system........	10.0	Including 95 per cent of stars
N. Distance of sun from center:		
1. Of system of globular clusters	200	69 clusters
2. Of system of B-type stars...	0.9	751 stars of Charlier's cluster
O. Width of equatorial segment...	35	Arbitrarily chosen; see note 1,p.5

definitely to be the greatest known. Further work on the colors and magnitudes, to test if this remote system is actually comparable to the nearer globular clusters, is described in the following paragraphs.

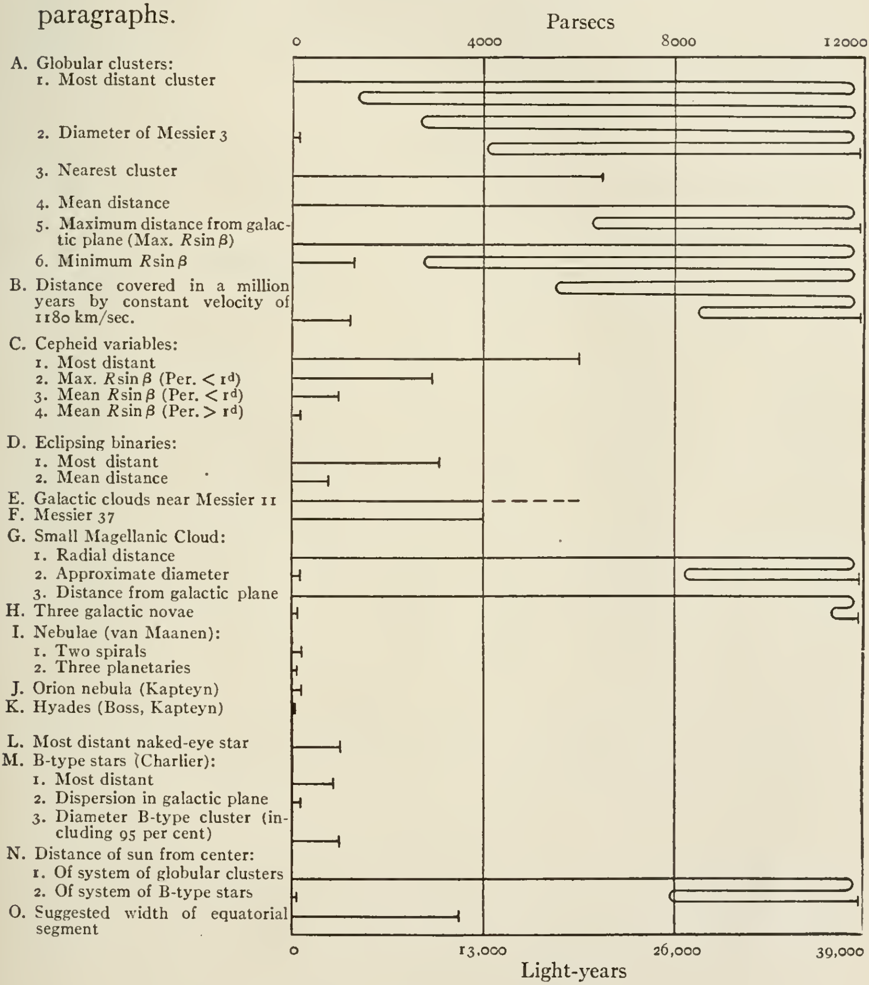

FIG. 1.—Comparison of distances in the galactic system

Since in N.G.C. 7006 the stars of zero absolute magnitude are probably of the nineteenth apparent magnitude, it is very difficult to extend the investigation of photo-visual brightness to any but the most luminous red giants. The extremely small apparent diameter also introduces difficulties of measurement and liability to systematic error if the central part is not carefully avoided.

Table II contains a list of all the stars between $0\overset{\prime}{.}3$ and $1\overset{\prime}{.}0$ from the center that could be measured on a photo-visual plate of 45 minutes exposure. Probably not more than two or three non-cluster stars are included.

TABLE II

MAGNITUDES OF THE BRIGHT STARS IN N.G.C. 7006

Star	Pg. Mag.	Pv. Mag.	Star	Pg. Mag.	Pv. Mag.
17	17.06	15.76	37	17.19	16.27
18	17.15	15.64	38	17.53	16.51
19	17.13	16.27	39	16.85	15.76
20	17.40	16.42	40	17.97	16.71
21	17.93	16.98	41	18.01	17.37
22	18.01	16.79	42	16.89	15.90
23	17.80	16.95	44	17.89	16.97
24	17.89	16.86	45	17.76	16.76
25	18.01	16.97	46	17.93	16.60
26	17.53	16.32	48	17.89	16.46
27	17.69	16.79	49	17.83	16.57
28	17.31	15.77	50	18.39	17.32
29	16.29	15.26	51	17.49	16.90
30	16.99	16.42	52	17.62	16.51
31	17.58	16.26	53	17.76	16.46
32	16.21	15.71	54	17.26	16.19
33	17.31	15.71	55	18.21	17.01
34	16.87	15.45	56	18.04	16.77
35	17.61	16.90	57	16.99	16.45
36	17.76	16.46			

The results are grouped in Table III according to photo-visual magnitude in order to show, as far as the data go, the typical decrease of color with absolute brightness. Star No. 32, being at some distance from the center and peculiarly blue for its magnitude, is probably not a member of the cluster and has been excluded from the table. Were it retained, the first mean magnitude would be 13.59, with a corresponding color-index of +1.23. The table also contains analogous data[1] for Messier 3 and 13. The frequency of stars of high luminosity is much the same in the three systems; further, the mean color-index is the same for the brighter stars and shows for all clusters a similar progressive decrease. Hence this very distant faint cluster of small angular diameter apparently differs no more in constituency and stage of development from the nearer systems than they differ among themselves. But as far as our present photometric study is concerned, N.G.C. 7006 is nearly two hundred thousand years younger than Messier 13—a significant phenomenon that will be considered further in the following paper.

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[1] One bright star in Messier 13, with color-index −0.52, is excluded.

The evidence of Table III bearing on the question of the scattering of light in space is of considerable interest. Granting the physical similarity of the three clusters, we find no appreciable difference in color on account of the much greater distance of N.G.C. 7006. The total effect of interstellar media on the color-indices of stars in this most distant system apparently does not much exceed a tenth of a magnitude. Therefore the "absorption coefficient," expressed as change of color-index for each parsec of distance, is

$$d < 0.000002 \text{ mag.},$$

a value but one-fifth the upper limit found from the study of nearby globular clusters and a hundredth the smallest value derived from the parallaxes and colors of isolated bright stars. We note, however, that the galactic latitude of N.G.C. 7006 is -20°, so that only 8 per cent of the light-path lies within the equatorial segment[1] devoid of globular clusters. If we assume that all the diminution

TABLE III

COMPARISON OF N.G.C. 7006 WITH MESSIER 3 AND 13

N.G.C. 7006			Messier 3			Messier 13		
Mean Pv. Mag.	No. of Stars	Mean Color-Index	Mean Pv. Mag.	No. of Stars	Mean Color-Index	Mean Pv. Mag.	No. of Stars	Mean Color-Index
15.56..	5	+1.37	12.59..	7	+1.30	12.11..	6	+1.31
16.02..	6	+1.14	12.90..	5	+1.18	12.47..	7	+1.14
16.41..	7	+1.04	13.10..	7	+1.61	12.72..	5	+0.94
16.55..	6	+1.16	13.43..	9	+1.12	12.87..	6	+0.82
16.82..	7	+0.96	13.70..	7	+0.99	13.05..	6	+0.92
17.08..	7	+0.95	(13.90..	13	+0.96)	13.14..	6	+0.93
16.46..	38	+1.09	13.17..	35	+1.15	12.72..	36	+1.02

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[1] The adopted width of the segment is 3500 parsecs; in Figs. 4 and 5 of the seventh paper the shaded area representing the mid-galactic segment is 4000 parsecs wide, but includes five clusters. For Messier 13 one-fourth of the light-path is within this region.

of light through scattering (if there be any at all) occurs within this region where stellar matter appears to be greatly concentrated, the value of d is still sufficiently small to be safely ignored in all ordinary stellar problems.

The magnitudes of Table III can be used for a new determination of the parallaxes of Messier 13 and N.G.C. 7006. The parallax of Messier 3, being one of the best determinations for any globular cluster, is adopted for the computation. Assuming that stars of the same absolute magnitude are involved in the final means for each cluster, we derive readily:

Messier 3,	$\pi = (0''.000072)$
Messier 13,	$\pi = 0.000089$
N.G.C. 7006,	$\pi = 0.000016$

The previously adopted parallaxes of the last two clusters are 0''.000090 and 0''.000015, respectively. It is to be noted that the present result is based upon photo-visual magnitudes, and that the five brightest stars of each cluster have not been excluded. The almost exact agreement with the former values merely shows that the similarity in the amount and frequency of color among the giant stars in clusters makes photo-visual as well as photographic magnitude a fairly definite criterion of distance.

A, 2. *The diameter of Messier 3.*—In the seventh paper of this series appears a discussion of the size of globular clusters. The occurrence of cluster stars far beyond the limits shown by the usual photograph has been inferred by Bailey and others from the distribution of cluster-type variables. We have recently found, however, that among galactic stars the variables of this type are peculiar in their occasionally great distance from the galactic plane, and for the five so far investigated the radial velocity is abnormally high. Is it possible that high velocity or some other cause has also scattered the short-period variables of clusters beyond the domain occupied by other cluster stars? To answer this question long-exposure photographs of the brighter systems are being made, with the clusters 15′ or 20′ out of center. The counts so far available show many faint cluster stars as far from the center as any of the variables. The angular diameter of Messier 15, for instance, appears to be not less than 35′, corresponding to a linear diameter of 150 parsecs. These counts indicate, first, that the stars of the outer parts of clusters may be systematically fainter, smaller, and less massive than those close to or at moderate distances from the center; secondly, that the space outside the equatorial galactic region may be occupied not only by occasional cluster-type variables but also by other isolated stellar bodies that possibly are relatively faint; for the time being, however, both suggestions must be regarded as tentative.

A, 3. *The nearest globular cluster.*—The parallax of ω Centauri ($\pi = 0\overset{''}{.}00015$) is uncertain because of the provisional nature of the magnitudes of the variables and bright stars. The adopted parallax of 47 Tucanae (N.G.C. 104) is nearly as large, but it is also lacking in precision, being derived from an uncertain part of the parallax-diameter curve.[1] The distances of both clusters can be very accurately determined when the variables have been further studied, but for the present an uncertainty of some 30 per cent may affect the adopted values. In general the largest as well as the very smallest cluster parallaxes are of relatively low accuracy, depending completely or in part upon the extremities of the curve relating parallax to diameter.

A, 5. *Maximum distance from the plane of the Galaxy.*—Five globular clusters are more than 20,000 parsecs from the galactic plane, but the small faint cluster N.G.C. 4147 in galactic latitude +78° is nearly twice as remote from the dense stellar regions as the next most distant system. Its peculiar position in space suggests the possibility of peculiarity in structure and content—perhaps in this case the methods of determining distance may not be strictly applicable. The adopted parallax appears to be of the right order, however, as the survey of the colors of the brightest objects in the cluster (Table IV) indicates that we are dealing with typical giants; but a deficiency in the number of giant stars is indicated by this table, and long exposures on fast plates suggest a deficiency of faint stars as well. Further, Table II of the seventh paper contains evidence that the angular diameter is somewhat small for the magnitude of the bright stars.

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[1] *Mt. Wilson Contr.* No. 152, Fig. 1.

This discrepancy between the values of the parallax of N.G.C. 4147 as based on the provisional magnitudes and on the estimates of diameter was considered unimportant until a verification of the earlier value of the mean magnitude was obtained through a recent study of additional plates. It now appears that the adopted parallax may be a few millionths of a second too small, and the distances given in the diagram and in Table I of this paper may be somewhat too large. Because of its high galactic latitude, however, the cluster will probably remain the most isolated of known systems, even when further work on the colors permits a definite revision of the parallax.

TABLE IV

COLORS AND MAGNITUDES OF GIANT STARS IN N.G.C. 4147

Star	Pv. Mag.	Color-Index	Star	Pv. Mag.	Color-Index
1	14.66	+0.99	18	15.74	+0.51
2	15.31	+0.71	22	13.59	+1.83
3	15.62	+0.76	23	15.92	+0.41
5	15.89	+0.08	24	15.33	+0.97
6	14.74	+1.08	29	15.95	+0.70
11	15.92	+0.69	30	15.81	+0.42
13	15.76	+0.52	35	15.74	+0.73
17	15.53	+0.76			

B. *Distance traversed in a million years by maximum velocity.*—The highest speed of translation so far recorded for any celestial object is 1180 km/sec.—the mean radial velocity of the spiral nebula N.G.C. 4594.[1] The corresponding entry in Fig. 1 permits an easy consideration of the possible past and future relationship of the various sidereal bodies. The greatest radial velocity recorded for a globular cluster is about one-third of the foregoing value and for isolated stars about one-half.[2] For discussions of this nature it is convenient that a velocity of a kilometer a second is very nearly equivalent to a velocity of a parsec in a million years.[3]

C. *Cepheid variables.*—The data for Cepheid variables of the galactic system are from Tables I and I *a* of the eighth paper of this series. The three most distant are VX Cygni, RU Boötis, and SW Herculis, the last two being cluster-type variables. The difference in the dispersion with respect to the galactic plane is well illustrated by C, 3 and C, 4 for Cepheids of short and long period, respectively.

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[1] Pease, *Mt. Wilson Communication*, No. 32; *Proc. Nat. Acad. Sci.*, **2**, 517, 1916.

[2] See Table I of the following paper.

[3] More exactly it is 1.02 parsecs.

The maximum $R \sin \beta$ for Cepheids with periods less than a day is the mean of the values for RU Boötis, SW Herculis, 169.1907 Leonis, RR and RU Canum Venaticorum; it is much greater than the adopted semi-width of the equatorial segment that is devoid of globular clusters. The corresponding value of maximum $R \sin \beta$ for periods greater than a day (excluding Z Canum Venaticorum) is 6.70.

D. *Eclipsing variables.*—With a few revisions based upon later orbital computations, the data for eclipsing binaries are taken from a paper by Russell and Shapley.[1] The five most distant variables are W Crucis, SS Carinae, UZ Cygni, RV Persei, and VV Cygni. The quantity designated as the average distance in the diagram in the *Publications of the Astronomical Society of the Pacific* for February, 1918, is the distance corresponding to the mean parallax—a quantity that, because of the wide range of values, is less than half the mean distance now computed.[2]

The parallaxes of eclipsing binaries, even of those with well-determined light-curves and spectra, are somewhat less accurately known than the parallaxes of average Cepheid variables. They depend upon the apparent magnitude, mass, surface-brightness, and radius of the brighter component of the eclipsing pairs, and in the evaluation of some of these quantities approximations are necessary. For the most favorable cases, however, the estimated error in the hypothetical parallaxes is not in excess of 25 per cent, and is essentially independent of the distance.

Direct measures of the parallax of several eclipsing stars, while yielding results of the same order of magnitude as the probable errors, serve to confirm the hypothetical values. Nearly all of the direct determinations have been made by various observers at the

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[1] *Astrophysical Journal*, 40, 417, 1914.

[2] In addition to the inclusion of more objects, the present diagram differs from the earlier one in several minor details.

Yerkes Observatory;[1] a summary of their results is given in Table V.

On the average the hypothetical parallaxes differ from the trigonometric values (corrected for comparison-star parallax by means of the Greenwich table)[2] by less than the probable error of the direct measures; systematically they are 0.″002 larger than the trigonometric results. The mean trigonometric parallax of β Persei, using all sources, is +0.″034±0.″011. For β Aurigae, an eclipsing binary not on the Yerkes list, the mean value of the parallax by three observers is +0.″034±0.″015; the hypothetical value is +0.″030, and if the star is a member of the Ursa Major group its parallax is +0.″025.[3] By entering these two mean values in Table V the small systematic difference disappears.[4]

TABLE V

COMPARISON OF HYPOTHETICAL AND TRIGONOMETRIC PARALLAXES OF ECLIPSING BINARIES (YERKES RESULTS)

Star	Measured by	Trigonometric Parallax (Abs.)	Probable Error	Hypothetical Parallax	Difference
β Persei	Lee	+0.″025	±0.″014	+0.″026	−0.″001
W Ursae Majoris	Lee	+0.011	±0.010	+0.020	−0.009
U Ophiuchi	Joy	−0.008	±0.004	+0.006	−0.014
Z Herculis	Van Biesbroeck	+0.031	±0.009*	+0.009	+0.022
RX Herculis	Van Biesbroeck	−0.006	±0.016	+0.006	−0.012
Y Cygni	Mitchell	+0.004	±0.010	+0.004	0.000
Mean			±0.0105		±0.0097

*The probable error is given as ±0.″010 in the collected results, p. 57, of *Yerkes Publications*, 4, Part I.

As a class the eclipsing variables are too remote for direct measures of parallax, but the foregoing comparison for the nearer ones shows that the method based upon orbital data is adequate to locate them in space with considerable certainty.

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[1] *Publications of the Yerkes Observatory*, 4, Part I, 1917.

[2] Dyson and Thackeray, *Monthly Notices*, 77, 14, 1916. The Greenwich values should all be decreased by 10 per cent if instead of 19.5 km/sec. we adopt for the velocity of the sun the value given in Sec. 2 of the sixth paper. The correction is unimportant in the present case.

[3] Erroneously printed as +0.″019 in *Astrophysical Journal*, 40, 425, 1914.

[4] *Note added to proof*. An overlooked parallax of u Herculis by Miller: Trig. π (abs.) = −0.″026, hyp. π = +0.″005.

E. *Galactic clouds.*—The study of the colors of stars near Messier 11 indicates the great extent of the galactic clouds in the line of sight;[1] the maximum distance so far estimated depends only on the limit of magnitude attainable on the photo-visual plates. Photographs of the field of Messier 11 are now available for the extension of this work to the fainter and more remote stars. It appears very probable, from the preliminary results, that isolated stars as distant as 30,000 parsecs may be found near the galactic plane in these southern star clouds.

F. *Messier 37 (N.G.C. 2099).*—This northern object, one of the richest of the bright open clusters, is situated near the galactic plane, almost diametrically opposite the point of concentration of globular clusters. A catalogue, now in manuscript, of the magnitudes and colors of several hundred of its stars will be published later as a paper of this series.

G. *Magellanic clouds.*—The adopted parallax of the Small Magellanic Cloud is that given in Table I of the seventh paper—a value undoubtedly of the right order but one which will be replaced by a value of much greater weight when the magnitudes of the variables have been referred to a standard system. The adopted mean angular diameter is $4^{\circ}.5$, representing the region throughout which the variable stars belonging to the cloud are distributed. We infer from the similarity of the provisional magnitudes of the variables that the distance of the two Magellanic Clouds is of the same order, but the Larger Cloud covers more than double the area.

H. *Galactic novae.*—The three objects for which directly determined parallaxes are available are Nova Persei No. 2, Nova Lacertae, and Nova Geminorum No. 2. The derivation of the adopted mean value of $+0''.009$ will be discussed by van Maanen in a forthcoming contribution.

I. *Spiral and planetary nebulae.*—The direct measurement of the parallaxes of nebulae will be considered by van Maanen in the same paper. He attaches little weight to the measured results for the spirals because of the non-stellar character of the central nuclei. One of the spirals is the Great Andromeda Nebula (N.G.C. 224);

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[1] The fifth paper of this series, *Mt. Wilson Contr.* No. 133, 1917.

the other is Messier 51 (N.G.C. 5194). The planetaries are N.G.C. 2392, 6720, and 7662. The adopted parallax of N.G.C. 6720, the ring nebula in Lyra, is the mean of the values by van Maanen and by Newkirk.

J. *The Orion nebula.*—On the basis of his exhaustive study of the motions of the B-type stars, Kapteyn adopts as the parallax of the "nebula-group" in Orion:

$$\pi = 0\rlap{.}''0054 \pm 0\rlap{.}''0009.$$

That the same parallax is the best available for the nebula itself can hardly be doubted.

It does not seem to have been pointed out that if the numerous variables of the Orion nebula are a part of the nebula-group we have in them a remarkably large assemblage of stars of low luminosity. Of more than 100 stars within two or three degrees of the trapezium that are supposed to be definitely variable, nearly all are fainter than the fourteenth photographic magnitude at maximum, and many never become brighter than the fifteenth magnitude. With a parallax of $0\rlap{.}''005$ the average absolute brightness at maximum is only about a tenth that of the sun, and at minimum less than a twentieth. Apparently they are dwarfs, while all other variable stars—Cepheids, eclipsing binaries, long-period variables (at maximum)—are typically objects of high luminosity.

The possibility that in the faint variable stars of Orion, which appear to be intimately associated with diffuse nebulosity, we have examples of stars in the pre-giant stage of stellar evolution, led to the initiation, nearly four years ago, of systematic observations of their magnitudes and colors. Such variables might be of considerable importance, for if Russell's theory of the evolutionary sequence of spectral types be accepted and we desire to find a record of the parentage and earliest development of the relatively young, highly luminous, low-temperature and low-density red giant stars, it seems obvious that we should look for (1) objects varying in light secularly or irregularly rather than with definite period and amplitude; (2) objects closely associated with the matter from which typical stars originate—nebulosity, we usually say, that is formless or in a quite chaotic state; (3) objects of faint absolute magnitude and low-surface temperature, therefore very red and possibly with striking peculiarities of spectrum. At least some of these properties are naturally thought of as attributes of the pre-giant stage; and the Orion variables appear upon first examination to be promising subjects for inquiry.

If, however, we cannot find satisfactory evidence of pre-giants among such stars as the Orion variables, the typical long-period variables, or the occasional irregular variables that are associated with variable nebulosity, there remain at least three ways (before we need abandon Russell's theory) in which we may attempt to account for the absence from our present records[1] of recognized visible forerunners of the low-density red giants. We may suppose (1) that the normal transformation of an invisible nebular mass to a highly luminous star is infrequent as compared with the rapidity of the change; astronomical history is so brief, on that hypothesis, that it contains as yet no definite record of a secularly brightening red star or of the occurrence of a red nova that maintains its maximum; or (2) that the phenomenon of rising luminosity, and hence of the birth of a typical giant star, no longer occurs, signifying that the stellar material is essentially exhausted in the sidereal universe we know; or (3) that the part of space where the phenomenon now occurs is remote from the region we ordinarily observe. Or stated more briefly we may suppose that the (1) *frequency*, (2) *epoch*, or (3) *place* of star-birth makes difficult or impossible our observation of the pre-giant stages.

The photographic observation of the magnitudes of the variables in Orion is very difficult because of the irregularity of the nebulosity and the uncertainties associated with the photographic study at Mount Wilson of southern objects in the winter season. Whether definite periodicity obtains for any of the variables has not yet been determined. Some of the stars previously suspected of variability may owe their apparent variations entirely to photographic phenomena; others at one time definitely variable now appear constant. The results so far derived for thirty variable stars near the trapezium are mainly negative: (1) the variables do not belong to the cluster-type; (2) they are not blue (apparently they are mainly red, but the nebulosity introduces systematic error of undetermined amount into measures of color); (3) they do not appear to show regularity, or great range, or close similarity to each other in color or range; (4) they are, of course, not undergoing a secular increase of brightness apparent within this short interval of time; (5) the variables as well as other faint stars are concentrated in the central part of the nebula; (6) in the dense nebulosity the light of very few faint stars appears to be actually constant—perhaps a spurious photographic effect.

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[1] The records of some globular clusters, and probably of the solar neighborhood as well, are sufficiently complete to show that such antecedents are not present as normal stars.

It seems impossible to say definitely as yet whether the variables are really associated with the nebula-group of bright stars and undergo intrinsic variations of light or are for the most part at a greater distance than the Orion stars and owe their variations to occultations by nebular matter. Most of the evidence, however, supports the view that the variables are typical dwarf stars within the nebula[1], deriving their irregular light-variations from contact with the irregular nebulosity, which recent spectroscopic work has shown to be moving differentially. Some strong points favoring this view may be inferred from the preceding paragraph, and together with some possible objections they will be more fully discussed when a detailed report is made on the magnitudes and colors. The similar, commonly-accepted hypothesis of nebular friction seems to account satisfactorily for the novae, where a much greater range of variation is involved.

L. *The most distant naked-eye star.*—The adopted lower limit of the parallax, $\pi = 0\rlap{.}''001$, of a hypothetical most distant naked-eye star depends upon the fairly safe assumption that the difference between apparent and absolute brightness, $m-M$, does not exceed ten magnitudes; that is, we assume that no first-magnitude star is brighter absolutely than -9, that no fifth-magnitude star is brighter absolutely than -5. The study of the giants in clusters, Kapteyn's work on B-type stars, and the results of direct measures of parallax support this assumption. The most relevant contribution to this question, however, is the very important statistical discussion by Dyson of the proper motions of one-sixteenth of all the stars brighter than the ninth magnitude.[1] He finds that brighter than the sixth magnitude there are in the whole sky only $15 \times 16 = 240$ stars more distant than 200 parsecs. The extreme limit[2] of 260 parsecs, to which naked-eye stars extend according to Dyson's formula, is only one-fourth of the limit assumed for the present illustration. We may note, however, that if a naked-eye Cepheid variable fainter than the fifth apparent magnitude should be found with period in excess of 32 days, its parallax would be somewhat smaller than the lower limit adopted above.

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[1] Recent studies of proper motion by van Maanen also strongly support this interpretation.

M. *Charlier's "Galaxy of B Stars"*.—A discussion of the distribution in space of all B-type stars in the Harvard catalogues has been made by Charlier,[3] who finds that they form a large cluster, the center of which is nearly 100 parsecs distant from the sun. As practically all the stars of all spectral types for which we know the position in space are within the bounds of this B-type cluster, it is chosen, in this comparison with the system of globular clusters, to represent the stellar system as ordinarily conceived.

N. *The eccentric position of the solar system.*—In the seventh paper it has been noted that the adopted distance to the center of the system of globular clusters is tentative, but probably of the correct order of magnitude.

Confirming Charlier's determination, either of the general direction or of the amount of the sun's displacement from the center of the local aggregation of stars, we have the results of Russell and Shapley on eclipsing binaries and Cepheids, of Walkey on 30,000 stars of the Harvard catalogues, and of Strömberg on second-type stars of known parallax and radial velocity.

SUMMARY

This paper contains a tabular and diagrammatic comparison of various sidereal distances. In addition to the material already available from the study of clusters and variables and from the results of other investigators, a considerable amount of new observational and computational work has been necessary for the discussion. The following statement summarizes the more important results, some of which are not specifically discussed in the text of the paper.

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[1] *Monthly Notices*, 77, 212, 1917.

[2] Dyson notes that the limit is too small; *ibid.*, p. 217.

[3] *Meddelanden från Lunds Astronomiska Observatorium*, Series 2, No. 14, 1916.

1. The most distant globular cluster now known, N.G.C. 7006, appears to differ from much nearer systems only in the matter of distance. In general appearance and in the relation of size to brightness it conforms with typical globular clusters; and in phenomena of color and the frequency of red giant stars it is very similar to Messier 3 and 13 (Table III).

2. The similarity in the frequency of colors for near and distant clusters shows that the selective scattering of light in space, if acting uniformly, affects the color-indices of stars by less than two-millionths of a magnitude for each parsec of distance. With any reasonable assumption as to the dependence of light-scattering on stellar concentration, interstellar media appear unimportant in their effect on the colors of stars brighter than the fifteenth magnitude.

3. The parallax of N.G.C. 7006 is 0″.000015 on the basis of diameter measures, 0″.000014 according to measures of photographic magnitude, and 0″.000016 if the mean absolute photo-visual magnitude of its brightest stars be assumed equivalent to the corresponding means for Messier 3 and 13.

4. Counts of stars on photographs of the edges of globular clusters show that faint stars as well as the typical variables are widely dispersed. Probably the diameter of every typical globular cluster exceeds twenty million astronomical units.

5. As far from the center as 15′, corresponding to a linear distance of about 65 parsecs, the elongation of Messier 15 is found to be in general agreement with the results for the central regions, both as to direction and amount.

6. The cluster N.G.C. 4147, which, of all known, is by far the most distant from the galactic plane, may differ considerably from typical globular systems in frequency of giant stars and perhaps in linear extent. A revision of the photographic magnitudes in this

cluster alters the previously adopted mean of the "25 brightest" stars by only five-hundredths of a magnitude.

7. A comparison of the direct and hypothetical parallaxes of seven eclipsing binaries shows an average difference of ±0.″01; the systematic difference is inappreciable.

8. Most if not all of the variable stars in the Orion nebula appear to be irregular in period and range; the color-indices of thirty situated in the denser nebulosity are large and positive. That the variables are actually associated with the nebula and therefore of faint absolute magnitude is not yet definitely proved.

MOUNT WILSON SOLAR OBSERVATORY
January 1918

ON THE GENERAL AURORAL ILLUMINATION OF THE SKY AND THE WAVE-LENGTH OF THE CHIEF AURORA LINE

By V. M. SLIPHER

Nature presents no sight so beautiful and wonderful as a brilliant display of the aurora. Records show that the aurora borealis has been observed since ancient times, and yet today it is not without mystery for many and is scientifically the least understood optical phenomenon of the earth. Quite naturally the spectrum of the aurora was studied by a wide circle of investigators, following the application of the spectroscope to astronomy. A bibliography of this spectroscopic work[1] alone would include upward of a hundred references and such names, well-known to astronomical literature, as Ångström, Young, Struve, Zöllner, Smyth, Vogel, Cornu, Holden, Tacchini, Secchi, Rowland, Copeland, Lockyer, Gyllenskiold, Huggins, Schuster, Pickering, Campbell, etc.

These early observations, made visually of course, revealed an interesting spectrum and established its general features. The observations could not be conducted deliberately with specially adjusted instruments and methods, on account of the character of the phenomena. Besides, the limited sensitiveness of the human eye left unexplored the more refrangible parts of the spectrum, and hence the complete investigation of the aurora spectrum depends upon the photographic method.

Professor Störmer's[2] very extensive observational and theoretical investigations into the nature of the aurora, and the late Professor

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[1] See Kayser, *Handbuch der Spectroscopie*, **5**, 47; and Frost-Scheiner, *Astronomical Spectroscopy*, p. 326.

[2] "Trajectories of Electric Corpuscles Applied to the Aurora Borealis and to Magnetic Disturbance," *Archiv for Mathematik og Naturvidenskab*, **28**, No. 2, 1906; see also *Archives des Sciences physiques et naturelles* (Genève), **24**, 5, 113, 221, 317, 1907, and **32**, 1911–12; and "Bericht über eine Expedition nach Bossekop zwecks photographischer Aufnamen und Höhenmessungen von Nordlichtern," *Videnskabs-Selskabets Skrifter, I, Math.-Naturv. Klasse*, 1911, No. 17, etc.

Birkeland's[1] vast work on polar magnetic storms and the related aurora, have added enormously to what we know in this field. Yet during this period our knowledge of the spectrum of the aurora has not been advanced proportionally, as it has hardly received its proper share of attention.

During the last few years I have done a little in the way of spectrographic observations which I shall give in this paper. These observations bear upon (1) the persistency of general auroral illumination of the sky, which is in continuation of the work briefly presented in *Lowell Observatory Bulletin*, No. 76, and (2) the wavelength of the chief aurora line in the yellow-green of the spectrum. A thorough investigation of the spectrum of the aurora can be done under conditions so much more advantageous at some northern station where brilliant displays of the aurora are frequent that undertaking it here is not considered.

Early in June, 1915, I recognized a faint line in the yellow region. This was on a negative which I had made, with an exposure of several nights, for the spectrum of the Milky Way, although the Seed 30 plate employed possessed only the ordinary emulsion, which is but slightly sensitive to the yellow of the spectrum. Hence the line in question must have been of pretty high equivalent brightness to have been recorded by the plate. I then undertook another spectrogram of the Milky Way, using a Cramer Isochromatic plate, the sensitiveness of which should easily record the line in question, suspected of being the aurora line, if of anything like the brightness indicated on the previous plate. However, this plate failed of its purpose in a singular manner. Its exposure, only partially completed, was continuing on the night of June 16, when there appeared that exceptional auroral display which was brilliant even at this low latitude (35°12′), and was the first aurora observed here in the history of the Observatory. Although the instrument was pointed toward the south, the light of the aurora of course got to the plate and strongly superposed itself upon the spectrum of the Milky Way and spoiled the plate for its intended purpose.

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[1] *The Norwegian Aurora Polaris Expedition 1902–1903* (Christiania, 1913); "Expédition norvégienne de 1899–1900 pour l'étude des Aurores boréales, etc.," *Videnskabs-Selskabets Skrifter, 1901;* and *Archives des sciences phys. et nat.* (4), 1, 512, 1896, and "Recherches sur les taches du soleil et leur origine," *Videnskabs-Selskabets Skrifter* (Christiania), pp. 2 and 167, 1899.

Since that time many spectrograms have been secured of the night sky to see whether it is always possible to record the chief auroral line and in that way definitely to recognize the presence of faint general auroral illumination of the night sky. In this work a small spectrograph of very high light-power (camera lens having speed F/1.9) has been employed, pointed directly to the region of the sky to be investigated, and generally left stationary during the exposure. In some instances an image-forming lens was placed before the slit. Recently, and especially for the long exposures for determinations of wave-length, the spectrograph has been directed upon the northern sky from the window of a basement room. Cramer Isochromatic plates have regularly been used, as they have a high sensitiveness for the yellow-green region of the spectrum, including the chief aurora line. In general each exposure has been continued through one night, although a number of plates were made of two exposures side by side for comparison of two different parts of the sky. In such cases the exposures were each for only one-half of the night. The presence of the moon does not seriously interfere with these observations, as the aurora line can be clearly seen, superposed upon the continuous spectrum of moonlight.

In the past three and one-half years something like one hundred spectrograms have been made of the night sky, and every one of these has recorded the chief aurora line. Thus during this period of time auroral illumination of the sky has been found to be present on every night that an exposure has been made for detecting it. The spectrograph therefore gives direct evidence of the existence of a permanent aurora.

The observations indicate an increase in the intensity of the aurora light as the zenith distance is increased. Also it is more intense in the sunrise and sunset parts of the sky than elsewhere. It is present in the southern as well as the northern sky, but seemingly less intense in the north than in the east and west. It is possible, however, that dawn and twilight may have influenced some of the plates, as a slight exposure of continuous spectrum of the sky would tend to strengthen the image of the aurora line. These observations were not made with great care in such respects, as they were not intended for the purpose of studying the variation of the light in different parts of the sky, although the value of observations carried out for quantitative results in that direction has been appreciated.

The close dependence of displays of aurora upon sun-spot activity, which for some time has been well established, suggests that there are variations in the intensity of this permanent, general, auroral illumination of the sky. Thus there is good reason to expect less general light of the aurora in the sky during the time of sun-spot minima than during sun-spot maxima.

To realize that it is astronomically of vital importance to know whether or not there exists a general, permanent, auroral illumination of the sky, it is only necessary to refer to the work that has been done in attempting to measure the brightness of the night sky in determining the total light of all the stars.[1] Newcomb, who seems to have been the first to take up this work, presents the needs and importance of investigation in this field in the following words:

> The total amount of light received from all the stars may serve as a control on theories of the structure of the universe, because the amount of light resulting from any theory should agree with the observed amount. It is also a quantity which we must regard as remaining constant from age to age. It seems possible to determine, not only its integral value for the whole sky, but its value separately in each region of the sky. For these reasons it must be considered as among the most important fundamental constants of astrophysics.

So much for the need of reliable measures of the brightness of the starlit sky and the total light of all the stars. That serious difficulties have been encountered and divergent results obtained in carrying out the necessary observations is evidenced by the following quotations from the recent paper on the subject by

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[1] Newcomb, "A Rude Attempt to Determine the Total Light of All the Stars," *Astrophysical Journal*, **14**, 297, 1901. Burns, "The Total Light of All the Stars," *ibid.*, **16**, 166, 1902. See also *The Observatory*, **33**, Nos. 420 and 421, 1910. Townley, *Publications of The Astronomical Society of the Pacific*, **15**, 13, 1903. Yntema, "On the Brightness of the Sky and Total Amount of Starlight," *Publications of the Astronomical Laboratory at Groningen*, No. 22, 1909. Fabry, "The Intrinsic Brightness of the Starlit Sky," *Astrophysical Journal*, **31**, 394, 1910.

Professor Charles Fabry (*loc. cit.*, p. 399), in which he concludes his discussion of his observations by saying:

> Upon the whole, therefore, the data actually obtained on the number of stars are very far from being in accord with the results of measures of intrinsic brightness. Is the disagreement a result only of inaccuracy of the measures and of the statistical data? That is not certain. If it were proved that the total intensity of the sky exceeds considerably the sum of the intensities of the observable stars, one could advance two hypotheses: either that there exists an immense number of stars too faint to be observed with our instruments, or that there exists throughout the sky a sort of continuous nebulosity giving a uniform brightness.

As an appendix to his paper, added later, he comments upon the extensive similar investigation of Yntema, thus:

> Very recently, Yntema has made a great many visual measures. The most important conclusion from his observations is that a great part of the light of the sky is of terrestrial origin. This result seems to be peculiar to the conditions under which Yntema has observed. No other observer has found, from one date to another, such large variations as those observed by him. The brightness of a square degree in the region of the celestial pole would equal 0.19 that of a star of magnitude 1, a result about four times as large as those found by other observers. . . . . The only conclusion to be drawn from these great discordances is that measures made in different epochs and in different places would be very useful.

Hence these spectrographic tests on a general illumination of the sky by aurora light serve to establish an important fact. While this disturbing factor in these important photometric researches has been photometrically a very troublesome one, it has yielded readily to spectrographic analysis. The case offers a good example of the directness and definiteness of solution sometimes given by the spectrograph to astrophysical problems.

These spectrographic results suggest the cause for the discrepancies in the measures of the brightness of the sky by the different observers referred to above. For example, it seems probable that Yntema, observing at a more northern station, encountered more intense auroral illumination of the sky than did the other observers, who were farther south. This could account for the greater brightness that he found for the polar region, and it is possible that the variation he found from date to date was due to occasional weak auroral displays not recognizable as such.

Newcomb, Burns, and Townley observed at a time nearly coinciding with a sun-spot minimum and Yntema nearer spot maximum than did Fabry. This would affect the observations if, as is to be expected, the intensity of the persistent aurora light is, like the aurora displays, greater during sun-spot maxima than during minima. Here it is well to call attention to the fact that Fabry observed photographically, and consequently his results should have been less affected by aurora light than if they had been obtained visually, since the yellow-green radiation of the aurora would affect the eye much, but the ordinary plate scarcely at all. However this may be, it is unfortunately not possible to correct these extensive observations on the total light of all the stars for the disturbing aurora light; but it seems clear that such observations in future will need to be made with care as to the presence of aurora light in the sky. Spectroscopic tests of the sky could be made to decide whether or not the night is a suitable one for the purpose, or as good as any to be had. Besides, the employment of a color-screen so adjusted as to absorb the auroral radiations is to be considered essential. To this end more should be known of the spectrum of aurora light, but a screen cutting out the chief line in the yellow would doubtless materially improve visual observations of the brightness of the light of all the stars.

The finding of the spectrograph substantiates Yntema's conclusion that "the light of the sky at night is composed of two parts, one reaching us directly from the stars, the other resulting from processes in the atmosphere. The latter termed 'earthlight,' is only partly due to diffused starlight. It seems probable that the rest, wholly or in part, is due to a permanent aurora." At that time Yntema was strengthened in his conclusion by reports of several observers[1] who had seen the chief aurora line in the spectrum of the sky on nights when they did not recognize any of the phenomena of an auroral display.

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[1] Ångström, *Die Spectralanalyse in einer Reihe von sechs Vorlesungen*, Braunschweig, 1870, p. 180. Vogel, *Astronomische Nachrichten*, **79**, 327, 1872. Campbell, *Astrophysical Journal*, **2**, 162, 1895; also *Lick Observatory Bulletin*, **5**, 46–47, 1908, and more recently *Publications of the Astronomical Society of the Pacific*, **29**, 218, 1917; Wiechert, *Physikalische Zeitschrift*, **3**, 366, 1902; and *Meteorologische Zeitschrift*, **19**, 315, 1902.

This subject invites much further spectrographic study with a view to more quantitative results. Simultaneous observations with duplicate instrumental means at two stations separated considerably in latitude should secure comparable material of the highest value in itself and in its bearing on other problems; and it is hoped that some observer stationed farther north will become sufficiently interested to take up the work of investigating the spectrum (which should not be difficult to one favorably located) and particularly in joining in a co-operative plan, possibly with this Observatory, in the further study of the general auroral illumination of the night sky.

WAVE-LENGTH OF THE CHIEF AURORA LINE

In following up the measurement of one of the earlier of the plates taken with low dispersion, which had indicated a longer wave-length for the chief line than the commonly accepted value of λ 5571, I measured several additional plates. These resulted in a considerably longer wave-length. While the scale of the plates was small, their individual results agreed satisfactorily, and the deviation from the old value seemed too great and persistent to be brushed aside as errors of observation.

It thus appeared desirable to determine the wave-length with more accuracy, and I adjusted a powerful three-prism spectrograph, with a 15-inch camera, for the desired region of the spectrum and made some plates with this high dispersion. The exposures were continued over many nights, the last one running considerably over one hundred hours. The instrument was in a basement room, of uniform temperature, and directed through a window to the northern sky a little way below and to the east of the pole. Three of these plates taken with higher dispersion were secured and each measured twice, with the results:

I	5578.06 5577.95	5578.01
II	5578.11 5578.00	5578.05
III	5577.99 5578.17	5578.08

Final mean = 5578.05

The sky spectrum served for comparison on these plates and was exposed briefly on a number of days during the period of the exposure to the aurora.*

The wave-length yielded by these high-dispersion plates is slightly greater than that shown by the small-scale ones, and greater by fully seven units than the usually accepted value. So large a deviation from the old value is very surprising and by present standards intolerable. I can find nothing wrong with the new value; the employment of the solar comparison spectrum precludes misidentification of the reference lines or their wave-lengths. The high-scale spectrograms show at a glance that the strong solar line λ 5573.075 falls well to the violet side of the image of the aurora line, the wave-length of which, hence, obviously cannot be anything like λ 5571, the previously accepted value. Spurious displacements in these spectrograms must, of course, be inappreciable in comparison, and we must attribute the discrepancy to the old visual observations, for we cannot assume a shifting aurora line. In this connection we should not lose sight of the fact that the visual observations were often made under very unfavorable conditions that permitted only estimate of the line's position or wave-length. There seems no escaping the conclusion from the high-scale spectograms that the wave-length of the line is substantially λ 5578.05.

Photographic observations of the spectrum of the aurora since 1900, by Paulsen,[1] Sykora,[2] Westman,[3] and Vegard,[4] included determinations of the chief line, but these do not incline strongly toward the value I have found. Values obtained visually by Vegard with a direct-vision spectroscope are: λ 5576.9, using helium reference lines, and λ 5573.7, referred to hydrogen lines. But his photographic value fell back to λ 5571.3, and he suggested λ 5572.5 as a mean result from his observations. Westman got λ 5572.6, Sykora λ 5570, and Paulsen's value seems indefinite, although I have not been able to consult his original paper. These observations, even those of Vegard, who apparently observed under unusually favorable circumstances, appear to have left the wave-length rather uncertain according to present-day standards.

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* Editorial Note.—The author has sent me a lantern slide of the spectrum of the aurora and sky which shows the green line very strongly, while the solar lines are too faint for successful reproduction. At his request I am glad to state that it is perfectly evident that the green line falls at a point of greater wave-length than the solar line λ 5573, in fact nearly 1 mm toward the red from that line on the scale of the slide. E. B. F.

[1] "Sur le spectre des aurores polaires," *Comptes rendus*, **130**, 655, 1900; *Rapp. Congr. Internat. de Physique* (Paris), **3**, 438–52, 1900; and *Rep. Brit. Asso.*, pp. 575–78, 1903.

[2] "Die Wellenlängen der photographisch erhaltenen Linien des Nordlicht-spectrums," *Astronomische Nachrichten*, **156**, 326, 1901.

[3] *Aurores boréales. Mission scientifique pour la mesure d'un arc de méridian au Spitzberg* (Stockholm), **2**, 1904.

[4] "Photographische Aufnahmen des Nordlichtspectrums mit einem Spectrographen von grosser Dispersion," *Physikalische Zeitschrift*, **14**, 677–81, 1913.

The new value of the wave-length resulting from my observations has not yet led to the identification of the chemical substance involved, and nothing promising falls near it. In this regard the old value offered more encouragement, as it chanced to come near a strong krypton line.[1]

Reference was made early in this paper to a spectrogram of the Milky Way which was exposing on the night of June 16, 1915, and so recorded the spectrum of the aurora of that night. The spectrograph was unfortunately directed toward the south and adjusted with a wide slit for the continuous spectrum of the Milky Way. Thus the conditions were unfavorable for a good spectrum of the aurora because it was, of course, blended with that of the Milky Way, although the latter was not very strong. It may be of interest, however, here to describe briefly the aurora spectrum. On this plate the chief aurora line is exceedingly intense, and a line near λ 3916 is fairly strong, while weaker ones are doubtless present near λλ 4277, 4180, 4450, and 3740. These wave-lengths are only approximate. Certainly the lines λ 3916 and λ 4277 have been recorded before, and very possibly the others, too.

In conclusion I wish to call attention to the evident need for a thorough investigation of the auroral spectrum and to emphasize the fact that, where brilliant auroras are frequent, valuable results should not be laborious of realization with a suitable spectrograph, i.e., one with moderately high angular dispersion and a fairly short camera.

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[1] C. Runge, "On the Spectrum of the Aurora," *Astrophysical Journal*, 18, 381, 1903.

Note added April 5, 1919:

Professor Frost has kindly called my attention to an extensive paper by J. Stark on the identification of the lines of the aurora spectrum, under the title: "Das Nordlichtspektrum ein Spektrum positiver Strahlen" (*Annalen der Physik*, No. 24, 598, 1917). It is to be noted that Stark, like Vegard (see previous reference), finds very close correspondence between the auroral spectrum and that of nitrogen, except in the case of the chief aurora line—and this line Stark regarded as very probably due to the pair of nitrogen lines 5560–5565 A. However, the new wave-length, 5578.05 A, for this line makes such an identification quite inadmissible, and implies a separate—non-nitrogen—origin for this line. Stark's work emphasizes still more the need for accurate knowledge of the other lines of the auroral spectrum, and it is to be hoped that this information will soon be available for such laboratory studies as Stark's on the origin of the features of the spectrum. These studies are of the highest importance in completing our knowledge of the aurora and in disclosing the nature of the upper air.

LOWELL OBSERVATORY, FLAGSTAFF, ARIZ.
March 3, 1919

SPECTRA AND ATOMIC NUMBERS OF THE ELEMENTS

By J. E. PAULSON

In the case of all spectra which exhibit series, it has long been known that there exists a certain relation between the frequency-difference of certain lines and the atomic weight of the substance. For elements belonging to the same group in the periodic system, the frequency-difference between the lines belonging to the first doublet or triplet of the principal series is approximately proportional to the square of the atomic mass of the element. As is known, the same difference exists between all the doublets or triplets of the subordinate series. In Table I are shown the values which Rydberg[1] has computed for $\frac{10^3\nu}{P^2}$, where ν indicates the frequency-difference and P the atomic weight.

TABLE I

	P	ν	$\frac{10^3\nu}{P^2}$		P	ν_1	ν_2	$\frac{10^3\nu_1}{P^2}$	$\frac{10^3\nu_2}{P^2}$
Li...	7.03	0?	0?	Mg.	24.38	40.91	19.85	68.8	33.4
Na..	23.06	17.19	32.3	Ca .	40.00	105.82	52.16	66.1	32.6
K...	39.14	57.85	37.8	Sr..	87.52	394.22	186.83	51.5	24.9
Rb..	85.44	235.98	32.3	Ba .	137.04	878.5	370.4	46.8	19.7
Cs...	132.88	553.87	31.6						
Cu ..	63.44	248.54	61.8	Zn .	65.38	388.97	189.82	91.0	44.4
Ag...	107.94	920.48	79.0	Cd .	112.08	1170.76	542.09	93.2	43.1
Au ..	197.25	3815.40	98.1	Hg .	200.36	4631.17	1767.09	115.4	44.0
Al...	27.08	112.02	152.8	O ..	16.00	3.70	2.08	14.5	8.1
Ga ..	69.9	823.6	168.6	S...	32.06	18.15	11.13	17.7	10.8
In...	113.4	2212.54	172.1	Se..	79.07	103.7	44.07	16.6	6.6
Tb ..	204.15	7792.63	187.0						

As will be seen from the table, this proportionality between ν and P^2 is not strictly constant. Since the frequency-difference can be expressed only approximately by use of the second power of the atomic weight, one naturally sets it proportional to an arbitrary power of the atomic weight, this power being different for each related group in the periodic system of elements. This is the method which has been used by Runge and Precht.[1] These authors represent the connection between ν and P in a system of co-ordinates, in which the variables are $\log \nu$ and $\log P$; and they find that the curves thus obtained are straight lines. On this basis they then proceed to compute the atomic weight of radium, a quantity at that time imperfectly known. But the rectilinearity of these curves is only, however, approximate, as one can easily see from a large-scale drawing.

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[1] J. R. Rydberg, *Rapport au Congrès international de Physique*, Paris, 1900.

Since no other relation between the spectra of elements and their other properties has yet been discovered, it has seemed to me worth while to study the question further. Since the atomic weight is surely complex in its nature and does not definitely determine the position of its element in the periodic system, it has seemed clear to me that the atomic weight is not the proper variable in which to express the properties of the elements. To start from the Atomic Numbers which are whole numbers and which definitely locate the elements in the periodic system would appear to be a better procedure. In what follows I shall employ the system proposed by Rydberg.[2] But in some cases it is quite possible that other values would have fitted in better. Rydberg's system is shown in Table II.

TABLE II

V	0	I	II	III	IV	V	VI	VII	VIII	–
G^1 ..	E(0)	H(1)								
	–(2)	–(3)								
G^2 ..	He(4)	Li(5)	Be(6)	B(7)	C(8)	N(9)	O(10)	F(11)		
	Ne(12)	Na(13)	Mg(14)	Al(15)	Si(16)	P(17)	S(18)	Cl(19)		
G^3 ..	Ar(20)	K(21)	Ca(22)	Sc(23)	Ti(24)	V(25)	Cr(26)	Mn(27)	Fe(28)	Co(29)
	Ni(30)	Cu(31)	Zn(32)	Ga(33)	Ge(34)	As(35)	Se(36)	Br(37)		
	Kr(38)	Rb(39)	Sr(40)	Y(41)	Zr(42)	Nb(43)	Mo(44)	–(45)	Ru(46)	Rh(47)
	Pd(48)	Ag(49)	Cd(50)	In(51)	Sn(52)	Sb(53)	Te(54)	I(55)		
G^4 ..	X(56)	Cs(57)	Ba(58)	La(59)	Ce(60)	Pr(61)	Nd(62)	–(63)		
	Sm(64)	Eu(65)	Gd(66)	Tb(67)	Dy(68)	Ho(69)	Er(70)	Tu(71)		
			Yb(72)	Lu(73)	–(74)	Ta(75)	W(76)	–(77)	Os(78)	Ir(79)
	Pt(80)	Au(81)	Hg(82)	Tl(83)	Pb(84)	Bi(85)	–(86)	–(87)		
	Nt(88)	–(89)	Ra(90)	–(91)	Th(92)	–(93)	U(94)			

My attempt has been to discover that function of the Atomic Numbers which will best represent the observed values of the frequency-difference ν. The first expression which suggests itself is that obtained by substituting the Atomic Number for the atomic weight, which at once leads to the formula

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[1] *Physikalische Zeitschrift*, 4, 285, 1903.

[2] *J. Ch. Phys.*, 12, No. 5, 1914.

$$\nu=[A \log N+B],^{1}$$

where A and B are constants. As a matter of fact, this equation represents the observed values generally just as well as, and in several cases better than, the corresponding equation using atomic weights

$$\nu=[A \log P+B].$$

The following formula, however, will give still better agreement, namely

$$\nu=[A \log (N+n)+B],$$

where n is a whole number, which may be positive or negative, and which is generally small. I am quite convinced that if one were to change the formula involving atomic weights in a corresponding way, so as to read

$$\nu=[A \log (P+n)+B],$$

one would also secure better agreement. Since it is difficult to assign any physical meaning to such an expression, I have not examined it further.

The following gives the result of my investigation, and also a comparison with the values obtained by use of the atomic-weight formula. The constants A and B have in each case been computed by the method of least squares. The value of n was first determined graphically.

Group I: 1. Li, Na, K, Rb, Cs.

$$\nu_N=[2.055461 \log (N-3)-0.819328]$$
$$\nu_P=[1.953599 \log P-1.393596]$$

TABLE III

	N	P	ν	ν_N	Diff.	ν_P	Diff.
Li.......	5	6.94	0?*	(0.63)	....	(1.78)	....
Na......	13	23.00	17.21	17.22	−0.01	18.48	− 1.27
K.......	21	39.10	57.00	57.66	+0.24	52.10	+ 5.80
Rb......	39	85.45	237.71	239.66	−1.95	239.99	− 2.28
Cs.......	57	132.81	554.10	551.50	+2.60	568.00	−13.90

* Not included in the calculation.

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[1] This symbol [ ] denotes the antilogarithm.

The computed value for lithium is, of course, too great and for the principal line λ 6708 would correspond to a difference of wave-length amounting to 0.28 A.

Group I: 2. Cu, Ag, Eu, Au.

$$\nu_N=[2.786693 \log (N-1)-1.721416]$$

$$\nu_P=[2.413119 \log P-1.951540]$$

TABLE IV

	N	P	ν	ν_N	Diff.	ν_P	Diff.
Cu...	31	63.57	248.13	248.24	−0.11	251.15	−3.02
Ag...	49	107.88	920.56	919.78	+0.78	899.89	+20.67
Eu...	65	152.0	....	(2050.47)	....	....	....
Au...	81	197.2	3817.11	3818.67	−1.56	3857.84	−40.73

For gold I have assumed the fundamental lines to be λλ 2676.06 and 2428.04, which yield values agreeing well with those computed. In the case of europium, no series have yet been found. Under the supposition that this element belongs to this group, I have computed the frequency-difference, ν, to be 2057.17. In a previous paper[1] I have listed some constant frequency-differences found in the europium spectrum, of which the most frequently recurring is 1669.71. Four doublets showing this difference involve eight of the strongest lines of this spectrum. It is not likely that this frequency-difference is identical with the one computed; and a second attempt to discover a frequency-difference agreeing better with the computed value has led to no favorable result. Possibly the discrepancy is due to a mistake in Rydberg's system.

Group II: 1. Mg, Ca, Sr, Ba, Yb, Ra.

$$\nu_{1N}=[2.163129 \log N-0.871542]$$

$$\nu_{2N}=[1.748748 \log (N-4)-0.459734]$$

$$\nu_{1P}=[1.756001 \log P-0.809218]$$

$$\nu_{2P}=[1.680803 \log P-1.006520]$$

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[1] *Astrophysical Journal*, 40, 298, 1914.

TABLE V

	N	P	ν_1	ν_2	ν_{1N}	Diff.	ν_{2N}	Diff.	ν_{1P}	Diff.	ν_{2P}	Diff.
Be...	6	9.1	....	....	(6.48)	....	(1.17)	....	....	....	....	....
Mg...	14	24.32	40.92	19.89	40.52	+0.40	19.45	+0.44	42.12	−1.20	21.04	− 1.15
Ca...	22	40.07	105.99	52.11	107.72	−1.73	54.38	−2.27	101.24	+4.75	48.70	+ 3.41
Sr....	40	87.63	394.44	187.05	392.57	+1.87	182.74	+4.31	400.02	−5.58	181.42	+ 5.63
Ba...	58	137.37	878.4	370.3	876.96	+1.44	371.35	−1.05	880.89	+2.49	386.27	−15.97
Yb...	72	172.0	....	....	(1399.84)	....	(555.73)	....	....	....	....	....
Ra...	90	226.4	....	....	(2268.33)	....	(837.95)	....	....	....	....	....

No triplets are known which correspond to the computed values for the three elements, Ba, Yb, and Ra.

Group II: 2. Zn, Cd, Gd, Hg.

$$\nu_{1N}=[3.522017 \log (N+17)-3.362998]$$
$$\nu_{2N}=[2.466934 \log (N+2)-1.499658]$$
$$\nu_{1P}=[2.220238 \log P-1.458622]$$
$$\nu_{2P}=[1.998697 \log P-1.357823]$$

TABLE VI

	N	P	ν_1	ν_2	ν_{1N}	Diff.	ν_{2N}	Diff.	ν_{1P}	Diff.	ν_{2P}	Diff.
Zn ...	32	65.37	388.91	189.78	388.96	−0.05	189.85	−0.07	377.02	+ 11.89	188.17	+ 1.61
Cd...	50	112.40	1171.05	541.86	1170.76	+0.29	541.51	+0.35	1243.27	− 72.22	550.85	− 8.99
Gd...	66	157.3	....	....	(2489.01)	......	(1049.59)	......	....	....	....	....
Hg...	82	200.6	4630.31	1767.19	4630.86	−0.55	1767.71	−0.52	4498.84	+131.47	1753.23	+13.96

Concerning ν_{1N}, graphical construction shows that an abnormally high value must be given to n, namely, $n=17$. This may appear questionable. The agreement is very good, however. The formula for ν_{1P}, on the other hand, shows great discrepancies. In case of the gadolinium spectrum, I have previously described[1] a connected system of lines of the type usually occurring in those spectra which are devoid of series. It is therefore doubtful whether this element belongs to the group. I have not found any triplet which corresponds with the predicted values.

Group III. B, Al, Ga, In, Tb, Tl.

$$\nu_N=[2.712737 \log (N+3)-1.355479]$$
$$\nu_P=[2.094583 \log P-0.955935]$$

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[1] *Physikalische Zeitschrift*, 16, 1915.

TABLE VII

	N	P	ν	ν_N	Diff.	ν_P	Diff.
B......	7	11.0	....	(22.76)	....	....	....
Al.....	15	27.10	112.07	112.14	− 0.07	111.06	+ 1.01
Ga.....	33	69.9	823.6*	(735.12)	....	....	....
In.....	51	114.8	2212.63	2208.26	+ 4.37	2284.46	− 71.83
Tb....	67	159.2	....	(4464.65)	....	....	....
Tl.....	83	204.0	7792.95	7803.78	−10.83	7616.88	+176.07

* Not sure and not included in the calculation.

The principal lines in the spectrum of boron, λλ 2497.80 and 2496.84, have a frequency-difference of 15.40, which possibly is identical with the computed one. In the Tb spectrum, no series have yet been found.

Group IV. O, S, Se, Te, Er, −(86)

$$\nu_{1N}=[2.208775 \log (N-3)-1.340342]$$
$$\nu_{2N}=[1.665132 \log (N-4)-0.855511]$$

$$\nu_{1P}=[2.130683 \log P-2.007606]$$
$$\nu_{2P}=[1.732346 \log P-1.613456]$$

TABLE VIII

	N	P	ν_1	ν_2	ν_{1N}	Diff.	ν_{2N}	Diff.	ν_{1P}	Diff.	ν_{2P}	Diff.
O......	10	16.00	3.38	2.76	3.36	+0.02	2.76	0.00	3.61	−0.23	2.97	−0.21
S.......	18	32.07	17.90	11.26	18.09	−0.19	11.30	−0.04	15.90	+2.00	9.90	+1.36
Se......	36	79.2	103.66	44.82	103.29	+0.45	44.75	+0.07	109.14	−5.58	47.40	−2.58
Te.....	54	127.5	....	....	(269.96)	....	(94.08)	....	....	....	....	....
Er.....	70	167.7	....	....	(493.23)	....	(149.37)	....	....	....	....	....
−(86)..	86	?......	....	....	(791.54)	....	214.41	....	....	....	....	....

No series are known in the spectrum of either tellurium or erbium.

UMEÅ, SWEDEN
October 1918

CORRELATION BETWEEN THE COLOR-SCALES OF ONE AND OF TWO DIMENSIONS

By J. G. HAGEN, S.J.

In a former number of this Journal (34, 261, 1911) the writer gave the history and the definitions of the two color-scales: the two-dimensional, which is used exclusively in England; and the one-dimensional, generally applied on the Continent.

1. At that time no comparison based on observations was possible, and so an attempt was made to co-ordinate the two scales from a purely mathematical point of view. The result is shown in Table I. It need only be mentioned that B stands for blue and S for somber or dark, and that in the compound letters the left-hand symbol has the adjectival meaning, e.g., YW = yellowish white.

TABLE I

White			Yellow			Orange			Red		
BW	W	YW	WY	Y	OY	YO	O	RO	OR	R	SR
−1	0	1	2	3	4	5	6	7	8	9	10

To make this table a practical one that can be used as a transition from one scale to the other would require, strictly speaking, that each observer express his estimates both by letters and by numbers. As this can hardly be expected, it only remains to construct a mean table by collecting the estimates of different observers and comparing letters with numbers.

2. This can now be done, since the observations of Franks, on the one hand, and of Krüger and Osthoff on the other, will appear in the forthcoming publication, No. IX, of the Vatican Observatory, which concludes the third volume of Series II.

There are 3162 stars observed by Franks on the areal, and by Krüger and Osthoff on the linear, scale. The result of the comparison is represented in Table II. The single-letter symbols of

Franks were written on separate sheets, and the corresponding numbers of Krüger or Osthoff, or both, were then collected and combined into mean values. The letter n denotes the number of stars in each case. An exception was made for the symbol W, which was compared with Osthoff's numbers only, exclusive of Krüger's, because Krüger saw the white stars more yellowish.

TABLE II

Yellowish Colors			Orange Colors			Peculiar Colors		
Fr.	K.-O.	n	Fr.	K.-O.	n	Fr.	K.-O.	n
W	1.99	553	$\overline{\text{OY}}^1$	4.23	4	B^1	2.01	7
Y^1	3.09	799	$\overline{\text{OY}}^2$	5.86	78	G^1	2.39	13
Y^2	4.96	653	$\overline{\text{OY}}^3$	6.42	72	$\overline{\text{YG}}^1$	2.82	55
Y^3	5.88	493	O^1	3.30	15	$\overline{\text{YG}}^2$	3.57	3
			O^2	6.35	253	R^1	3.40	6
			O^3	6.70	137	$\overline{\text{OR}}^1$	3.35	2
			$\overline{\text{OR}}^3$	7.71	17	V^2	1.50	2
Total	....	2498	....	....	576	....	....	88

3. It now remains to examine the effect of saturation indicated by exponents in the two-dimensional scale. This question is of special interest, because it touches the radical difference between the two scales, at least from yellow as far as red.

Leaving out for the moment the peculiar colors, we collect the letter-symbols that correspond nearly to the same number, and combine these numbers into mean values. We thus obtain Table III. The second line of the table is marked normal, because in the linear scale the shadings of yellow are treated as normal steps or grades. It will be noticed that Franks has not estimated any star as purely red.[1]

A number of conclusions may be drawn from Table III:

a) The W-estimates of Franks coincide nearly with those of Potsdam. According to the *Generalkatalog*, p. xxi, the symbol W corresponds to the number 2.4 of Osthoff. This means that

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[1] Apart from variables, the catalogues of K.-O. contain a few stars between the color-grades 8 and 9, e.g., K 844, 1079, O 2264.

Osthoff discerned the shadings of yellow more minutely than any other observer.

b) The normal colors of Franks for all the tones have the index 3. This fact is important, as it shows that the four degrees of saturation proposed by Smyth and retained generally are only theoretical, and that practically the normal shading is 3, which, by the way, is an improvement on the theory (see the writer's article quoted above, p. 272).

TABLE III

Krüger-Osthoff	2.0	3.2	4.6	5.9	6.4	6.7	7.7	—
Normal colors...	W	Y^1	Y^2	Y^3	$\overline{OY}^3$	O^3	$\overline{OR}^3$	R^3
Shifted colors...	....	O^1	$\overline{OY}^1$	$\overline{OY}^2$	O^2	....	....	....

c) Lowering the exponent shifts the letters from right to left, i.e., from red to white. Now the fact that the shifted symbols all contain orange, which is a tone and not a shading, teaches an important lesson: What the areal scale expresses as a shading is in the linear scale a different tone. To explain more in particular, O^2 is meant to be a pale orange; now a pale orange seems to be necessarily a yellowish orange and hence a tone different from orange. Again, $\overline{OY}^2$ is a pale orange yellow; what else can it be than a pure yellow i.e., a lower tone in the scale? Similarly, the symbols O^1 and $\overline{OY}^1$ are supposed to designate very faint tints of orange. Can the most experienced observer distinguish them from shadings of yellow? Krüger and Osthoff could not, nor can the writer.

It seems then that the symbols in the last line of Table III have only a theoretical meaning, like the exponent 4, and that practically the symbols that stand in the vertical columns of Table III are identical.

d) Comparing Tables I and III, we notice that the normal colors in the second line of Table III have a shorter range than the letter-symbols of Table I, both in form and in substance. In form, because they contain eight symbols against ten in Table I; and in substance, because the range from W to OR^3 is 5.7 steps, while in the numerical scale there are 7.5 grades from W to $\frac{1}{2}$ (RO+OR).

The question whether a shorter scale is preferable or not is far less important than an attempt to establish a closer agreement between the color-scales. Two simple changes in the areal scale would make the correlation almost perfect. The first is that permutations be admitted in the compound letter-symbols; and the second that the exponents be replaced by the letters W and S, as in Table I. More detailed suggestions were proposed by the writer in the article quoted, p. 273.

e) Finally, as regards the peculiar colors of Table II, they lie outside the numeral scale and, in fact, could not be expressed in its digits. This circumstance forms a decided advantage of the letter-scale, although it cannot be called an essential deficiency of the number-scale. Both Krüger and Osthoff maintain that those peculiar colors, especially the tints of blue and green, are seen only in double stars, and that observations of this kind in single stars are always traceable to some error. The mean of the last column K.-O. in Table II is 2.73, i.e., on the eyes of Krüger and Osthoff those 88 stars of Franks made the average impression of a pale yellow. In whatever manner this question may be decided, let both scales, the literal and the numeral, exist side by side and supplement each other's defects.

One paramount advantage of the number-scale is asserting itself at the present time, when numerous observations of Nova Aquilae are being reduced and published, to which a color-correction is, or should be, applied for each observer. A linear formula like this,

$$\text{Mag.}=x+y\ \text{Grade}+z\ \text{Color},$$

is usually sufficient. The unknown coefficients are obtained from the comparison-stars, for which the photometric magnitude, the scale of grades, and the color are supposed to be known. A solution by least squares will furnish the zero point x, the step value y, and the color-coefficient z.

Evidently a literal scale cannot lend itself to the reduction, unless it be transformed into numbers by some means like Table II.

Vatican Observatory, Rome
February 1919

Reviews

Geschichte und Literatur der veränderlichen Sterne. Band I. G. Müller und E. Hartwig.

A free translation of the full title of this work is: "History and Literature of the Light-Changes of the Stars Recognized as Variable up to the End of 1915, as well as a Catalog of the Elements of their Variations," edited under instructions from, and at the cost of, the Astronomische Gesellschaft, by G. Müller and E. Hartwig (Leipzig, Poeschel and Trepte, 1918. Vol. I).

The first catalogue of variable stars appeared in the *Philosophical Transactions* for 1786, written by Pigott and containing "observations and remarks" on 12 known and 38 suspected variables. The work under review describes 1687 variables, of which 838 are found in the first volume of xvi+401 folio pages. The stars are arranged in order of right ascension and the hours 0 to 14 inclusive appear in this volume. The second volume will contain the stars in the remaining hours, and also a list of probably variable but as yet unconfirmed stars, mostly found on the photographs taken at the Harvard College Observatory and at Heidelberg. The third volume will have the catalogue of latest elements of variation and a number of auxiliary tables, but no graphic light-curves.

This great work was planned in 1901 when Chandler announced that his series of catalogues of variables could no longer be continued. The Gesellschaft appointed as a committee: Dunér, Müller, Oudemans, and Hartwig. Kempf was added to the number after the deaths of Dunér and Oudemans. The scope of the work grew during years of planning till it finally included everything that could be classed as historical about all the known variable stars. No wonder that the mere title is so ponderous as to be even top-heavy, when the plan was so inclusive and the contents were made to measure up to the plan.

This review must be limited mainly to the history of the undertaking, which is described in 16 folio pages of introduction. A collection was first made of all the known literature bearing on variables, and a card catalogue was made by individual stars. This literature was examined with great care by Müller, Boegehold, Hartwig, Zinner, Lehnert, Schulz, and Pračka. Efforts were made to collect as much unpublished material as possible, but the editors admit that many important series, especially from American observers, were not available to them. On the other hand, of the three sources considered the most valuable, the *Astronomische Nachrichten*, the *Astronomical Journal*, and the *Harvard Annals*, two are American.

After three years of labor the literature was assembled and the card catalogue brought up to date. Since the completion of the work as planned was evidently beyond the powers of the commission alone, co-workers were secured, including Beljawsky, Van Biesbroeck, Boegehold, Ebell, Fagerholm, Graff, Guthnick, Lehnert, Pračka, Rosenberg, and Zinner. The stars were divided among these co-workers, copies of the card catalogue being furnished them by Frau Müller, and the actual work of arranging the material for publication was begun. In the published work the reports on the individual stars are signed by the author's initials.

The commission felt obliged to suspend the rule requiring two witnesses to a star's variability before it was admitted to the catalogue, and included many stars vouched for by a single reputable observer, if the change observed visually amounted to half a magnitude; or, if found from photographs, if it changed a full magnitude. They speak of the difficulty of forming a judgment on account of the failure of the discoverer to publish his original comparisons. They bemoan the fact that many valuable series of observations remain so long unpublished; and, oddly enough, mention Hartwig, one of their own number, as an offender in this respect.

The important question of reform in the nomenclature of variables is discussed at length; with the final decision to adhere to the old system, with amendments to make it last for a decade or two at most, rather than to break boldly with tradition and inaugurate a lasting system.

A compromise of somewhat similar nature is made in the matter of color. Osthoff's visual scale is adopted and tables are given for converting to his standard the systems of Chandler, Innes, Krüger, Müller and Kempf, Nyland, Pračka, and Yendell. No mention is made of color-index or the ever-present changes in color with variation in light.

The spectral classification is taken from *Harvard Annals*, Vol. 56, No. 6, which is Mrs. Fleming's "Stars Having Peculiar Spectra."

Great care is taken in the matter of accurate positions, and authorities are quoted for each star. The statement is made that "precise positions are valuable not only for certain identification but especially for the determination of proper motions."

As 838 stars occupy 401 pages, an average of about half a page is devoted to each star. A translation of the matter given for a sample star, SS Cassiopeiae, follows, which will give a better idea of the work than a verbal description.

3. SS Cassiopeiae ($0^{\mathrm{h}}4^{\mathrm{m}}24^{\mathrm{s}}$ $+51^{\circ}0'.6$). Not found in the B.D.

Position determined by Hartwig (*V.J.S.*, 43, 72). Chart by Hartwig (*Bamb. Veröff.*, II, Band 1, 7).

(* $11^{\mathrm{m}}.6$ pre. 6^{s}, $2'.8$ S.—* $11^{\mathrm{m}}.4$ fol. 3^{s}, $1'.6$ S.—* $11^{\mathrm{m}}.1$ fol. 5^{s}, $3'.3$ S)

Discovered 1905 by Fleming on the Draper-Memorial plates (*Harv. Circ.*, 98). Spectrum Md. A range of photographic magnitude from $9^{\mathrm{m}}.0$ to $11^{\mathrm{m}}.5$ was found on 5 chart plates between the years 1898 and 1902. Variability confirmed by Seares and by Hartwig. From photometer measures in 1905 and 1906 Seares deduced the approximate elements: Max. = 1905 Aug. 24 (2417082) $+141^{\mathrm{d}}$E. He found the visual range to be about 3 magnitudes, the light-curve nearly symmetrical about a sharp maximum. Hartwig found from observations between 1906 and 1910 a value of $139^{\mathrm{d}}.6$ for the period, with a range between $8^{\mathrm{m}}.5$ and $11^{\mathrm{m}}.7$.

Literature.—Pickering, Notice of Discovery by Fleming (*Harv. Circ.*, 98, and A.N. 4027).—Seares, 2 Max. 05 Aug. 24 ($8^{\mathrm{m}}.0$) and 06 Oct. 20 ($8^{\mathrm{m}}.7$) from photometer measures 05 Aug. 7 to Sept. 19 and 06 June 11 to 07 March 1. Statement of days when the variable was at or below the limit of the $7\frac{1}{2}$-inch equatorial. Approximate elements (*Laws Bulletin*, 10).—Hartwig, Individual Estimates and Deduced Magnitudes on 18 Days in the Years 1906–1910. 3 Max. 06 Oct. 20: ($8^{\mathrm{m}}.5$), 07 March 5: ($8^{\mathrm{m}}.5$), 07 July 20 ($8^{\mathrm{m}}.5$) and 2 Min. 07 Oct. 3 ($11^{\mathrm{m}}.7$), 08 July 1 (11.7). Chart of the neighboring stars (*Bamb. Veröff.*, II, Band 1, 7. See also A.N. 4212, Notation).—Graff, 4 estimates 06 July 29—07 Sept. 1, color 7, first and last estimates near the Min., third near the Max. (A.N. 4719). Color-estimates from 2 observations 6.0 (A.N. 4709).

The typography, as might be expected, is excellent. A single detail perhaps merits criticism, the use of lower case superior *m* ($^{\mathrm{m}}$) to denote both minutes of right ascension and magnitude, frequently on the same line.

J. A. P.

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ON THE USE OF GRATING SPECTRA FOR DETERMINING SPECTRAL TYPE AND ABSOLUTE MAGNITUDE OF THE STARS

By BERTIL LINDBLAD

PRELIMINARY NOTICE

In a recent paper[1] I have tried to give an exposition of the method of deriving the photographic effective wave-lengths in grating spectra, especially used by Hertzsprung[2] and by Bergstrand[3] for the determination of the color and spectral type of the fixed stars. The instrument employed was the twin 6-inch astrograph at the Upsala Observatory with Zeiss triplet lenses (aperture 15 cm, focal length 150 cm). The constant of the wire grating in front of one of the objectives was 1.3422 mm. This instrument had previously been used for determining effective wave-lengths by

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[1] *Arkiv för Matematik, Astronomi och Fysik utgifvet af K. Svenska Vetenskaps akadamien*, **13**, No. 26, 1918.

[2] *Bulletin astronomique*, **25**, 5, 1908; *Publ. des Astrophysikalischen Observatoriums zu Potsdam*, **22**, No. 1, 1911; *Astrophysical Journal*, **42**, 92, 1915; *Contributions from Mount Wilson Solar Observatory*, Nos. 100 and 101.

[3] *Astronomische Nachrichten*, **177**, 241, 1908; *Nova Acta R.S. Scient. Upsal.*, Serien IV, **2**, No. 4, 1909, also in *Comptes Rendus*, **148**, 1079, 1909.

Bergstrand and Lindblad[1] for fixed stars, and by Lundmark and Lindblad[2] for some spiral nebulae and globular clusters.

One of the chief results of the investigation was that the effective wave-length λ_c showed a very close relation to the spectral type defined in the Harvard system. On the whole the effective wave-length seemed to be perfectly analogous to the color-index. The difference in λ_c between classes B and K amounts to about 22 $\mu\mu$, the mean error for a single measured image being ± 2 $\mu\mu$. But it was also evident that for the later spectral types the color was closely related to the absolute magnitude of the star, a fact which previously has been more or less clearly expressed by Kapteyn, Van Rhijn, Adams, and Monk.[3] The spectral type being the same, a star will be redder the smaller its absolute magnitude, i.e., the greater its luminosity. From the colors of Osthoff[4] and the list of parallaxes prepared by Walkey[5] I found for the coefficient $\gamma = \frac{\Delta C}{\Delta M}$, where C and M denote color and absolute magnitude, the value $-0^{\mathrm{c}}.12$, corresponding to $-0^{\mathrm{M}}.053$ in color-index. From the effective wave-lengths for 26 stars in the catalogue of 500 parallaxes given by Adams and Joy[6] I found $\gamma = -0.93$ $\mu\mu$ for the types G0 to K5, corresponding to $\gamma = -0^{\mathrm{M}}.052$ in color-index. The effect is so great that it seems quite possible to compute the absolute magnitude by independent determinations of color and spectral type. The effective wave-lengths for the stars from the list of Adams and Joy could be arranged in two distinct series, one representing the giants, the other the dwarfs. Thus it seems as if we had here a valuable complement to the method of Adams and Joy for the determination of absolute magnitudes.

Now the question arises, What may be the cause of this relation between color and absolute magnitude? It seems most reasonable to assume that we have here to deal with an effect of a selective absorption, more pronounced for the shorter wave-lengths, in the vast atmospheres of the immense stars which have been characterized as giants. The spectral lines which define the spectral type, for instance in the Harvard system, are dependent on the temperature and composition of the photospheric layers of the star; the color, at least for the later spectral types, in addition depends on the dimensions of the overlying atmosphere. Thus the photospherical conditions and the dimensions of a star seem to be uniquely determined by the spectral lines and the color in combination.

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[1] *Arkiv för Mat., Astr., och Fysik*, 11, No. 17, 1916.

[2] *Astrophysical Journal*, 46, 206, 1917; *Astronomische Nachrichten*, **205**, 161, 1917.

[3] *Astrophysical Journal*, 44, 45, 1916; *Mt. Wilson Contr.* No. 119, 1916. See also a recent paper of Seares, *Publications of the Astronomical Society of the Pacific*, **30**, 122, 1918.

[4] *Astronomische Nachrichten*, **153**, 141, 1900.

[5] *Journal of the British Astronomical Association*, Appendix to **27**, 1917.

[6] *Astrophysical Journal*, 46, 313, 1917; *Mt. Wilson Contr.* No. 142, 1917.

But have we really exhausted all the possibilities of the grating spectra by measuring the effective wave-length that is the center of the density in the photographic image of the spectrum? There is at least another quantity that may be measured as well, and that is the length of the spectrum. In the investigation mentioned above I have measured the length of the spectra for some stars of different spectral types.[1] Thereby it was found that the outer limit of the spectra was tolerably constant, but that the inner limit varied rather strongly. Thus the spectra were much shorter for stars of the later types than for those of the earlier. But since the outer limit only varies slightly, and besides is often hard to define, it is of no use to measure it, and in this way we have only the inner limit left. This inner limit defines the shortest wave-length in the spectrum of a star that has had any effect on the photographic plate during a particular exposure. In the following I will call this wave-length simply the minimum wave-length, λ_{min}. For good images of grating spectra taken with the astrograph of the observatory of Upsala this minimum wave-length is rather sharply defined and easy to measure. It is determined in a manner analogous to that used for the effective wave-length. The distance between the inner limits of the two spectra, s_{min}, is measured with the micrometer of the measuring machine. Then for the center of the plate we have, if f is the focal length, c the grating constant:

$$\lambda_{\text{min}} = \frac{c}{2f} s_{\text{min}}$$

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[1] *Loc. cit.*, p. 6.

One of the greatest merits of the grating spectra is that there are good controls on the quality of the images. The chief control is the symmetry of the two spectra, not only the symmetry of the outlines, but also of the distribution of intensity within the two images. A second control is the regularity of the central image. As a third control one has the intensity and sharpness of the blackening of all the three images.

Naturally the minimum wave-length as well as the effective wave-length depends on the instrumental conditions, the telescope employed, the grating constant, and the properties of the plates. The sharp limit is produced by several causes beside the character of the energy-curve of the spectrum, as the sensibility-curve of the plates, the absorption in the lenses, and the secondary spectrum of the objective, i.e., the variation of the focal length with the wave-length. The limit is broad and well defined for the white stars; when λ_{min} increases for the redder stars the limit is drawn to a point, and for extremely red stars, the giants of type M, it is rather diffuse.

The minimum wave-length is dependent on the intensity of image, and must be defined for a certain intensity. As parameter of correction I have chosen the same as for the effective wave-length, the diameter D of the central image. The normal intensity is also the same, $D=0.14$ revolutions of the micrometer, corresponding to 0.07 mm. The correction seems at first to diminish for the redder stars; it is greater for λ_{min} than for λ_c, but is well defined and lies within reasonable limits.

Thus we have found in the minimum wave-length a new and easily measurable color-equivalent, determined from the shortest possible wave-lengths in the spectrum. The question now is what we may gain by this new color-equivalent. In the foregoing we have seen that the later spectral types do not uniquely determine the color of a star, this being moreover in a certain degree dependent on the absolute magnitude. In order to simplify the reasoning we may say that there are two different series of stars, giants and dwarfs, corresponding to two distinct series of color. In each of these series taken alone the spectral type and the color advance with falling temperature; in the investigation previously mentioned it was found that for the stars of Wilsing and Scheiner,[1] in all probability mostly giants, the relation between temperature and effective wave-length could be represented by the equation:

$$\lambda_c = 403.4\ \mu\mu + 11.4 \cdot \frac{10^4}{t}.$$

According to the law of Wien this means that there is a linear relation between λ_c and the wave-length for the maximum of energy in the spectrum. The difference between the two series is probably caused by a different absorption in the stellar atmospheres. As a type for these atmospheres we may take that of the sun. From the observations of Vogel the coefficient of transmission for different wave-lengths has been determined by Seeliger.[2] From these values of the coefficient of transmission I have computed in Table I the coefficient of absorption n, defined in the equation:

$$I = I_0 e^{-ns},$$

where I is the intensity of light for a certain wave-length, s the number of solar atmospheres which correspond to the distance covered in the absorbing medium.

The law of Planck for the intensity of radiation of a black body may be written for short wave-lengths and not too high temperatures:

$$I = C\lambda^{-5} e^{-\frac{c}{\lambda t}}.$$

If the beam of light also has to travel through an absorbing medium, we have

$$I = C\lambda^{-5} e^{-\frac{c}{\lambda t}}. \quad e^{-ns}.$$

If n were proportional to $1/\lambda$, the effects of temperature and absorption would be of the same quality, and would produce analogous changes in the intensity distribution of the spectrum. Then theoretically we could gain nothing by the introduction of a new color-equivalent such as the minimum wave-length, for then it would be impossible to separate the effects of the two variables—temperature and absorption—even if the new color-equivalent were determined from another part of the spectrum, for instance as here from the shortest possible wave-lengths. Now according to Müller,[1] Abney, and Langley the coefficient of absorption for the atmosphere of the earth increases with a much greater speed than the function C/λ. According to Table I it seems as if the same were in some degree true for the atmosphere of the sun, at least for the photographic part of the spectrum. In the last column C/λ is made equal to n for $\lambda = 470\ \mu\mu$, which is very near the outer limit of the grating spectra. At all events we have some reasons to believe that λ_{min} may be considered as an *absorption-equivalent;* it is a priori probable that λ_{min} is more sensible for absorption than λ_c, which on the whole principally has the character of a temperature-equivalent.

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[1] *Publ. des Astrophysikalischen Observatoriums zu Potsdam*, **19**, 1909.

[2] *Sitzungsber. der K. Bayer. Akad. der Wiss. Math. Phys. Kl.*, **21**, 247–72,, 1891.

TABLE I

Wave-length	Coefficient of Transmission	n	$\frac{C}{\lambda}$
662 $\mu\mu$	0.77	0.26	0.32
579	0.66	0.42	0.37
513	0.63	0.46	0.41
470	0.64	0.45	0.45
443	0.57	0.56	0.48
409	0.54	0.62	0.52

But before we go further in the theoretical discussion I will refer to the results obtained in a preliminary investigation of the subject. For a sequence of thirty-five stars, chosen to represent different spectral types and absolute magnitudes, I have determined λ_c and λ_{min}. Twenty-five of the stars were taken from the catalogue of Adams and Joy, and as a complement five stars of earlier types were measured. In addition four stars of c-character in Miss Maury's classification have been included. According to Hertzsprung[2] the stars of this character are of a very high luminosity; from the catalogue of Adams and Joy it is seen that their absolute magnitudes are on an average smaller than those of the other giant stars. To the c-stars I have added δ Cephei; though only being of

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[1] *Photometrie der Gestirne*, p. 139.

[2] *Zeitschrift für wissenschaftliche Photographie*, 3, 429; 5, 86.

ac-character in Miss Maury's classification, there cannot be any doubt about its exceedingly great luminosity; its absolute magnitude (apparent magnitude in the distance 10 parsecs) may be estimated to -2^{M}.[1]

The telescope and grating were those mentioned above, the plates were Imperial S.S., Emulsion 9629A. These plates seem to be less rapid than those employed in the foregoing investigation of the emulsion 9220A. As developer Rodinal was used. A good control on the quality and development of the plates would be to take some control stars on each plate, but this would involve a considerable increase of labor. In this case I have generally taken many stars of different kinds on the same plate. A systematic difference between the different plates has not been discovered. The distribution of the stars between the plates is shown in Table II. The numbers refer to the first column of Table IV.

TABLE II

Plate Number	Date 1918	Stars
1	Sept. 14	2, 5, 6, 7, 8, 17, 18, 22, 29, 34
2	Sept. 20	32
3	Sept. 21	26, 27, 28, 35
4	Sept. 21	15, 30, 31, 33
5	Sept. 24	10, 11, 12
6	Sept. 26	13, 14, 16, 19, 24, 25
7	Sept. 27	3, 4, 9, 19, 20, 21, 23
8	Oct. 4	1

The effective wave-lengths were determined in the usual way. After a plate was measured in one position it was turned 180° and measured again. The correction for the intensity of image, measured by the diameter D of the central image, was taken from the investigation previously referred to, but for safety the limits were taken more close to the normal diameter. The correction for the zenith-distance was also taken from the paper mentioned.

The minimum wave-length was measured according to the principles mentioned above. As far as possible the measurements were performed without knowledge of either the spectral type or the luminosity of the star in question. The distance between the inner limits of the two spectra was measured with a double thread of the microscope of the measuring-machine, and in both positions of the plate. A rigorous control was applied to the quality of the images, of which only the best ones were measured.

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[1] Hertzsprung, *Astronomische Nachrichten*, **196**, 201, 1914.

Table III shows the correction of $\lambda_{\min}$ for the intensity of image. I have found three distinct types for the effect in question, corresponding to three different groups of stars. The first group contains all stars with $\lambda_{\min} < 378\ \mu\mu$; the second consists of the dwarfs with $\lambda_{\min} > 378\ \mu\mu$; the third is formed by the giants from the catalogue of Adams and Joy with $\lambda_{\min} > 378\ \mu\mu$. Obviously the magnitude of the correction diminishes with increasing redness of a star. In consequence of this it is probable that the effect, at least for the first group, is principally produced by the same cause as the usual widening of stellar disks with increasing time of exposure, for otherwise we should have to expect a less marked effect for the white stars than for the redder. The choice of the diameter D of the central image as parameter of correction, which might be open to some objections, thus seems more legitimate.

TABLE III

(Unit of D=0.01 of a rev. of the micrometer)

Diameter	12	13	14	15	16
Group I	−4.6 $\mu\mu$	−2.1 $\mu\mu$	0.0 $\mu\mu$	+2.1 $\mu\mu$	+4.4 $\mu\mu$
Group II	−2.9	−1.4	0.0	+2.2	+5.0
Group III	−1.2	−0.2	0.0	0.0	0.0

For exceedingly red stars, $\lambda_c > 435\ \mu\mu$, the magnitude of the effect increases considerably; in this way we have a fourth type of the effect, which is here conditioned by the less rapid fall of the spectral intensity on the side of the shorter wave-lengths. This type of the effect seems also to hold true for the *c*-stars of the later types. The correction used for the stars in question is: $\Delta\lambda = 3.5\ \mu\mu\ (D-14)$, where D is expressed in hundredths of a revolution of the micrometer.

Then in order to derive the definitive values of $\lambda_{\min}$, when they are greater than $378\ \mu\mu$, we must know something about the absolute magnitudes of the stars. As shall be mentioned in the following, this will be possible by a combination of λ_c with a preliminary value of λ_{min}. On the other hand the appearance of the effect for a certain star may give some information about its luminosity.

TABLE IV

No.	Star	App. Mag.	R.A. 1900	Decl. 1900	Spectrum	Abs. Mag.	λ_c	λ_{min}	*n*
							$\mu\mu$	$\mu\mu$	
1 ..	τ Cygni	3.8	$21^h 10^m.8$	$37° 37'$	F1	+1.9	419.3	375.4	3
2 ..	σ Pegasi	5.3	22 47.3	9 18	F4	+3.9	419.4	374.0	1
3 ..	Boss 5772	6.1	22 18.8	20 21	F5	+4.5	419.9	375.5	2
4 ..	Groom.3357	6.6	20 56.1	39 51	F7	+4.6	422.3	373.4	2
5 ..	Boss 5920	6.5	22 53.5	8 50	G0	+4.4	420.6	374.5	1
6 ..	Lal. 47207	6.2	23 59.6	34 6	G0	+4.4	422.5	375.0	1
7 ..	85 Pegasi	5.9	23 56.9	26 33	G1	+5.8	419.2	375.3	1
8 ..	η Pegasi	3.1	22 38.3	29 42	G2	+1.2	425.4	379.9	2
9 ..	ϵ Androm.	4.5	0 33.3	28 46	G3	+1.3	423.1	379.2	4
10 ..	Fed. 4371	7.5	23 1.2	67 52	G5	+7.0	421.2	376.0	2
11 ..	η Piscium	3.7	1 26.1	14 50	G5	+0.3	426.5	385.2	1
12 ..	Boss 6097	6.6	23 39.9	55 15	G6	+0.7	425.8	383.5	1
13 ..	24 Vulpec.	5.4	20 12.5	24 22	G7	+0.7	429.1	386.8	2
14 ..	ϵ Cygni	2.6	20 42.2	33 36	G8	+0.8	428.4	386.5	2
15 ..	η Cephei	3.6	20 43.3	61 27	G9	+2.4	429.6	380.3	2
16 ..	Lal. 39704	7.0	20 29.3	41.33	K0	+6.4	428.6	380.5	1
17 ..	Pi 22^h 214	6.5	22 40.9	29 55	K0	+4.4	425.9	382.3	2
18 ..	ι Cephei	3.7	22 46.1	65 40	K0	+1.1	427.7	383.5	3
19 ..	ϵ Pegasi	2.5	21 39.3	9 25	K0	−0.3	435.7	385.6	3
20 ..	Lal. 38683	7.3	20 6.6	15 53	K1	+6.7	432.3	379.7	1
21 ..	Boss 5602	6.5	21 41.8	25 6	K3	+1.5	436.2	384.2	1
22 ..	Boss 5868	6.1	22 39.6	38 56	K4	+0.9	438.8	386.6	1
23 ..	Boss 5976	5.6	23 8.5	56 37	K5	+6.6	429.7	381.2	4
24 ..	61^1 Cygni	5.6	21 2.4	38 15	K7	+7.9	430.4	382.8	2
25 ..	61^2 Cygni	6.3	21 2.4	38 15	K8	+8.7	430.5	379.9	1
26 ..	β Cephei	3.3	21 27.4	70 7	B1; IIIa	....	411.1	373.0	4
27 ..	o Herculis	3.8	18 3.6	28 45	A; VIIb	....	415.7	371.4	2
28 ..	δ Cygni	3.0	19 41.9	44 53	A; VIIb	....	414.8	374.7	3
29 ..	α Pegasi	2.6	22 59.8	14 40	A; VIIIb	+0.35	412.6	374.0	2
30 ..	ϵ Cephei	4.2	22 11.3	56 33	A5; Xa, b	+3.80	418.0	374.5	1
31 ..	4 Lacertae	4.6	22 20.4	48 58	B9; VIc	....	414.9	376.0	3
32 ..	α Cygni	1.3	20 38.0	44 56	A2p; VIIIc	....	413.2	376.4	1
33 ..	*d* Draco.	5.0	18 30.9	56 58	F8p; XIIIc	....	423.8	380.7	2
34 ..	ρ Cass.	5.0	23 49.4	56 57	G4p; XIIIc	−0.4	428.7	383.5	1
35 ..	δ Cephei	var.	22 25.4	57 54	G0; XIVac	....	423.9	378.7	2

As the material is too scanty for an accurate determination of the effect, I have not generally used corrections exceeding 5 $\mu\mu$.

For the influence of the zenith-distance I have assumed the same values of the correction as for the effective wave-length. It is evident that λ_{min} is much less sensible for changes in the spectral intensity for stars of the earliest spectral types, and therefore I have not applied the correction for stars with λ_{min} less than 375 $\mu\mu$. It may be that the correction for the zenith-distance ought to be a little greater than I have assumed here, but in most cases it is too small to be of any importance.

Table IV contains the results of the measurements. The first column contains the number of the star, the second to fifth the name of the star, its visual magnitude, R.A., and Decl. The sixth and seventh contain the spectral type and absolute magnitude according to Adams and Joy (spectrum estimated). For the last ten stars the spectral type is due to Harvard; for α Pegasi and ε Cephei the absolute magnitudes are computed from the parallax-list prepared by Walkey. The last columns contain the effective and minimum wave-lengths and the number of images from which λ_{min} has been derived. In computing definitive values of λ_{min} the images with $D < 12$ are generally excluded.

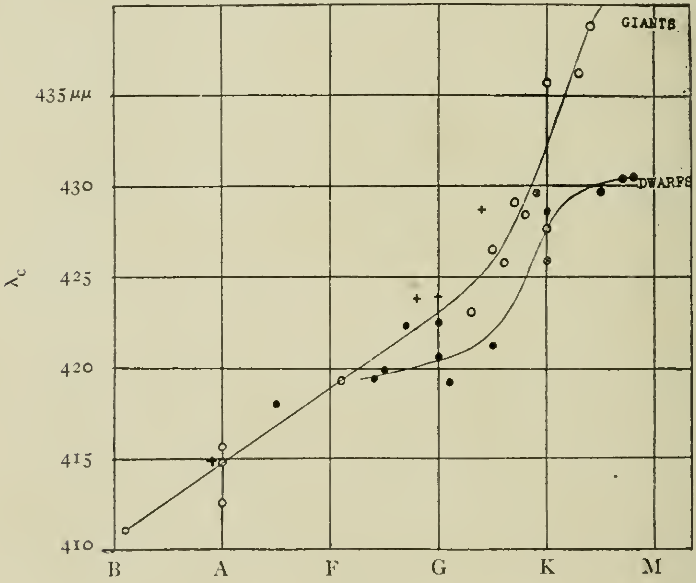

Fig. 1

Fig. 1 shows the relation between the effective wave-length λ_e and the spectral type. The open circles indicate giants, the black circles dwarfs, the circles with a cross represent an intermediate type. The five *c*-stars are represented by crosses; as mentioned above, they may be considered as the most pronounced giants. The early-type stars with character *a* and *b* are designated as giants, except ϵ Cephei.

The character of the diagram is precisely the same as had been found in the previous paper.[1] Five stars of the giant series are common for the two investigations. From these stars it is seen

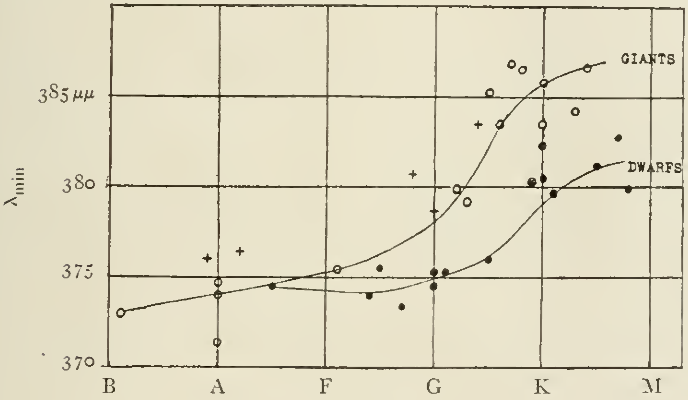

FIG. 2

that the effective wave-lengths for the Imperial plates of the emulsion 9629A are on an average 3.0 $\mu\mu$ smaller than those measured on plates of the emulsion 9220A. This correction seems to hold unchanged through the whole spectral series. For the coefficient

$$\gamma_c = \frac{\Delta\lambda_c}{\Delta M},$$

where M is the absolute magnitude, I find the value $-0.87\ \mu\mu$ from the stars of the spectral types G0 to K8. In the previous paper I found $\gamma_c = -0.93\ \mu\mu$ for the types G0–K5.

In Fig. 2 the relation between the minimum wave-length $\lambda_{\min}$ and the spectral type is shown. It is at once evident that the difference between giants and dwarfs is far more marked than for λ_c.

$$\gamma_{\min} = \frac{\Delta\lambda_{\min}}{\Delta M}$$

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[1] *Loc. cit.*, pp. 45 f., 73.

is found to be $-1.10\ \mu\mu$ for the types G0 to K8, mean type G9. Thus the coefficient γ has a somewhat higher value than for λ_c, though the differences in $\lambda_{\min}$ between different spectral types are generally less than the corresponding differences in λ_c. It is evident that $\lambda_{\min}$ has a lower limit at about $373\ \mu\mu$, which seems to be reached already by some dwarfs of types F–G. This limit is caused by the fact that the maximum of energy in the spectrum begins to fall in too short wave-lengths, outside the limit of the sensibility of the plate. It is moreover very probable that the appearance of Fig. 2 is in a high degree conditioned by the instrumental means, for instance the secondary spectrum of the objective. A consequence of the lower limit of $\lambda_{\min}$ must be that for the early types the effect of different luminosity is diminished. That nevertheless there are some possibilities for a successful application of the method, at least for the greater luminosities, is indicated by the high values of $\lambda_{\min}$ for the two *c*-stars 4 Lacertae and α Cygni. The mean error of $\lambda_{\min}$ for a single measured image I have found to be about $\pm 1.3\ \mu\mu$.

Fig. 1 and Fig. 2 illustrate the relations and fundamental differences between the three spectral symbols, the spectral type determined from the lines in the spectrum, and the color-equivalents λ_c and $\lambda_{\min}$. The spectral lines define the photospheric conditions of a star; then according to Fig. 2 the minimum wave-length seems to determine the absolute magnitude with a fair degree of accuracy. For the real color-equivalent λ_c the effects of absorption are on the whole of a subordinate importance. This leads us to the important conclusion that the effective wave-length λ_c may be used as a substitute for the spectral class in the determination of absolute magnitude by means of $\lambda_{\min}$. The relation between λ_c and $\lambda_{\min}$ is shown in Fig. 3. The differences between giants and dwarfs are still well marked. Thus when the absolute magnitude is known from a combination of $\lambda_{\min}$ and λ_c in the way indicated by Fig. 3, then λ_c may be used to determine the exact spectral type by means of Fig. 1. In this way we should have determined both spectral type and absolute magnitude with quantities measured directly on the grating spectra *and without any reference to the spectral lines.*

If this principle can be realized in practice with the necessary accuracy also for relatively long exposures, it would be of great importance, because the grating spectra can be photographed with relatively small instruments for stars of very faint magnitudes, for which the spectral lines cannot be determined even with the largest instruments. At any rate Fig. 2 represents a method for deriving absolute magnitudes of later-type stars which promises to be very convenient in practice, and seems to be a valuable complement to the method of Adams and Joy.

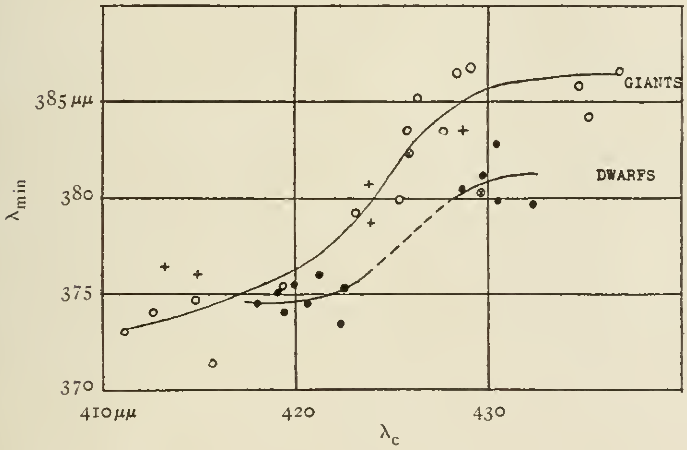

Fig. 3

There is, however, another important conclusion to be drawn from Figs. 1 and 3. Hertzsprung[1] has found that the effective wave-lengths of absolutely faint stars have a superior limit, which seems to be reached at the absolute magnitude +8 (app. mag. in the distance 10 parsecs). Now the two stars in the system of 61 Cygni, of the spectral types K7 and K8, are of the absolute magnitudes 7.9 and 8.7; their effective wave-lengths have been found to be 430.4 μμ and 430.5 μμ. If the division of the stars in giants and dwarfs, as it is shown in the catalogue of Adams and Joy, holds through the whole stellar system, then from the dwarf series in

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[1] *Astrophysical Journal*, **42**, 111, 1915; *Mt. Wilson Contr.* No. 101.

Fig. 1 and 3 we may conclude in accordance with the facts mentioned *that stars with* $\lambda_c > 433\ \mu\mu$ *are all giants.* $\lambda_c = 433\ \mu\mu$ corresponds to about $+1.2$ in color-index.

Further investigations on the subject will be continued at the observatory of Upsala.

Astronomical Observatory, Upsala
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COLOR-SENSITIVENESS OF PHOTO-ELECTRIC CELLS

By T. SHINOMIYA

In various photometric measurements the photo-electric cell proves to be a simple, reliable, and accurate instrument. It has been shown in a previous paper by J. Kunz that cells can be constructed in which for a constant potential-difference the photo-electric current is proportional to the intensity of light over a wide interval of illumination. P. Guthnick and R. Prager,[1] Elster and Geitel, Pohl and Pringsheim, have already studied the color-sensitiveness of photo-electric cells throughout the spectrum. The curves given by Guthnick and Prager contain, however, only a few points, which hardly suffice to determine the maximum sensitiveness and the nature of the curve with much accuracy. Moreover, the question arises as to how the colloidal modification of the alkali hydrides affects the color-sensitiveness. The following report contains the first measurements made in the Physical Laboratory of the University of Illinois. In Table I the dimensions of the cells are given. The anode rings had a cross of very fine platinum wire, so as to make the electrostatic field between the sensitive alkali surface and the anode as uniform as possible. The beam of light was allowed to enter the photo-electric cell through a narrow

TABLE I

K	Diameter of Bulbs	Diameter of Anode Rings
K	4.2 cm	2.2 cm
K-H No. 1	5.7	3.3
K-H No. 2	5.0	2.3
K-H No. 3	6.0	2.7
Na	4.1	2.2
Na-H	4.1	2.2
Rb	4.1	2.2
Rb-H	4.1	2.2
K-H Pyrex glass	3.1	1.3

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[1] *Veröffentlichungen der Sternwarte zu Berlin-Babelsberg,* 1, 1, 1914.

opening, the rest of the inner surface of the bulb being coated with alkali metal.

The source of light was a 15-ampere nitrogen-filled tungsten lamp made by the General Electric Company. The light was condensed on a slit of width 0.7 mm by a system of condensing lenses. By another lens the image of the slit was thrown on a screen and a prism placed in this beam of light. The position of minimum deviation was chosen for the green-blue light. A pretty pure spectrum was obtained, and all reflected and stray light was eliminated by five diaphragms placed in suitable positions. The whole optical system was contained in a box of thin board blackened inside. A table, carrying a slit *S*, the thermo-couple, and the box with the photo-electric cell could be moved by means of a fine screw along a scale *M* rigidly attached to a pier. In this way the slit was brought to any desired position of the spectrum and allowed the monochromatic beam of light to fall either on the thermo-couple or on the photo-electric cell. The scale was read outside the closed frame.

The scale *M* was calibrated in terms of wave-length by a Hilger spectrometer. The incident light was not absolutely monochromatic. A mean wave-length was used. As the sensitiveness of the photo-electric cell changes in a continuous way through the spectrum, this small deviation from the ideal condition has little effect on the final curve.

The energy of the light passing through the slit *S* was measured by means of a thermo-couple furnished by W. W. Coblentz from the Bureau of Standards. The slit of the thermopile was a little narrower than the slit *S*, so that the former was always covered by light. The thermo-electric current was measured with a high-sensitive moving-coil galvanometer (figure of merit $2.7 \cdot 10^{-9}$). The galvanometer and the wires connecting it with the thermo-couple were completely covered with thick boards of wood in order to keep the temperature constant and to prevent air currents. The galvanometer deflections were quite steady and could be repeated with ease.

The thermo-couple was calibrated by means of the light from a Hefner lamp. The lamp was placed at a distance of 50.1 cm from the slit. The corresponding deflection was 154.6 mm. The energy radiated from a Hefner lamp[1] was taken as 8.3 ergs per second per square centimeter at a distance of 1 meter. At 50.1 cm it is about 33 ergs. This flow of energy produced a deflection of 154.6 mm. Therefore the deflection of 1 mm corresponds to the flow of energy of 0.214 ergs per second per square centimeter. The area of the slit *S* admitting the light to the photo-electric cell was 0.272 cm and the energy passing through that slit per 1 mm deflection of the thermo-couple galvanometer was $0.214 \cdot 0.272 = 0.0581$ ergs.

The galvanometer, which measured the photo-current, had 480 ohms internal resistance, a critical damping resistance of 9600 ohms, and a sensitiveness of 4.10^{-10} amperes for a scale distance of 1.25 meters. To safeguard against short circuit a resistance of 100,000 ohms was inserted in the photo-electric circuit. A shutter in front of the photo-electric cell was opened and closed by the observer at his observing position by means of pulleys and strings.

At first the characteristic curve of each cell was determined. The source of light was of course kept as constant as possible. A small deviation from the constant 15 amperes made itself felt readily in the galvanometer deflection. These deflections were, however, quite steady and could easily be repeated as long as the source of light was constant.

The characteristic curves are shown in Fig. 1, in which the abscissae represent voltages, the ordinates currents. K-H No. 1 and Rb-H appear more sensitive than the alkali metals K and Rb respectively; on the other hand, Na-H and K-H No. 2 and K-H No. 3 are less sensitive for higher voltages than the pure metals. All cells of course contained argon. K-H No. 3 was the cell which J. Stebbins and J. Kunz used in the photometry of the corona at the solar eclipse of June 8, 1918.

The characteristic curve having been determined, a constant potential-difference was chosen lying on the horizontal branch of that curve. This potential-difference and the heating current of the source of light were kept very constant during the time in which the final readings for the sensitiveness of a given cell through the spectrum were taken. The photo-electric cell with the slit was moved first in one direction, then in the opposite direction, through the spectrum. For the same position the deflection of the galvanometer was the same.

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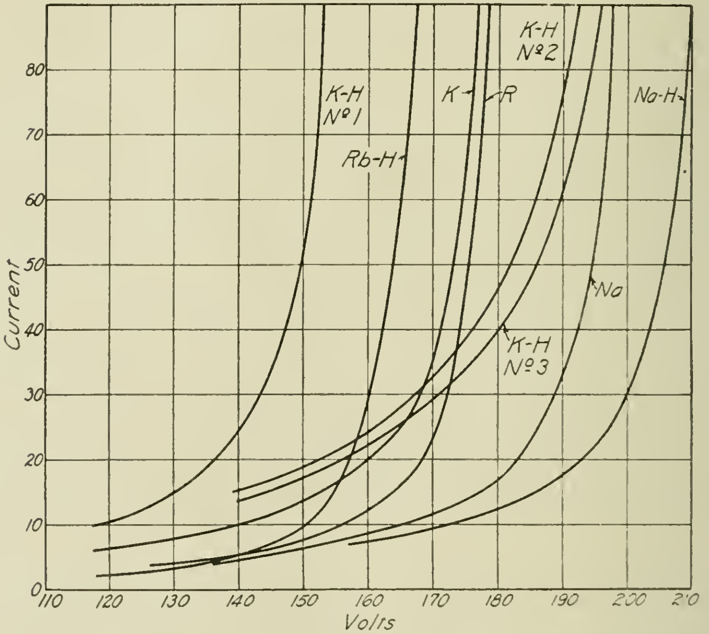
[1] E. L. Nichols, *Physical Review*, 21, 169, 1905.

A photo-electric cell of a pyrex glass tube was constructed. The deflections even for a small potential-difference were very variable, and it was impossible to get a curve for its sensitiveness. The other cells, however, were constant. Only when the potential-difference approached the critical value did the deflections become a little unstable.



Characteristic Curves

FIG. 1

Table II contains the readings of the scale M, the wave-length in angstroms, the deflections of the galvanometer measuring the energy of the beam of light, and the photo-electric currents as galvanometer deflections. The numbers below the alkali indicate the potential-difference applied to the cell when the readings were taken. A deflection of 102 mm of the photo-electric galvanometer means a current of $102.4 \cdot 10^{-10} = 4.08 \cdot 10^{-8}$ amperes. The corresponding deflection in the thermo-couple circuit was 6.8 mm, i.e.,

TABLE II

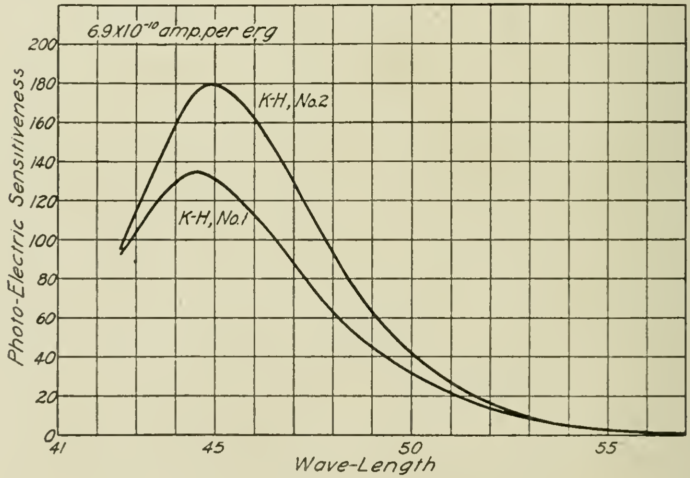
Scale	Wave-Length	Deflection of Light Energy	K-H No. 1 129	K-H No. 2 153	K-H No. 3 160	K 149	Na 142	Na-H 165	Rb 140	Rb-H 118
40...	6800	104.0	2.5	2.5	3.1	2.9	1.9	1.5	0.8	0.7
45...	6427	76.0	3.0	3.1	3.5	3.2	2.1	1.8	1.0	0.9
50...	6092	57.0	4.1	4.2	4.4	5.0	2.6	2.1	1.4	1.0
52...	5950	50.8	....	....	....	......	......	......	......	......
55...	5778	42.8	7.0	7.0	6.8	7.9	3.2	3.1	2.9	2.9
57...	5667	38.3	....	....	8.4	......	......	......	......	......
60...	5515	32.1	12.0	12.0	12.3	11.7	5.1	6.3	8.6	10.1
62...	5440	28.9	....	....	16.6	......	......	......	......	15.0
65...	5321	23.9	21.0	21.4	24.3	15.2	14.6	13.2	20.9	22.7
67...	5247	21.1	25.5	27.2	30.5	17.0	23.1	17.0	28.0	27.1
70...	5160	18.0	32.8	38.9	39.9	19.9	29.8	22.9	36.0	33.2
72...	5084	16.0	37.5	47.3	46.0	21.6	52.2	26.6	39.1	35.6
74...	......	....	....	....	....	......	......	......	41.8	36.4
75...	4985	13.4	45.5	61.0	54.2	23.7	68.9	30.7	42.0	37.0
76...	......	....	....	....	....	......	......	......	42.1	36.8
77...	4921	12.0	50.8	70.0	58.6	24.8	77.5	32.8	42.0	35.9
80...	4838	10.3	57.5	82.0	63.8	25.7	87.0	33.8	38.2	34.0
82...	4791	9.2	61.2	90.4	66.2	25.9	90.5	33.9	34.9	31.3
85...	4711	7.9	66.8	98.0	....	25.8	93.0	33.2	29.1	28.0
86...	......	....	68.1	....	68.0	25.5	......	......	......	......
87...	4666	7.2	69.1	101.6	....	......	92.8	32.5	25.8	25.6
88...	......	....	69.9	101.8	68.4	25.0	......	......	......	......
90...	4599	6.2	69.9	100.6	67.0	24.0	89.5	31.0	20.1	21.4
91...	......	....	69.1	....	....	......	......	......	......	......
92...	4557	5.5	....	95.9	63.3	22.9	86.0	29.8	17.0	18.4
95...	4499	4.9	62.5	85.0	57.0	20.8	79.1	28.0	12.8	15.0
97...	4460	4.2	57.5	76.0	51.3	19.5	74.2	26.7	10.5	12.9
100...	4417	3.7	48.7	60.2	43.0	17.8	66.0	24.7	8.1	10.5
102...	4374	3.4	43.1	51.2	37.6	16.4	61.0	23.3	6.9	9.5
105...	4324	3.1	35.0	39.3	30.2	14.6	54.1	21.2	5.2	6.9
107...	4294	2.9	30.1	32.9	26.0	13.5	49.2	19.9	4.3	5.8
110...	4254	2.6	23.5	25.0	20.3	11.8	42.9	17.7	3.7	4.2

TABLE III

MAXIMUM SENSITIVENESS OF 8 CELLS

Cell	K-H No. 1	K-H No. 2	K-H No. 3	K	Na	Na-H	Rb	Rb-H
Potential-difference.........	129	153	160	149	142	165	140	118
Wave-length $\mu\mu$	446	449	448	437	439	430	478	468
Max. photo-sensitiveness....	$9.3 10^{-8}$	12.4	8.3	3.3	11.7	4.7	2.6	2.4

the energy incident on the photo-electric cell per second was $0.058 \cdot 6.8 = 0.394$ ergs; hence one erg of light of wave-length 464 $\mu\mu$ produces a current of $1.22 \cdot 10^{-7}$ amperes. For the same cell, K-H No. 2, the maximum sensitiveness is found at 449 $\mu\mu$ and is $1.24 \cdot 10^{-7}$ amperes per erg. In this way the numbers of Table II were used for each cell and the system of curves was obtained as shown in Fig. 2, in which the abscissae represent wave-lengths in $\mu\mu$, and the ordinates multiplied by $6.9 \cdot 10^{-10}$ give the currents in ampere per erg.



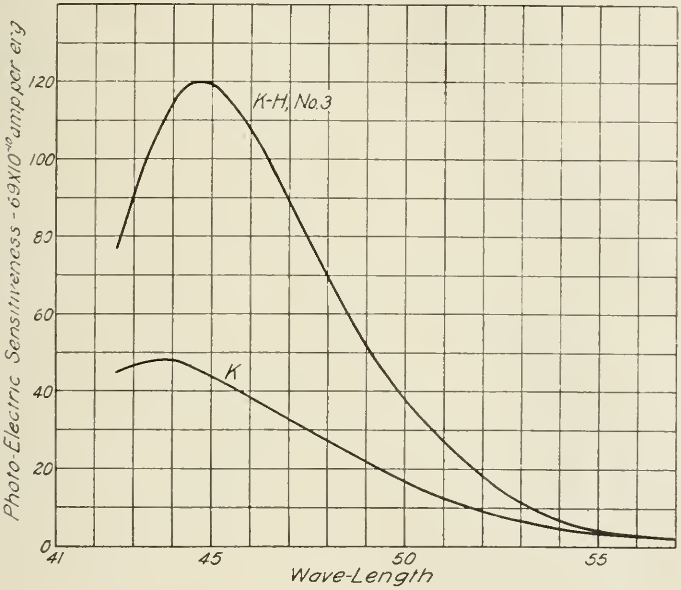
Sensitiveness for K-H No.1 & K-H No.2

FIG. 2

Pohl and Pringsheim, who measured the sensitiveness of sodium cells, give 2.10^{-12} amperes per "Lichteinheit" for a wave-length of 436 $\mu\mu$. We compared also the measurements of Elster and Geitel with our own results; with a few exceptions there is good agreement in the nature of the curves.

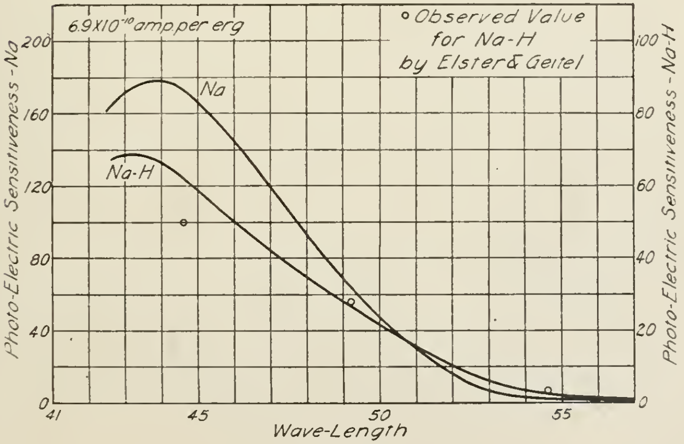
SUMMARY

Eight cells of potassium, sodium, and rubidium were examined. The maximum sensitiveness is of the order of magnitude of 10^{-7} ampere per erg by using rather low potential-differences. By an increase of the voltage much higher values can be obtained, but the deflections become less stable. Three potassium-hydrogen cells have about the same sensitiveness at the same wave-length. This maximum appears to be shifted toward longer wave-lengths with



Sensitiveness for K-H No.3 & K

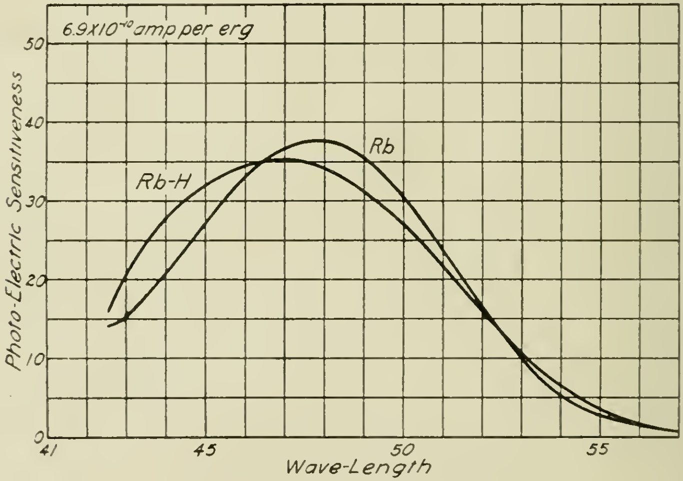
FIG. 3



Sensitiveness for Na-H & Na

FIG. 4

respect to the maximum of pure potassium. Moreover, the sensitiveness-curve of K-H is more pronounced than that of pure K, which is flatter. On the contrary, the H compounds of Na and Rb shifted the maximum toward shorter wave-lengths as compared with the pure metals, and the sensitiveness-curves of the alloys are flatter than those of the metals themselves. This strange difference in the properties of K, on one hand, and Na and Rb, on the other, requires a continued study of a larger number of cells. This study will also include lithium and caesium and will be extended in the ultra-violet region. It would be interesting to know whether the hydrogen affects the limit for long wave-length of the photo-electric effect of alkali metals. For comparative photometry the color-sensitiveness of the photo-electric cell, of the photographic plate, and of the human eye is required.



Sensitiveness for Rb-H & Rb

FIG. 5

The writer wishes to thank Professor A. P. Carman for the facilities of the laboratory, and to acknowledge his indebtedness to Jacob Kunz, under whose direction this work was carried out, for his many valuable suggestions concerning the conduct of the experiment.

Physical Laboratory
University of Illinois, Urbana
March 12, 1919

STUDIES BASED ON THE COLORS AND MAGNITUDES IN STELLAR CLUSTERS

TWELFTH PAPER: REMARKS ON THE ARRANGEMENT OF THE SIDEREAL UNIVERSE[1]

By HARLOW SHAPLEY

I. THE GENERAL GALACTIC SYSTEM

1. *Introduction.*—A fairly definite conception of the arrangement of the sidereal system evolves naturally from the observational work discussed in the preceding *Contributions.* We find, in short, that globular clusters, though extensive and massive structures, are but subordinate items in the immensely greater organization which is dimly outlined by their positions. From the new point of view our galactic universe appears as a single, enormous, all-comprehending unit, the extent and form of which seem to be indicated through the dimensions of the widely extended assemblage of globular clusters. The fundamental nature of the galactic plane, in the dynamical structure of all that we now recognize as the sidereal universe, is manifested by the distribution of clusters in space. Near this plane lie the celestial objects that we customarily study. The open clusters, the diffused and planetary nebulae, the naked-eye stars, most variables, the objects that define and compose the star streams—all of these appear to be far within a relatively narrow equatorial region of the greater galactic system, a region in which globular clusters are not found. The Orion nebula and even the Magellanic clouds are miniature organizations in this general scheme, and undoubtedly are dependents of the Galaxy.

The adoption of such an arrangement of sidereal objects leaves us with no evidence of a plurality of stellar "universes." Even the remotest of recorded globular clusters do not seem to be independent organizations. The hypothesis that spiral nebulae are separate galactic systems now meets with further difficulties. So long as the high velocities of nebulae were unapproached by the motions of other objects and the maximum luminosity attainable by stars was beyond estimate, and so long as the diameter of the galactic system was thought to be only a thousand light-years or so, we had a fairly plausible case for the "island universe" hypothesis. But now we must consider radial velocities of several hundred kilometers a second as quite possible for objects in our own system; we must assume a moderate upper limit of luminosity, perhaps even for the most massive of novae; and any external "universe" must now be compared with a galactic system probably more than three hundred thousand light-years in diameter. As seen from the center of the galactic system, globular clusters would be distributed in the sky much as the spirals are when observed from the earth.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 157.

It is probable that the further accumulation of observations will modify to some extent the views outlined above and discussed more fully in the following pages. The present data may in some cases be susceptible of alternative interpretation, or possibly the conclusions may be questioned in the belief that the material is insufficient. But the greater part of the hypothesis proposed is merely the most direct and simple reading of recent observations.

2. *Outline of interpretation.*—The suggested plan of the galactic system may be concretely formulated through the following series of propositions; some of them are later amplified in so far as seems necessary; for others the discussion of preceding contributions will suffice. A few of the statements are obvious corollaries, while those designated B and F, in some of their details, may be less easy to maintain. Taken altogether they attempt to establish a general idea of the arrangement, extent, and constituency of the system of stars and nebulae.

A. The globular clusters are a part of the galactic system and knowledge of their distances seems at present to afford the best way to fathom the system.

B. The system of globular clusters, which is coincident in general, if not in detail, with the sidereal arrangement as a whole, appears to be somewhat ellipsoidal. The longest axis of the ellipsoid lies in the galactic plane and passes the sun at a distance of approximately three thousand parsecs. Its nearest point is in galactic longitude 240°, nearly coincident with the direction of the center assigned to the local system of stars. See F and Fig. 1.

C. The center of the sidereal system is distant from the earth some twenty thousand parsecs in the direction of the constellation Sagittarius; it lies in the galactic plane, which dynamically and statistically appears to be the symmetrical plane of the entire sidereal universe as now known. As seen from the sun the thinnest part of the Milky Way lies in Gemini, Taurus, and Auriga—a region rich in bright open clusters close to the galactic plane.

D. The axes of the system in the galactic plane and perpendicular to it may not differ greatly; but the gravitationally important equatorial segment, which apparently contains most of the stars, is at least thirty times as extended in the plane as at right angles thereto.

E. The equatorial region appears to be uninhabitable by compact systems, such as globular clusters, notwithstanding the greater abundance there of stellar material.

F. The stars in the neighborhood of the sun (practically all that go into our catalogues of spectrum, position, and motion) appear to compose (1) a large, open, moving subordinate group, and (2) a part of the surrounding and interpenetrating star fields of the equatorial segment of the greater galactic system. The center of the local system is in the direction of the constellation Carina, nearly at right angles to the direction of the center of the general galactic system, but less than one two-hundredths as far away. The plane of symmetry and condensation of the local cluster is inclined to the galactic plane about 12°; the center of the cluster is north of the galactic plane, and the sun is north of both planes.

G. The volume of space occupied by stars brighter than the sixth apparent magnitude, some of which, being absolutely very bright, are extremely distant as compared with the majority of naked-eye stars, is at most only a hundred-thousandth of the volume occupied by the other parts of the galactic system.

3. *Relation of present interpretation to earlier hypotheses.*—In order to show where the earlier working hypotheses stand with

respect to the interpretation now offered, it may be of interest to note the development, during the course of this work on clusters and variable stars, of the ideas concerning the relation of globular clusters to the galactic organization. Until the last year or so most students of stellar problems believed rather vaguely that the sun was not far from the center of the universe, and that the radius of the galactic system was of the order of 1000 parsecs. From the earlier observational data Seeliger and Newcomb derived a fairly central position for the sun. Hertzsprung[1] in 1906 estimated the "Dimensionen" of the visible Milky Way system to be of the order of 2000 parsecs, and some years later Walkey,[2] from consideration of extensive distributional data, estimated a distance of about seventeen hundred parsecs for the galactic main stream. In 1914, referring to the apparently lens-shaped sidereal system, Eddington[3] wrote, "There is little evidence as to the sun's position with respect to the perimeter of the lens; all that we can say is that it is not markedly eccentric"; and the diameter of the whole system (possibly excluding the peripheral ring of galactic clouds) was placed at some two or three thousand parsecs, with emphasis on the uncertainty. For a later computation Eddington[4] assumed the distance of the Milky Way to be 2000 parsecs.

The work on the hypothetical parallaxes of Cepheids and O-type stars by Hertzsprung, and of eclipsing binaries and Cepheids by Professor Russell and the writer, began to give concrete numerical expression to the distances of remote galactic objects, and in 1914 we have the statement:[5] "Our 'universe' of stars must be some thousands of light-years in diameter," but the computed radius of 2500 parsecs was reduced to 1200 by allowing for a presumably reasonable and necessary scattering of light in space. The necessity for such a correction seems now definitely to have vanished, but the general conception of the size of the stellar system has not materially changed. In his valuable memoir[1] on the B-type stars Charlier states:

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[1] *Zeitschrift für wissenschaftliche Photographie*, 5, 106, 1907.

[2] *Monthly Notices*, 74, 655, 1914.

[3] *Stellar Movements and the Structure of the Universe* (London, 1914), p. 32.

[4] *Ibid.*, p. 261.

[5] Russell and Shapley, *Astrophysical Journal*, 40, 434, 1914.

> The luminosity of these stars is, indeed, so great that a star of this type, situated—as far as can be concluded—at the limits of our stellar universe, is scarcely fainter than the eighth magnitude. We are thus in the position to get, with the help of the B stars, what might appropriately be called a skeleton image of the Milky Way.

Only one star out of the 800 in Charlier's list is more distant from the sun than 800 parsecs, and a heliocentric sphere of 500 parsecs radius contains 95 per cent of the total number. Again, Charlier states that "the center of this cluster [of B-type stars], which may be assumed to coincide with the center of our stellar universe, is situated . . . . in the Constellation Carina" at a distance of 88 parsecs.[2]

In view of these prevailing beliefs, the working hypothesis that a globular cluster is a wholly distinct stellar system was quite appropriate when in 1915 it was found that the Hercules cluster (Messier 13) is possibly several hundred parsecs in diameter. It appeared then that the galactic system might be a large but subordinate unit, eccentrically situated with respect to the greater aggregation of globular clusters. The great distances of the clusters, the similarity of their stars, in many properties, to those of the Galaxy, and finally the discovery of galactic planes in a number of them, tended to emphasize further the comparability of clusters and our own surrounding galactic system of stars.

Two difficulties stood in the way of a definite hypothesis—the comparatively high condensation at the centers of globular clusters and the uncertainty relative to the constituency and distances of open clusters in the galactic clouds. The study of Messier 11 and of the dense star clouds in its neighborhood revealed the presence of faint blue stars—objects which, unless abnormal in size, must be quite similar in absolute magnitude to members of Charlier's cluster of B-type stars, but at least twenty times as distant as the limits of his group.

[1] *Meddelanden från Lunds Astronomiska Observatorium*, Series 2, No. 14, p. 103, 1916.

[2] *Ibid.*, p. 104

Evidence was also found that the Milky Way clouds near Messier 11 were greatly extended in the line of sight. For a number of other parts of the Milky Way the study of stars in clusters and clouds indicated analogous conditions, and the conviction grew that the galactic system had an extent of at least 15,000 parsecs along its plane. This left little occasion for the direct comparison with globular clusters, the diameters of which were found by further study to be of the order of 150 parsecs. As a consequence, their relation to the general system was quite uncertain until the present determination of parallaxes and the discussion of the distribution in space indicated the position of globular clusters in the arrangement of sidereal objects and suggested that the actual diameter of the galactic system is of the order of 100,000 parsecs.

4. *The plane of symmetry and the equatorial segment.*—In the figures and discussion of the seventh paper of this series the dependence of globular clusters upon a larger symmetrical organization is definitely shown. Apparently there is no occasion to doubt the identity of the plane of symmetry in this system of globular clusters with the galactic plane defined by stellar condensation and the Milky Way. An agreement within a degree or two of the poles of two distinct and unrelated planes is far too unlikely for consideration. Further, we now know that in distance along the plane many of the globular clusters are nearer to us than stars in the open galactic clusters; and also we note that the distribution of Cepheid variables, as shown in Fig. 3 of the eighth paper, bridges the gap between the distant Milky Way clouds and the local system of stars.

That the equatorial segment is populated by stars throughout its whole extent seems very probable; both the arrangement of the clusters and the appearance of the Milky Way agglomerations support this view. But, owing to the evidence of rifts and divisions in the galactic clouds, it is impossible to suppose uniform stellar distribution or even to assume that the stars everywhere are concentrated progressively toward the medial plane. Stars of the galactic branch in Ophiuchus do not necessarily lie farther from the plane than the semi-width of the equatorial segment, for to be outside this region devoid of globular clusters the parallax of the stars in the cloud must be less than 0".0001. Globular clusters that appear involved in the edges of the star clouds are almost without exception small and faint, and because of their greater radial distances are therefore clear of the equatorial region that contains the majority of known sidereal objects.

5. *The Milky Way and its asymmetry; regions of maximum star density.*—According to the present view of the galactic system the phenomenon of the Milky Way is largely an optical one. Although the existence of local and occasionally very extensive condensations of Milky Way stars is not denied, the conception of a narrow encircling ring is abandoned. The Milky Way girdle is chiefly a matter of star depth, and its long recognized weakness between longitudes 90° and 180° is now taken to be a reflection of the eccentric position of the sun.

On the basis of the third and fourth diagrams of the seventh paper we estimate provisionally that the limit of the Galaxy is three times greater in longitude 325° than in the opposite direction. This does not require an impossible difference of stellar density in the two directions, even if there is a considerable condensation toward the center. A star of a given absolute luminosity situated in the galactic plane would appear less than two and a half magnitudes fainter at the boundary of the system beyond the center than at the opposite point, which is nearest the sun. The remarkable one-sidedness of the Milky Way has been little considered heretofore in works on stellar distribution. Nort,[1] in studying the Harvard map, has made an important beginning by showing that the star density is four or five times greater in the direction of the southern star clouds than in some of the shallower galactic regions of the north.

The surpassing stellar density in the direction now assigned to the center of the galactic system is particularly remarked by Chapman and Melotte[2] in their study of the Franklin-Adams plates. They state that one plate with center in $\alpha = 18^h$, $\delta = -30°$ covers the Sagittarius region of the Southern Milky Way, and the star clouds on limited portions of it are so thick that in the case of twelve out of the twenty-five areas counted on it, it was found impossible to count every star shown; the images of the faintest stars in these regions merged into one another forming a continuous gray background. On every other plate of the Franklin-Adams series even the faintest star images shown were separate and distinct, and the counts included all stars visible. The extreme richness of the Sagittarius region may be judged of, then, when it is noticed that the incomplete counts on it show far more stars than are found in any other part of the Milky Way.

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[1] *Recherches Astronomique de l'Observatoire d'Utrecht,* 8, 113, 1917.

[2] *Memoirs of the Royal Astronomical Society,* 60, Part IV, p. 168, 1915.

The fathoming of the sidereal universe need not long depend on globular clusters alone. If the nearest part of its boundary in the general direction of Auriga and Gemini is not more distant than 30,000 parsecs, no stars in that locality with absolute magnitude of zero or brighter will be fainter than the apparent magnitude 17.5. B-type stars will therefore contribute in future measurement of the extent of the system; and the Cepheid variables fainter than the fourteenth magnitude will in time be fully as valuable as the globular clusters in outlining the diameter and contour of the equatorial segment. As a ready qualitative check of the direction and distance of the center, the blue stars in the Milky Way should persist to a fainter magnitude in the southern sky than in the direction of the anti-center.

The possibly ellipsoidal form of the system of globular clusters is indicated in Fig. 1, which gives a projection on the galactic plane of the 60 clusters for which $R \sin \beta < 15{,}000$ parsecs. If the elongation be accepted as a real characteristic of the stars also, it is evident that the apparently densest star regions, depending on the faintness of the stars involved in the estimate, may lie in a longitude differing considerably from that of the center. The general direction of the galactic center is clearly toward the dense star clouds of Sagittarius and Scorpio; but the adopted galactic longitude, 325°, and the corresponding equatorial co-ordinates of the center, $\alpha = 17^{h}.5$, $\delta = -30°$, are necessarily approximate.

The statistical center derived by Charlier[1] from B-type stars is in Carina, in longitude 236°, a result referring entirely to the local group (within 500 parsecs of the sun) and not influenced by the arrangement of the general system. Strömberg,[2] from bright stars of the redder spectral types, finds the dynamical center in longitude 257°. Nort,[1] using stars to the eleventh magnitude on the Harvard map of the sky, gets farther outside the bounds of the local cluster and obtains a maximum stellar density in the Milky Way between longitudes 280° and 290°; he finds a density but one-fifth as great in longitude 120°, the direction of the anti-center. Chapman and Melotte,[2] working to the still fainter limit of the Franklin-Adams plates, find in the clouds of Sagittarius the only region too dense for counting.

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[1] *Meddelanden från Lunds Astronomiska Observatorium*, Series 2, No. 14, 1916.

[2] *Astrophysical Journal*, 47, 33, 1918.

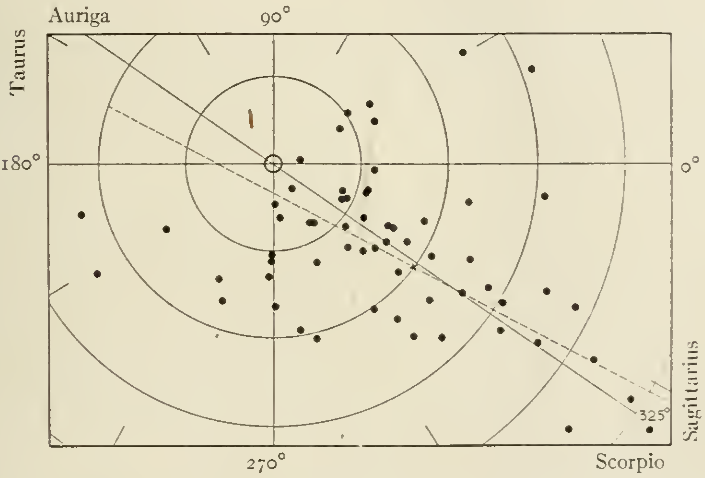

FIG. 1.—The system of globular clusters projected on the plane of the Galaxy. The galactic longitude is indicated for every 30°. The "local system" is completely within the smallest circle, which has a radius of 1000 parsecs. The larger circles, which are also heliocentric, have radii increasing by intervals of 10,000 parsecs. The dotted line indicates the suggested major axis of the system, and the cross the adopted center. The dots are about five times the actual diameters of the clusters on this scale. Nine clusters more distant from the plane than 15,000 parsecs are not included in the diagram.

This progressive increase of the longitude of maximum star density from 236° to 325° (with the increasing predominance of the general system over the local group), and the appearance to be expected of the star clouds in the directions of the two centers, are both in striking agreement with Gould's observations of the brightness of the Milky Way:[1]

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[1] *Op. cit.*

[2] *Ibid.*

> Its brightest portion is unquestionably in Sagittarius [the galactic center]; that in Carina [the local center] being slightly inferior to this as regards intrinsic brilliancy, although far more magnificent and impressive on account of the great number of bright stars with which it is there spangled.

6. *Absence of clusters from the equatorial segment and its dynamical significance.*—In an earlier paper we have remarked that the absence of globular clusters from the mid-galactic region is revealed by the galactic latitudes as well as by the distances from the plane. For the latitudes, however, the wide avoidance of that region is not so evident, because of the apparent nearness to the plane of some very remote systems. The relative frequency of galactic latitudes, plotted in Fig. 2, shows an equatorial belt 10° in width, which is as yet completely empty of known globular clusters.

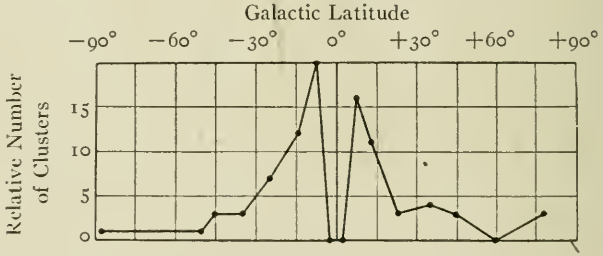

Fig. 2.—Frequency of galactic latitudes, illustrating the mid-galactic region devoid of globular clusters.

It is clear from the figure that any reasonable change in the adopted position of the galactic pole would neither eliminate the segment nor modify its coincidence with the mid-galactic stellar region. It also appears that the region cannot be explained away by supposing the data incomplete through either the failure to record visible globular clusters or the obstruction of light in the equatorial segment. In support of this statement it will suffice to point out that already at a distance of 12,500 parsecs from the sun the absence of clusters is more than chance. Then, first, the suggestion that clusters have been overlooked is absurd, for some of the stars in globular systems at that distance would be photographically brighter than the fourteenth magnitude. That is, the overlooked clusters would be more conspicuous than Messier 3. In the second place, if general scattering of light is to account for the absence of clusters, typical stars with negative color-indices should not be found in the Milky Way when fainter than the fifteenth magnitude, for at the distance of 12,500 parsecs $m-M=+15.5$. But such objects have been observed; and, moreover, it is highly improbable that the millions of Milky Way stars fainter than the eighteenth magnitude are all dwarfs.

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[1] *Uranometria Argentina*, p. 370, 1879.

We have referred to the equatorial segment as a region of avoidance. Slipher's observations of radial velocities suggest that the term is scarcely appropriate, since seven out of ten systems are approaching the sun (and probably the galactic region) with conspicuous velocity.[1] The segment seems rather to be a region of attraction and demolition, and if further work on radial velocities confirms the present indication of systematically high speeds of approach, a factor of very great importance in the evolution of the galactic system will be established.

Table I contains information relative to the ten clusters of measured radial velocity. The observed values in the third column and the computed quantities based upon them in the last column are admittedly provisional. The observed photographic magnitudes in the fourth column, taken from Table II of the seventh paper, show that two of the largest velocities pertain to the faintest and probably most difficult objects. The last column gives the observed radial velocity, V_p (expressed in the unit of 100 parsecs per million years), divided into the radial distance.

The present evidence suggests that globular clusters as a class may be falling rapidly into the regions rich in stars. None of the clusters in Table I is necessarily receding from the plane, for the positive velocity of Messier 5 is far within the uncertainty of the observation, and Messier 9 and 28 are in such low galactic latitudes that the radial component is no test of the true direction of motion perpendicular to the Milky Way. With the possible exception of Messier 5, therefore, all these clusters, with latitudes high enough

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[1] *Popular Astronomy*, **26**, 8, 1918.

to make it probable that the radial and perpendicular components have the same sign, are approaching the galactic plane with a speed that unless rapidly diminished will bring most of them to the dense stellar regions in less than 150 million years (cf. last column of Table I). Such an interval of time seems relatively short in the history of a stellar system, and hence these negative velocities make still more remarkable the absence of globular clusters from the equatorial segment.

TABLE I

RADIAL VELOCITIES OF GLOBULAR CLUSTERS

CLUSTER		RADIAL VELOCITY	MEAN PG. MAG. 25 BRIGHTEST STARS	GALACTIC		DISTANCE (UNIT IS 100 PARSECS) R	DISTANCE FROM GALACTIC PLANE $R \sin \beta$	$\frac{R}{V_p}$ (UNIT IS A MILLION YEARS)
N.G.C.	Messier			Long. λ	Lat. β			
		km						
5024...	53	−170	15.07	307°	+79°	189	+186	110
5272...	3	−125	14.23	8	+77	139	+135	110
5904...	5	+ 10	13.97	333	+45	125	+ 88	....
6205...	13	−300	13.75	26	+40	111	+ 71	40
6333...	9	+225	15.61	334	+ 9	250	+ 39	....
6341...	92	−160	13.86	35	+34	123	+ 69	80
6626...	28	0	14.87	336	− 7	185	− 23	....
6934...	....	−350	15.78	20	−20	333	−114	90
7078...	15	− 95	14.31	33	−29	147	− 71	150
7089...	2	− 10	14.61	22	−37	156	− 94	1600

7. *Possible explanation—a provisional hypothesis.*—There appears to be no very obvious or reasonable escape from the conclusion that some clusters at least are destined to enter the mid-galactic region. Revolution around the whole system is dynamically improbable; moreover, the present distances from the plane are small as compared with the mean equatorial diameter of the cluster system. With such great velocities and masses it seems incredible that clusters would not easily pass through the stratum of stars. Further observation may reveal survivors receding from the galactic plane, and we may be able to examine them for the effects of the passage.[1]

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[1] The galactic latitudes of the large and small Magellanic clouds are 32° and 44°; their velocities of recession, according to observations by Wilson at the D. O. Mills Observatory, probably are 260 km/sec. and 150 km/sec., respectively. Reasonable estimates of the distance (cf. eleventh paper) give 70 and 130 as corresponding values of R/V_p in millions of years.

There is, however, suggestive though incomplete evidence that globular clusters are disrupted upon approaching the mid-galactic regions. As a general but not infallible rule, the clusters at moderate or high distances from the Galaxy are compact; the nearer ones are open. There is, for example, the well-known contrast between ω Centauri and 47 Tucanae, the two brightest and nearest systems, the former 1800 and the latter 4700 parsecs from the plane. Of the five systems within 2000 parsecs of the galactic plane four are decidedly open for globular clusters (Table II). The fifth, Messier 62, shows a structural irregularity that may be an indication of rupture; its parallax, depending only on its diameter, is perhaps inaccurate because of asymmetry.

TABLE II

Globular Clusters Nearest the Galactic Plane

N.G.C.	Galactic Latitude	$R \sin \beta$ (Parsecs)	Remarks
4372	− 9°	−1800	Large, faint, and little condensed. "Rather faint at center."—Melotte.
5139	+16	+1800	ω Centauri. Not strongly condensed.
6266	+ 7	+1900	M. 62. Well condensed but probably the most asymmetrical of globular clusters. See Bailey, *Harvard Annals*, **76**, 74, 1916.
6397	−12	−1700	Large cluster and one of the nearest. "Stars rather scattered."—Melotte.
6656	− 9	−1300	M. 22. The most open of large globular clusters.

The distribution of stars in open galactic clusters is possibly a second indication of the dissipation of globular clusters in the Milky Way. The diameter of some open clusters appears to be quite comparable with that of globular clusters. Several of them contain mainly stars that are highly luminous and probably of great mass. Counts in the surrounding star fields have shown in certain cases (Messier 67, Messier 11, *h* and χ Persei,[1] the Pleiades[2]) that the cluster stars are sparingly scattered over an area several times greater than that of the nucleus.

[1] Inferred from radial velocity results by Adams and van Maanen, *Astronomical Journal*, **27**, 187, 1913.

[2] Trümpler, *Popular Astronomy*, **26**, 9, 1918.

As a provisional working hypothesis, which is supported by numerous observed conditions and may be further tested without difficulty, it is suggested that the obvious dynamical equilibrium of a globular cluster has been acquired originally at a great distance from external perturbing forces and is an extremely delicate adjustment that quickly breaks down under stresses such as those prevailing in the galactic region; further, that faint stars in globular clusters are of small mass and of more than average velocity (analogy with galactic stars), and in their orbital motions frequently attain great distances from the center. When the globular cluster approaches a disturbing body as massive as the general galactic system, such stars are of course most readily lost and intermingled with galactic stars.[1] On the other hand, the massive cluster stars, which are mostly of high luminosity, having low peculiar velocities and maintaining in their subsystem a high degree of stability, retain their organization longer in a disrupting field and, except for rare and accidental encounters with galactic stars, undergo as a single unit the general perturbations of the galactic system. It appears necessary to suppose that the gravitational or electrical field near the plane of the Milky Way is by itself more potent in the acceleration of stellar velocities than are near approaches; otherwise it seems impossible that through chance dynamical encounters a massive globular cluster, highly organized and presumably in a steady state, should be so speedily dissipated and transferred into an open galactic group.

8. *Remarks on the contraction theory of stellar evolution.*—In a preceding paragraph we have alluded to the scale of time in the development of the galactic system. An observational contribution to this problem, and more directly to the problem of the speed of evolution of spectral type, is afforded by the distant clusters. Recent discussion by Lindemann, Joly, Véronnet, Eddington, and Jeans, relative to the probable age of the earth and the sun from the standpoint of the contraction theory, emphasizes the persistent disagreement between this theory and the data of geology and physics in estimating the interval of time since the deposition of

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[1] A somewhat similar problem has been examined analytically by Jeans; in fact, he may have had this identical hypothesis in mind (*Monthly Notices*, 76, 560, 1916).

the earliest sedimentary rocks. Lindemann and Jeans have argued that allowance for known radioactive or even for all electrical properties of matter cannot have prolonged the apparent constancy of solar radiation sufficiently to meet geological requirements. Specifically, Eddington computes that the duration of the total giant stage of a star, if contraction is its source of energy, is not likely to exceed one hundred thousand years; the development from a giant M to a giant G should be less than twenty-five thousand years. To obtain an appreciably slower development we must call upon sources of energy now unrecognized.

Giant stars of various spectral types are numerous in globular clusters and are easily studied in relation to the foregoing problem. We recall that, owing to the finite velocity of light, our investigations of clusters do not relate to contemporaneous conditions, but rather to events of the remote past. Stars in the nearest globular cluster are actually two hundred centuries older at present than they appear in our records, and the light we now receive from the most distant systems has been en route about two hundred thousand years. A significant consequence of these unlike distances, thanks to the structural uniformity of stars and of globular clusters, is that we are able to compare phenomena separated by enormous intervals of time. Instead of having to test stellar development observationally through the dissimilar records of one century or possibly two, with globular clusters we may make a test under identical conditions of climate and instrumental equipment at intervals up to more than a thousand centuries.

The clusters range in distance from twenty thousand to more than two hundred thousand light-years. The farther away the younger they are in our study and the less developed are their stars. (We assume, of course, that distance from the earth is totally irrelevant in the birth time of a globular cluster, though distance from the galactic center may not be.) The results at hand, though preliminary, are apparently decisive; and, as regards definite indications of evolution, are uniformly negative. We note, in particular, the conclusions reached in the eleventh paper, that no measurable change has occurred even in the 180,000 years separating the phenomena recorded for the nearby Hercules cluster and for the most distant system known.

It would have been impressive to find with decreasing distance and increasing age a progressive change in stellar conditions, thus enabling us to use this time scale to its greatest advantage. Perhaps, however, our negative result may be quite as important—our evidence, that is, of essential contemporaneousness from cluster to cluster. The result suggests the insufficiency of the contraction theory, thus supporting the geological time scale on the earth; it contributes observationally to the growing belief that stellar radiation involves sources of energy that are as yet wholly unknown; and, by indicating that an interval of 100,000 years is insufficient to show appreciable stellar development, it tends to alter our conceptions of the speed of evolutionary change.

II. THE LOCAL SYSTEM

9. *Discrepancy of centers—its interpretation.*—As has been stated above, the galactic center deduced from the arrangement of clusters, and roughly corroborated by the density of the Milky Way clouds, is about 90° from the center recently assigned by Walkey and Charlier on the basis of statistical discussions of the bright stars near the sun. The new position is not far distant, however, from the direction toward and from which catalogued stars show a preferential drift. To investigate this discrepancy of centers and the possible meaning of the star drifts in the mechanics of the general galactic system we are led to inquire what part the local stars play in the larger scheme. Let us consider the stellar phenomena manifested in our neighborhood from the standpoint of the greater galactic organization and think of the local stars as forming a group which is imbedded in an extensive stellar stratum and subordinate to the gravitational influences of the major system. Such a procedure brings us to the following hypothesis, which promises fairly satisfactory interpretations of a number of observational results, and appears to harmonize some of the earlier theories as to the systematic motions of the stars:

In the neighborhood of the sun (within 1000 parsecs) the stars fall into two categories: (1) Members of the general galactic system, having motions, distribution, frequencies of spectral type, and absolute magnitude, much the same as are believed to prevail throughout the greater part of the equatorial segment; these we shall call field stars. (2) Members of a moving local system of limited extent; these we shall call cluster stars.[1] The cluster is receding in a nearly radial direction from the center of the galactic system and is surrounded by and intermingled with the field stars. It is concentrated toward a plane of symmetry, which is inclined about 12° to the galactic plane as defined by remote non-cluster stars and the clouds of the Milky Way. Its motion of translation apparently is parallel to the Galaxy rather than to its own equator; and there is, in addition, an internal motion in the cluster with respect to its center which appears to be about fifty parsecs north of the true galactic plane. The sun, evidently a field star, is at some distance laterally from the center of the cluster and a few parsecs above its central plane of symmetry. Cluster stars comprise nearly all of the B-type and a decided majority of the A-type stars brighter than the seventh magnitude and a large percentage of the redder stars within three or four hundred parsecs of the sun. The field stars within this limit include very few B's, a small percentage of A's, and possibly a majority of the redder spectral types. Cepheids, N-type stars, most of the O-type stars, the Ursa Major group, planetary nebulae, and stars of peculiarly high random velocity are probably all members of the field, which is, in general, a heterogeneous mixture of many groups and tendencies.

10. *The local cluster.*—In discussing the foregoing hypothesis the most important object will be to demonstrate the existence of a large, local, definitely organized cluster; for when we have satisfied ourselves that there is such a group moving in the neighborhood of the sun, and have admitted the reality of an extensive and populous galactic system, the present hypothesis appears to afford the most simple and natural explanation of the observed motion and distribution of local stars. As an analogy suppose we consider

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[1] For a slight modification of the following theory of star-streaming see Section IX of *Mt. Wilson Contr.* No. 161.

the compactly organized galactic cluster Messier 11, which is deeply immersed in the rich star clouds of the Milky Way. Unless it be completely at rest with respect to its surroundings, its motion, as seen from a random-moving field star within its bounds, would give rise to exactly such phenomena as the systematic stellar motions we observe in our neighborhood.

We start from the well-known observational results that stars of spectral type B are much alike in actual luminosity, that they are much brighter than the average star, and that with decreasing apparent magnitude they do not increase in number as rapidly as other types of stars. A consideration of the star ratio shows that beyond the distance corresponding to apparent magnitude 7.5 the B stars must be very infrequent.[1] Charlier, computing the average luminosity of the early subclasses of this type, has derived the distance and distribution in space of all such stars contained in the Harvard catalogues and has given striking evidence that they form a large system consisting of about eight hundred stars, among which the sun is eccentrically situated. These two results by themselves seem to show the existence of an important local aggregation. But there is much additional evidence.

11. *Central plane of the B stars and its relation to the Galaxy.*—The first problem to be investigated is whether the system of bright B stars is other than a normal arrangement of galactic stars in the extensive equatorial segment. To determine the position of the central plane of the B-type group Charlier used 751 stars (without differentiation as regards distance from the sun) and obtained for the co-ordinates of the pole: $\alpha = 184\overset{\circ}{.}3$, $\delta = +28\overset{\circ}{.}7$. This position he adopts as the pole of the Milky Way,[2] considering it in satisfactory agreement with the pole of the plane defined by the Milky Way clouds: $\alpha = 191\overset{\circ}{.}2$, $\delta = +27\overset{\circ}{.}4$. Apparently he identifies the "Galaxy of B stars" with the entire stellar universe—a procedure that cluster studies obviously do not support, for only a few of the stars in Charlier's group are more distant from the sun than five hundred parsecs.

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[1] Cf. discussion by Seares, *Publications of the Astronomical Society of the Pacific*, 30, 114, 1918.

[2] *Op. cit.*, pp. 40 and 43.

If, however, the difference in the positions of the two poles be real, an inclination of the central plane of the B-type system to the Galaxy is indicated, and we have an additional argument for its existence as a distinct cluster. The diagrams in Charlier's memoir show, moreover, that the adopted plane is far from satisfying the distribution of the nearer B stars. In Fig. 3, which is based on data (Table III) derived from Charlier's tabulated results, the median values of Z, distance from the adopted central plane, are plotted for successive intervals of Y, that component of the distance in the plane which is perpendicular to the direction of the center from the sun. The area of each of the plotted points is proportional to the number of stars entering the median value and illustrates the rapid falling off of star density beyond a certain distance. The unit used by Charlier, the siriometer, is nearly equivalent to five parsecs or sixteen light-years. A cross indicates the position of the sun. The broken horizontal line represents the projection of the galactic plane adopted by him. The majority of B stars obviously lie below it, proving the sun's position to be above the cluster's central plane. The full horizontal line indicates the central plane adopted by Charlier; but considering only the inner part of the cluster we observe that the sun is much nearer to the central plane. The inclined broken line shows the projection of the galactic plane as derived from the stars of the Milky Way clouds; its inclination and its distance below (south of) the B

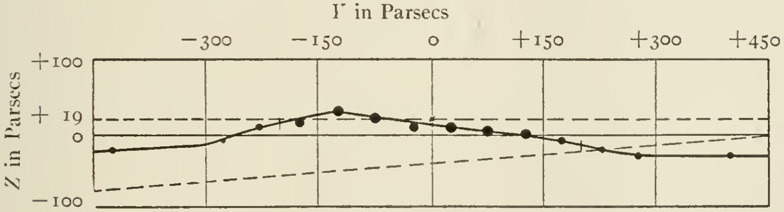

FIG. 3.—Projection of the local cluster of B stars on a plane perpendicular to the Galaxy and to the projection, on Charlier's adopted plane, of the line joining the sun and the center of the cluster. The center of the system of B stars, as derived by Charlier, is at the origin of co-ordinates. The cross shows the position of the sun. The true galactic plane is indicated by the inclined broken line. Vertical lines across the curve show the limits of distance adopted for the solution represented by Fig. 4.

cluster and the sun are derived from the results mentioned on a following page. The figure suggests in general that a few B stars of the Galaxy are sparingly intermixed with a definite and limited cluster.

TABLE III

Intervals of *Y* in Siriometers	Number of Stars	Median *Z* in Siriometers
≦ −60	41	−4
−59 to −50.........	16	−2
−49 “ −40.........	52	+2
−39 “ −30.........	59	+3
−29 “ −20.........	72	+6
−19 “ −10.........	80	+4
− 9 “ 0.........	83	+2
0 “ + 9.........	88	+2
+10 “ +19.........	76	+1
+20 “ +29.........	70	0
+30 “ +39.........	60	−2
+40 “ +49.........	37	−4
+50 “ +59.........	36	−6
≧ +60	35	−6

12. *Summarized discussion of the properties of the local cluster.*—The procedure from this point involves much computational work and statistical discussion, and for the present the most relevant results can best be given in a summarized form.

a) Charlier's determination of the mean absolute magnitudes of B stars is compared in Table IV with averages derived from the parallaxes given by Kapteyn in *Mount Wilson Contributions*, Nos. 82 and 147. While systematically fainter, Kapteyn's mean values

TABLE IV

SPECTRAL CLASS	KAPTEYN		CHARLIER		DIFFERENCE
	Number of Stars	Absolute Magnitude	Number of Stars	Absolute Magnitude	
B0*............	36	−0.34±0.23	27	−0.56	$+0\overset{\mathrm{M}}{.}22$
B1............	23	−2.00±0.16	26	−2.76	+0.76
B2............	36	−1.21±0.16	40	−2.76	+1.55
B3............	148	−0.38±0.06	157	−1.10	+0.72
B5............	67	−0.21±0.09	104	−1.29	+1.08

* In Kapteyn's lists B stands for B0. The lack of homogeneity in this subtype, noted by both investigators, may be due to the inclusion of some (fainter stars) of the general class B with the true B0 stars.

verify the similarity of the subclasses B0, B3, and B5, and the much greater average luminosity of B1 and B2.

b) If we desire to limit our investigation of the B stars to the denser part of the group (say, to approximately two hundred parsecs from the sun, as indicated in Fig. 3), we should include only those brighter than apparent magnitude 4.0 for types B1 and B2, and brighter than 5.5 for types B0, B3, and B5. Such data permit a solution for the equatorial plane of the B cluster that is not seriously influenced by outside stars.

c) From the several lists in *Harvard Annals*, **56**, the galactic longitudes and latitudes have been discussed for 356 B-type stars falling within the foregoing limits of spectrum and brightness. The results, collected in Table V, give for the pole of the central plane of the flattened B-type cluster: $\alpha=178^\circ$, $\delta=+31\overset{\circ}{.}2$.

TABLE V

Intervals of Galactic Longitude*	No. of Stars	Median Galactic Latitude	Mean Galactic Latitude	Residual from Curve
0°– 9°	15	+10°	+12°	+ 2°
10 – 19	13	+ 6	+ 8	− 1
20 – 29	19	+ 8	+11	+ 3
30 – 39	26	+ 8	+ 5	− 1
40 – 49	16	+ 2	− 5	−10
50 – 59	22	+ 4	+ 7	+ 4
60 – 69	22	+ 2	+ 3	+ 2
70 – 79	15	+ 4	− 1	0
80 – 89	26	− 1	− 3	− 0
90 – 99	13	− 3	− 2	+ 2
100 –109	14	−19	−17	−11
110 –119	24	− 8	− 4	+ 4
120 –129	20	−10	− 9	+ 0
130 –139	39	−16	−10	+ 1
140 –149	18	− 2	−11	+ 0
150 –159	14	− 6	−10	+ 2
160 –169	20	−16	−10	+ 2
170 –179	20	−16	−14	− 3

* Diametrically opposite regions are combined for this table.

d) The definiteness with which this secondary Galaxy is determined is shown in Fig. 4, where mean galactic latitudes are plotted against galactic longitude. The semi-amplitude of the resulting sine-curve yields 12° for the inclination of the two planes, and the nodes are in galactic longitudes 70° and 250°. The dip of the central plane due to the sun's position to the north is 5°. If stars in the constellation of Orion were deleted from the discussion, the result would not be materially affected. The average deviation of a mean value from the curve is $+2.^\circ7$.

e) A discussion of all A-type stars brighter than the sixth apparent magnitude confirms the foregoing results, showing that other spectral classes are involved in the cluster. The inclination derived from A stars, however, is only about 4°, probably an indication that many general galactic stars are included and also that the concentration to the plane of the cluster may not be so pronounced as for B-type stars.

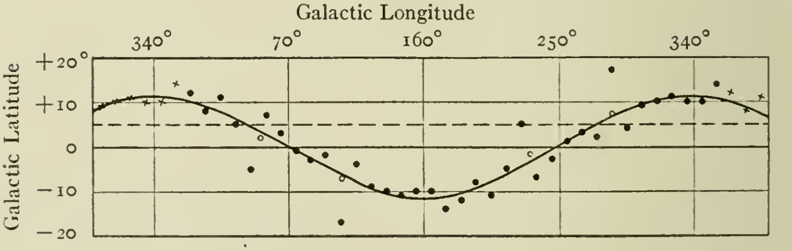

FIG. 4.—Solution for the inclination of the local cluster to the galactic plane

f) We have further suggestions that a majority of A stars probably belong to this limited cluster, first in Plummer's discussion of the preferential motion shown by their radial velocities and second in the observations of Boss and Eddington that the mean parallactic motion of these stars is the same in low and high galactic latitude although the density is twice as great near the Galaxy.

g) In Kapteyn's result that the B stars are members of Stream I we also have an indication that some stars of all the later types belong to the cluster.

> The work thus far done proves the continuity of the series of the B and A and the second-type stars. . . . . Some B stars at least must be regarded as members of the Second Stream. It is now found that the remainder—the overwhelming majority—belong to the First Stream.[1]

Apparently, then, the membership of Stream I is limited wholly to the cluster.

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[1] Kapteyn, *Mount Wilson Annual Report*, 1916, p. 256.

h) Gould's belt of bright stars, as revised by Newcomb,[1] is nearly coincident with this secondary Galaxy. A definite trace of it is given by all naked-eye stars when considered together, irrespective of type (lucid red and yellow stars contribute little because of their uniform distribution in the sky); stars of the ninth magnitude, however, give the same pole for the Milky Way plane as is given by galactic clouds (Newcomb).

i) Cepheid variables extend far outside the limits of the local cluster, and the pole of their plane of concentration is identical with that for the galactic clouds (Fig. 5). They afford the best determination yet available of the sun's distance north of the galactic plane—about sixty parsecs. A systematic error in the absolute magnitude of Cepheids would affect this distance proportionately, but would not alter the general conclusions. The N-type and most O-type stars are sufficiently remote and concentrated to the plane to afford a check of the sun's displacement, if reliable individual parallaxes become available. The poles of their planes of concentration are apparently identical with the pole derived from the galactic clouds.[2]

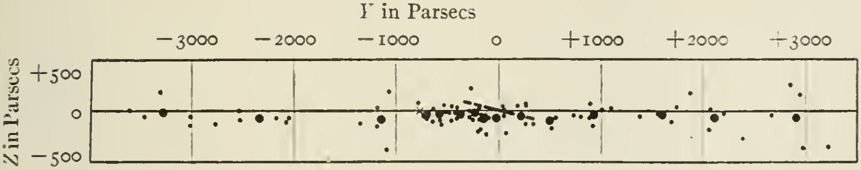

FIG. 5.—Projection of the positions of Cepheid variables on the same plane as in Fig. 3 the sun being at the origin. The small dots represent individual stars, the cross a special group in Carina, and the large dots mean values. The inclined line in the center represents the projected plane and the probable extent of the local cluster of B stars.

j) The center of the B-type cluster, adopting Charlier's values of absolute magnitude, is more than fifty parsecs north of the galactic plane, making the sun less than ten parsecs north of the central plane of the B stars (Fig. 3). For this last quantity Charlier derived 19.2 parsecs, but his work involves extra-cluster stars that apparently are concentrated to the true galactic plane. With Kapteyn's absolute magnitudes of the B's the distance of the cluster's central plane below the sun is still less, and the sun's distance from the local center is less than fifty parsecs.

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[1] *Carnegie Institution of Washington, Publication No. 10*, 1904.

[2] Hertzsprung, *Astronomische Nachrichten*, **192**, 261, 1912.

k) The evidence that the local cluster is a dynamical unit and has conspicuous internal motion superposed upon a translational velocity includes: (1) the behavior of the star ratio for spectral types B and A; (2) the greatly flattened form of the system, at least for the B stars; (3) Kapteyn's announcement of the acceleration of Stream I;[1] (4) Strömberg's evidence from second-type stars of a rotation around a center approximately coincident in direction with the center of the cluster as derived from B stars;[2] and (5) the deviations of the vertices of the various groups of B stars, depending upon position in the sky. A study of the last three points, if not too much constrained by the magnitude of the observational errors, may show whether the internal motion is approximately radial or is a rotation in one general direction or in more than one.

12. *Consequences of motion—star-streaming.*—The foregoing abbreviated account seems to show conclusively that an open cluster surrounds the sun, and that it is comprehensive enough to include stars as numerous and varied as the components of a star stream. The continuity of the galactic star fields beyond the cluster's limits is strongly indicated by the arrangement in space of Cepheid variables (Fig. 5) and other types of distant stars, but especially by the distribution of the faint stars of the Bonner Durchmusterung (Seeliger, Newcomb) and by the increasing galactic concentration with decreasing apparent brightness of still fainter stars (Kapteyn, Seares, Van Rhijn). As seen from the solar system, therefore, the local cluster, if in motion, must inevitably produce star-streaming analogous to that observed; hence it is a natural assumption that the recorded systematic motions of the local stars are due wholly to such a cause. That the translation through the star field is approximately radial is of some dynamical significance, and that it is in the plane of the Galaxy was to be expected. The internal motion, on the other hand, probably

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[1] *Mount Wilson Annual Report*, 1916, p. 255.

[2] *Mt. Wilson Contr.* No. 144; *Astrophysical Journal*, 47, 7, 1918.

has a maximum component in the plane of the cluster. Unfortunately the small inclination may make difficult the separation of the two motions.

The present viewpoint of star-streaming seems to be greatly strengthened by the way in which important observed facts agree with its requirements. We have mentioned the form and limits of the cluster, its stellar content, its motion parallel to the galactic plane, the probable acceleration of Stream I, and the distribution of A-type stars. Two or three other points may be referred to here.

a) Since the plane of concentration of field stars lies south of the sun (Fig. 5), the theory requires that stars of Stream II should be relatively less numerous in northern than in southern galactic latitudes. In agreement with this Hough and Halm have found, from both radial velocities and proper motions, that the stars of Stream II are more than twice as frequent near the southern galactic pole as near the northern.[1] This point appears to be of high importance.

b) As the cluster is of limited extent the proportion of field stars should increase with increasing distance from the center. The ratio of the number in the first stream to that in the second is $\frac{3}{2}$ for naked-eye stars (excluding type B); for fainter stars Dyson and Thackeray[2] find the ratio $\frac{4}{3}$. Stars as distant as 1000 parsecs presumably would show only accidental traces of Stream I.

c) The declination of the apex of solar motion should be sensitive, on this cluster hypothesis, to the character and average distance of the stars involved in its determination. Thus the low declinations derived from stars of early types, of large proper motions, of faint absolute magnitude, of bright apparent magnitude, all indicate that the apex with respect to the galactic field lies considerably north of the value commonly used. Dyson and Thackeray's[3] work on Groombridge stars and Comstock's[4] results from the very faint "optical" companions of double stars perhaps most nearly indicate the "true" direction of solar motion.

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[1] *Monthly Notices*, **70**, 85, 568, 1910; Halm. *ibid.*, **71**, 610, 1911.

[2] *Ibid.*, **77**, 590, 1917.

[3] *Ibid.*, **65**, 428, 1905.

[4] *Astronomical Journal*, **28**, 54, 1913.

In many of its features the present hypothesis makes no claim to novelty. It includes, indeed, a patchwork of many preceding suggestions, but is of course entirely independent in that the problem has been approached from the point of view of the recently acquired ideas on clusters and on the arrangement and constituency of the larger galactic system. Among earlier deductions and speculations related to this subject we note that Gould, forty years ago, thought his "Belt" of bright stars (inclination 20°, longitude of node 90°) represented a local cluster of possibly four hundred members; but Newcomb's later analysis attributed the whole phenomenon to a few chance aggregations. Hertzsprung and Charlier have found the deviation of the galactic pole for B stars, the former suggesting that the result might be significant. Perrine has noted that one of the vertices lies in the general direction of regions rich in globular clusters. An early hypothesis by Hough and Halm tentatively identified Stream II with the Milky Way stars. Turner's gravitational hypothesis of the streams assigned the center of the universe to the line of star-streaming. Jeans has considered theoretically several different possibilities, some of which resemble the present suggestions. But in developing this interpretation the obligation is by far the greatest to the various discussions of theories and observational data by Eddington and Kapteyn.

If a theory similar to the present one survives the test of future observation and analysis, it will be of interest to note how closely it approaches Kapteyn's original statement of star-streaming, so far as local phenomena are concerned. The "two cloud" hypothesis of the streams—the "dualistic" conception of the universe, as Charlier calls it—would be a more literal view of the conditions near the sun than that given by the ellipsoidal theory of streaming —the "monistic" conception. But so far as we now see the latter may apply better to the galactic system as a whole, and we also have a case for the ellipsoidal theory in the internal motions of the local cluster.

Mount Wilson Observatory
April 1918

THE HELIUM SPECTRUM AND THE UNIT ELECTRICAL CHARGE

By FERNANDO SANFORD

In his paper in *Physikalische Zeitschrift*, 4, 400 (*Gesammelte Werke*, p. 80), Ritz concludes that all spectral series may be expressed by the formula

$$\pm\gamma = N\left(\frac{1}{p^2}-\frac{1}{q^2}\right),$$

where γ represents the wave-number per centimeter, N the Rydberg constant 109675, and p and q the roots of certain transcendental equations. In the case of the Balmer series this equation takes the form

$$\pm\gamma = N(1/4-1/m^2).$$

Ritz concludes that the constant N is strictly universal, and hence that the wave-numbers of the lines of all spectral series may be expressed in terms of this constant.

In a paper entitled "The Astronomical Atom and the Spectral Series of Hydrogen"[1] the present writer has shown how, on the assumption that radiating electrons are revolving in elliptical or circular orbits about a central positive charge, the central positive charge for any wave-length may be calculated in terms of the wave-length and a numerical constant, viz., $Q=2.882\cdot 10^{-12}/\sqrt{\lambda}$, where Q is the central charge. It was also shown in the same paper that the Rydberg constant N is the wave-number of an electron revolving about a central positive charge $Q=2e=9.548\cdot 10^{-10}$ E.S.U.

This being the case, it should be possible to calculate the effective central charge corresponding to any line of any series to which the Ritz formula may be applied in terms of the charge $2e$ by means of an equation of the form $Q=2e\sqrt{1/p^2-1/q^2}$. This was done for all the known series in hydrogen in the paper referred to above. It is the purpose of the present paper to show that it may also be done for the six known series in the helium spectrum.

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[1] *Astrophysical Journal*, 48, 1, 1918.

The helium series are usually separated into two groups, sometimes classified as the helium and the parhelium groups. Each group consists of a "principal" series and two "subordinate" series. The two subordinate series of the helium group are made up of double lines, while the corresponding series of the parhelium group consists of single lines. In the present paper the mean wave-length of a pair is taken as the wave-length for which the central charge is computed, though since the value of some of the constants used in calculating our numerical constant $2.882\cdot10^{-12}$ is known to only three significant figures, it usually makes no difference in our computed value of Q which wave-length of a close doublet is used.

FIRST SUBORDINATE SERIES, HELIUM GROUP

There is a long series in helium classed by Kayser as the first subordinate series of the helium group, or Group I. This series corresponds to the Balmer series in hydrogen, but whereas in the Balmer series the wave-length of any line may be computed from the equation $1/\lambda=N(1/4-1/m^2)$, the corresponding equation for the first subordinate series of helium is

$$1/\lambda=N(1/3.756-1/m^2)\text{ or }1/\lambda=N\left[\frac{1}{(1.938)^2}-\frac{1}{m^2}\right].$$

In order to compute the unit electrical charge by means of this equation, we write

$$Q=2.882\cdot10^{-12}/\sqrt{\lambda}=2e\sqrt{1/(1.938)^2-1/m^2}.$$

The values of $2e$ derived from this equation for each line of the series under consideration are given in Table I.

FIRST SUBORDINATE SERIES, PARHELIUM GROUP

The series of the parhelium group which corresponds to the above series consists of twelve lines which correspond in wave-length with the lines of the Balmer series even more closely than do the lines of the series given in Table I. The value of e may be

TABLE I

UNIT ELECTRICAL CHARGE FROM FIRST SUBORDINATE SERIES OF HELIUM

m	λ	$\frac{2.882\cdot 10^{-12}}{\sqrt{\lambda}}$	$2e$
3	5876	3.758	9.538
4	4472	4.310	9.546
5	4026	4.542	9.548
6	3820	4.663	9.548
7	3705	4.735	9.546
8	3634	4.780	9.547
9	3587	4.812	9.548
10	3555	4.834	9.548
11	3531	4.850	9.547
12	3513	4.862	9.546
13	3499	4.872	9.547
14	3488	4.880	9.548
15	3479	4.886	9.547
16	3472	4.891	9.548
17	3466	4.895	9.547
18	3461	4.899	9.549
19	3457	4.902	9.549
		Mean value of $e=4.7735$	

calculated from these lines by the equation $Q=2e\sqrt{1/4.038-1/m^2}$; or, to put the equation in the form used by Ritz, $Q=2e\sqrt{1/(2.0094)^2-1/m^2}$, where, as before, $Q=\frac{2.882\cdot 10^{-12}}{\sqrt{\lambda}}$. The results of this calculation are given in Table II.

TABLE II

UNIT ELECTRICAL CHARGE FROM FIRST SUBORDINATE SERIES, PARHELIUM

m	λ	$\frac{2.882\cdot 10^{-12}}{\sqrt{\lambda}}$	$2e$
3	6678	3.527	9.546
4	4922	4.108	9.546
5	4388	4.351	9.550
6	4143	4.477	9.546
7	4009	4.551	9.546
8	3927	4.599	9.548
9	3872	4.632	9.548
10	3834	4.655	9.548
11	3806	4.667	9.540
12	3785	4.681	9.544
13	3769	4.690	9.540
14	3756	4.698	9.540
		Mean value of $e=4.7725$	

As is well known, there is in the hydrogen spectrum another series, known as the Pickering series, which converges to the same wave-length, and hence to the same central charge, as does the Balmer series. The wave-numbers of the lines of this series are expressed by the Ritz equation $1/\lambda = N(\frac{1}{4} - 1/(m+0.5)^2$ and their central charges by the equation $Q = 2e\sqrt{\frac{1}{4} - 1/(m+0.5)^2}$.

The two second subordinate series in the helium groups correspond to the Pickering series in hydrogen in that each series converges to the same wave-length and the same central charge as does the first subordinate series of its group.

SECOND SUBORDINATE SERIES, HELIUM GROUP

This series, as generally given, consists of thirteen lines, though the line for which $m=3$ does not fit Rydberg's equation and the value of e calculated from this line differs by a little more than 4 per cent from its true value. The line for which $m=4$ also gives a value of e almost 1 per cent too great.

The wave-lengths of these lines and the value of $2e$ as calculated from them are given in Table III.

TABLE III

UNIT ELECTRICAL CHARGE FROM SECOND SUBORDINATE SERIES, HELIUM

m	λ	$\frac{2.882\cdot10^{-12}}{\sqrt{\lambda}}$	$2e$
3	7066	3.428	9.965
4	4713	4.195	9.643
5	4121	4.486	9.585
6	3868	4.631	9.567
7	3733	4.715	9.556
8	3652	4.767	9.553
9	3599	4.802	9.542
10	3563	4.827	9.549
11	3537	4.843	9.530
12	3517	4.858	9.546
13	3502	4.870	9.549
14	3491	4.876	9.546
15	3481	4.882	9.544
		Mean value of $e=4.79$	

The values of $2e$ are calculated from the equation

$$\frac{2.882\cdot 10^{-12}}{\sqrt{\lambda}}=2e\sqrt{\frac{1}{(1.938)^2}-\frac{1}{(m-0.4)^2}}\,.$$

SECOND SUBORDINATE SERIES, PARHELIUM GROUP

There is also a second subordinate series in the parhelium group which converges to the same wave-length and the same central charge as does the first subordinate series. The value of e may be calculated from this series by using the equation

$$\frac{2.882\cdot 10^{-12}}{\sqrt{\lambda}}=2e\sqrt{\frac{1}{(2/.0094)^2}-\frac{1}{(m-0.15)^2}}\,.$$

The results of this calculation are given in Table IV.

TABLE IV

Unit Electrical Charge from Second Subordinate Group, Parhelium

m	λ	$\frac{2.882\cdot 10^{-12}}{\sqrt{\lambda}}$	e
3	7282	3.377	4.782
4	5050	4.056	4.772
5	4438	4.326	4.773
6	4169	4.463	4.783
7	4024	4.543	4.772
8	3936	4.594	4.775
9	3878	4.628	4.775
10	3838	4.652	4.776
11			
12	3788	4.683	4.773
13	3771	4.693	4.772
		Mean value of $e=4.775$	

THE PRINCIPAL SERIES OF HELIUM

The two principal series of helium not only converge toward a different wave-length and central charge from the subordinate series, as does the principal series of hydrogen, but they converge toward different central charges in the two helium groups. Thus the principal series of Group I converges toward a central charge $Q=2e\sqrt{\frac{1}{(1.69)^2}}$, and the principal series in the parhelium group converges toward a charge $Q=2e\sqrt{\frac{1}{(1.85)^2}}$. The principal series in hydrogen converges toward a charge $Q=2e\sqrt{\frac{1}{(1.5)^2}}$.

Both the principal series in helium seems to have the same value for $1/q^2$ in the Ritz equation. This value may be taken as $1/q^2 = 1/(m-.06)^2$. These values are only approximate, but are as close as are needed in calculating the value of e.

The principal series in the helium group gives the more consistent values of e, but with the exception of the first two lines of the parhelium series the values of e calculated from this series agree satisfactorily. The values of e as calculated from the different lines of these series are given in Tables V and VI.

TABLE V*

UNIT ELECTRICAL CHARGE FROM PRINCIPAL SERIES OF HELIUM, GROUP I

m	λ	Q	e
2	10830	2.77	4.776
3	3889	4.62	4.772
4	3188	5.10	4.770
5	2945	5.305	4.770
6	2829	5.415	4.775
7	2764	5.48	4.773
8	2723	5.52	4.775
9	2696	5.545	4.772
10	2677	5.560	4.768
11	2663	5.580	4.773
12	2657	5.594	4.772
13	2645	5.600	4.774
		Mean value of $e=4.7725$	

*The equation used in this computation is $Q=2e\sqrt{\frac{1}{(1.69)^2}-\frac{1}{(m-.06)^2}}$

TABLE VI*

UNIT ELECTRICAL CHARGE FROM PRINCIPAL SERIES OF PARHELIUM

m	λ	Q	e
3	5016	4.069	4.843
4	3965	4.577	4.795
5	3614	4.794	4.783
6	3448	4.908	4.778
7	3355	4.976	4.775
8	3297	5.019	4.772
9	3258	5.049	4.773
10	3231	5.070	4.774
11	3212	5.085	4.772
12	3197	5.097	4.772
13	3177	5.110	4.775
	Omitting the first two lines, $e=4.775$		

* Calculated from the equation $Q=2e\sqrt{\frac{1}{(1.85)^2}-\frac{1}{(m-.06)^2}}$

STANFORD UNIVERSITY
March 1919

PHOTO-ELECTRIC MEASURES OF THE LIGHT OF NOVA AQUILAE NO. 3

By JOEL STEBBINS AND ELMER DERSHEM

We did not learn of the appearance of the Nova until the morning of June 9, when Mr. Stebbins at Rock Springs heard by telephone from Mr. Frost that a first-magnitude new star had been discovered by Mr. Barnard at Green River the previous night. A telegram was promptly sent to Urbana, and by special effort a photometric measure was secured on June 9. The night of June 8 had been clear throughout at Urbana, and a series of photo-electric observations of stars on the regular program was carried on until about fifteen hours local time. The observers then closed up the dome and walked away toward the northwest, missing the opportunity of measuring the Nova at the critical time. The disappointment of not seeing the Nova on the first night was compensated in part by the following ten nights being clear, but we shall always regret that some accident did not bring the Nova to our attention on June 8.

In its present condition the photo-electric photometer, attached to the 12-inch refractor, gives satisfactory measures of stars as faint as photographic magnitude 5.5. Although stars fainter than this can be observed, the measures become slow and burdensome because of the extra precautions necessary. We have tried objects as faint as magnitude 7.5 and got measurable effects, but there is no need of pushing things to the limit when there are so many brighter stars available. In the six months from June 9 to December 10 the Nova was well within the working range of the apparatus, and measures were secured on 79 nights, every clear night except two when the photometer was out of order. From June to the middle of September the observations were all made by Mr. Dershem, and after that time the work was divided between us. We are indebted to Miss Iva Hamlin for most of the reductions.

The magnitudes, which are derived with a potassium photoelectric cell, are very near to photographic magnitudes. The color-equation of our apparatus is 0.86 magnitude; i.e., a star of spectrum K0 is measured 0.86 magnitude fainter than an A0 star of the same visual brightness.

The photo-electric measures are most satisfactory when the stars to be compared do not differ more than half a magnitude in brightness, and it is always aimed to choose stars as nearly equal as possible. However, as suitable comparison stars are often difficult to find, we have been using, for some time, a series of shade glasses to increase the working range of the instrument. These shades range in absorptive power from about half a magnitude to three magnitudes, and are all nearly neutral in tint. They were made from some plane dark spectacle glasses which we managed to gather up in war time. In comparing two stars of unequal brightness, one star of the two is always observed through the same shade, so that the effect of the shade need be known only approximately, since only variations of light are studied.

TABLE I

Absorption of Shade Glasses

Star	Magnitude	Spectrum	I	II	III
ζ Persei	2.91	B1	$0^{\mathrm{M}}.78$	$1^{\mathrm{M}}.66$	$2^{\mathrm{M}}.80$
γ Pegasi	2.87	B2	0.77	1.61	2.81
α Andromedae	2.15	A0	0.79	1.65	2.81
α Aquilae	0.89	A5	0.80	1.72	2.85
δ Aquilae	3.44	F0	0.79	....	....
α Persei	1.90	F5	0.83	1.76	2.84
γ Cygni	2.32	F8	0.84	1.80	2.85
α Cassiopeiae	2.47	K0	0.84	1.81	2.79
α Arietis	2.23	K2	0.84	1.83	2.81
β Andromedae	2.37	Ma	0.86	1.86	2.80
β Pegasi	2.61	Mb	0.86	1.85	2.78

For the work on Nova it was necessary to know the absorption of each shade, and when we came to make tests we found marked differences for stars of various spectral types. In all strictness the absorption of each shade should be known for every wave-length, but for our purpose it is sufficient to measure the absorption under the working conditions of the instrument. In Table I are given the measures for the three shades, I, II, and III. The result for each star is the mean of four determinations, made on two or more nights.

Although ten stars are scarcely sufficient for a definitive measure, they indicate very well the differences of the shades for the spectral classes, but there are no doubt peculiarities of the individual stars. Since the tests were made we have learned from Mr. W. S. Adams that γ Cygni has a peculiar spectrum, which might have been suspected from the shade tests. In fact, by using strongly colored shades it would be quite feasible to work out a method of measuring quantitively the spectral class of a star. From smooth curves, drawn to represent the results in Table I, the values in Table II were derived.

TABLE II

ADOPTED VALUES OF SHADE GLASSES

Spectrum	I	II	III
Bo	$0^{\mathrm{M}}.77$	$1^{\mathrm{M}}.63$	$2^{\mathrm{M}}.80$
Ao	0.79	1.68	2.82
Fo	0.81	1.73	2.84
Go	0.82	1.78	2.84
Ko	0.84	1.82	2.82
Mo	0.86	1.86	2.79
Nova	0.79	1.68	2.82

It turns out that the difference of absorption between spectral classes B and M is $0^{\mathrm{M}}.09$ for shade I, and $0^{\mathrm{M}}.23$ for shade II, while for shade III the variation is smaller. These differences were not suspected at all when the observations of the Nova were being made, and it is necessary to know the equivalent spectrum of the Nova on any date in order to apply the proper shade correction. However, we were careful not to put in too many complications, and for the first month shade III was always used, followed by II, and then I as the star grew fainter. Rather than to attempt to bring the Nova into the spectral class of other stars we have uniformly used the shade values for class Ao for the Nova. This may introduce a variable systematic error, especially with shade II, as the Nova changed in spectrum, but the error will not be large, probably less than a tenth of a magnitude. When the Nova was observed both with and without a shade on the same night no great difference was noted.

In the matter of comparison stars, there was the choice of using only one or two objects, which would give the most accurate results, provided the comparison stars themselves did not vary; or several stars could be included, each being used when the correction for atmospheric extinction was small. We chose to use several stars, since an unknown or irregular variation of a single comparison object might vitiate an entire series of measures.

In Table III are given the comparison stars, also their adopted "photo-electric" magnitudes, which were determined directly by suitable measures here. In order to compare these with visual magnitudes we may apply the correction in the fourth column, which is simply 0.86 times the color-index of the corresponding spectrum class, as given by Professor E. C. Pickering.[1] The resulting "photo-electric visual" magnitudes should be directly comparable with the Harvard visual magnitudes, and the differences are given in the last column.

TABLE III

COMPARISON STARS

Star	Photo-electric Magnitude	Spectrum	Color-Correction	Photo-electric Visual Magnitude	Harvard Visual Magnitude	H.−O.
α Aquilae	1.01	A5	+ $^{\mathrm{M}}.12$	0.89	0.89	$0^{\mathrm{M}}.00$
α Aquilae, II	2.73	...	...	...	...	...
θ Aquilae	3.19	A0	.00	3.19	3.37	+ .18
δ Aquilae	3.64	F0	+ .24	3.40	3.44	+ .04
γ Aquilae	3.83	K2	+ .92	2.91	2.80	− .11
α Aquilae, III	3.84	...	...	...	...	...
η Serpentis	4.03	K0	+ .86	3.17	3.42	+ .25
δ Aquilae, I	4.46	...	...	...	...	...
4 Aquilae	5.04	B5	− .10	5.14	5.04	− .10
5 Aquilae	5.82	A0	.00	5.82	5.68	− .14

The starting-point for the photo-electric magnitudes was fixed provisionally by adopting 1.01 for α Aquilae, which comes from the visual magnitude of 0.89. The systematic difference shown in the table is so small that we have let the magnitudes remain as they are.

In Table IV are our observations of the Nova. When a shade was used on Nova the correction is noted in the third column, a blank meaning no shade. In the fourth column each observation is usually the mean of three sets of measures, sometimes only two

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[1] *Harvard Annals*, 80, 151, 1917.

TABLE IV

OBSERVATIONS OF NOVA AQUILAE No. 3

Date	G.M.T.	Shade	Difference of Magnitude	Magnitude of Comparison Star	Magnitude of Nova	Remarks
1918 June 9...	18^h47^m	−2.82	−1.48	3.83	−0.47	½ set, clear
	18 57	−2.82	−1.55	3.84	−0.53	½ set, between clouds
10...	16 39	−2.82	−0.81	4.03	+0.40	
	17 00	−2.82	−0.80	4.03	0.41	
	17 47	−2.82	−0.71	3.84	0.31	
	18 13	−2.82	−0.72	3.84	0.30	
	18 58	−2.82	−0.50	3.64	0.32	
	18 58	−2.82	−0.91	4.03	0.30	
	19 57	−2.82	−0.37	3.64	0.45	
	21 14	−2.82	+0.48	3.19	0.85	Nova fainter
11...	17 38	−2.82	−0.61	4.03	0.60	
	18 05	−2.82	−0.50	3.84	0.52	
	18 38	−2.82	−0.27	3.64	0.55	
	18 38	−2.82	−0.66	4.03	0.55	
	19 10	−2.82	−0.51	3.84	0.51	
	20 17	−2.82	+0.20	3.19	0.57	
	20 36	−2.82	+0.22	3.19	0.59	
12...	16 13	−2.82	−0.32	4.03	0.89	
	17 22	−2.82	−0.28	4.03	0.93	
	17 56	−2.82	−0.12	3.84	0.90	
	18 32	−2.82	+0.12	3.64	0.94	
	18 32	−2.82	−0.28	4.03	0.93	
	19 08	−2.82	−0.07	3.84	0.95	
	19 33	−2.82	+0.13	3.64	0.95	Very poor
	20 22	−2.82	+0.56	3.19	0.93	
	20 41	−2.82	+0.58	3.19	0.95	
13....	16 07	−2.82	−0.04	4.03	1.17	
	16 33	−2.82	+0.01	4.03	1.22	
	17 56	−2.82	+0.34	3.64	1.16	
	17 56	−2.82	−0.02	4.03	1.19	
	18 29	−2.82	+0.20	3.84	1.22	
	18 58	−2.82	+0.39	3.64	1.21	
	19 20	−2.82	+0.43	3.64	1.25	
	19 59	−2.82	+0.38	3.64	1.20	
	20 25	−2.82	+0.83	3.19	1.20	
14....	20 11	−2.82	+0.61	3.64	1.43	Poor sky, extinction 0.10
15....	16 13	−1.68	−0.67	4.03	1.68	Discordant, poor sky
	16 39	−1.68	−0.64	4.03	1.71	Discordant, poor sky
	17 59	−1.68	+0.60	2.73	1.65	Discordant, poor sky
	19 24	−1.68	−0.28	3.64	1.68	Discordant, poor sky
	19 51	−1.68	−0.29	3.64	1.67	Discordant, poor sky
	20 22	−1.68	+0.15	3.19	1.66	Discordant, poor sky
	20 49	−1.68	+0.11	3.19	1.62	Discordant, poor sky, extinction 0.10
16....	17 31	−1.68	−0.46	4.03	1.89	
	17 55	−1.68	+0.82	2.73	1.87	
	18 17	−1.68	+0.82	2.73	1.87	
	19 14	−1.68	−0.09	3.64	1.87	
	19 14	−1.68	−0.50	4.03	1.85	
	19 55	−1.68	−0.09	3.64	1.87	

TABLE IV—*Continued*

Date	G.M.T.	Shade	Difference of Magnitude	Magnitude of Comparison Star	Magnitude of Nova	Remarks
June 17....	16^h15^m	−1.68	−0.32	4.03	+2.03	
	17 11	−1.68	−0.36	4.03	1.99	
	17 29	−1.68	−0.33	4.03	2.02	1 set
	17 29	−1.68	+0.05	3.64	2.01	1 set
18....	16 30	−1.68	−0.27	4.03	2.08	
	16 52	−1.68	−0.30	4.03	2.05	
	17 47	−1.68	+1.04	2.73	2.09	
	19 22	−1.68	+0.02	3.64	1.98	½ set
	19 22	−1.68	−0.30	4.03	2.05	½ set
	20 10	−1.68	+0.08	3.64	2.04	
	20 33	−1.68	+0.55	3.19	2.06	
21....	19 03	−1.68	+0.28	4.03	2.63	Poor, smoke?
	19 28	−1.68	+0.63	3.64	2.59	Poor, smoke?
22....	17 16	−1.68	+0.48	4.03	2.83	½ set, worth something
23....	18 20	−0.79	−0.42	4.03	2.82	
	18 38	−0.79	−0.08	3.64	2.77	1 set
30....	15 38	....	−0.65	4.03	3.38	
	17 15	....	−0.73	4.03	3.30	
	17 42	....	−0.35	3.64	3.29	
	19 37	−1.68	−0.02	5.04	3.34	
July 1....	15 45	....	−0.81	4.03	3.22	
	16 09	....	−0.85	4.03	3.18	1 set
	17 11	....	−0.42	3.64	3.22	
	18 26	....	−0.41	3.64	3.23	
	19 54	−1.68	−0.20	5.04	3.16	
3....	16 57	−0.79	−0.25	4.03	2.99	
	17 29	−0.79	+0.12	3.64	2.97	
	18 57	−1.68	−0.38	5.04	2.98	
5....	16 53	−0.79	−0.02	4.03	3.22	
	17 26	−0.79	+0.39	3.64	3.24	
8....	15 23	−0.79	+0.26	4.03	3.50	
	16 24	−0.79	+0.23	4.03	3.47	
	17 50	....	−0.11	3.64	3.53	
10....	15 54	....	−0.17	4.03	3.86	
	17 43	....	+0.22	3.64	3.86	
	19 19	−0.79	−0.39	5.04	3.86	
11....	15 51	....	−0.20	4.03	3.83	
	17 21	....	+0.20	3.64	3.84	
	17 55	−0.79	−0.40	5.04	3.85	
	19 17	−0.79	−0.41	5.04	3.84	Very poor, smoke
12....	15 52	....	−0.08	4.03	3.95	
	16 44	....	+0.23	3.64	3.87	
	17 35	−0.79	−0.36	5.04	3.89	
	19 36	−0.79	−0.36	5.04	3.89	
18....	15 06	−0.79	+0.48	4.03	3.72	
	16 58	....	+0.08	3.64	3.72	
	17 57	−0.79	−0.57	5.04	3.68	
	19 22	−0.79	−0.56	5.04	3.69	
19....	14 58	....	−0.15	4.03	3.88	
	16 24	....	+0.18	3.64	3.82	
	17 24	−0.79	−0.40	5.04	3.85	

TABLE IV—*Continued*

Date	G.M.T.	Shade	Difference of Magnitude	Magnitude of Comparison Star	Magnitude of Nova	Remarks
July 20....	15^h21^m	....	+0.08	4.03	4.11	
	16 37	....	+0.38	3.64	4.02	
22....	14 54	....	+0.22	4.03	4.25	Moon
27....	14 55	....	−0.32	4.03	3.71	
	16 08	....	+0.04	3.64	3.68	
	17 06	−0.79	−0.56	5.04	3.69	
	18 16	−0.79	−0.55	5.04	3.70	
29....	15 32	....	−0.17	4.03	3.86	
	15 56	....	+0.20	3.64	3.84	
	16 15	....	+0.24	3.64	3.88	
	17 17	−0.79	−0.38	5.04	3.87	
30....	14 41	....	−0.14	4.03	3.89	
	15 53	....	+0.26	3.64	3.90	
	17 11	−0.79	−0.35	5.04	3.90	
31....	14 38	....	−0.02	4.03	4.01	
	15 47	....	+0.37	3.64	4.01	
	16 40	−0.79	−0.20	5.04	4.05	
	18 26	−0.79	−0.17	5.04	4.08	
Aug. 1....	15 40	....	+0.84	3.64	4.48	
	16 07	....	+0.01	4.46	4.47	Poor
	16 42	−0.79	+0.26	5.04	4.51	
	17 01	−0.79	+0.23	5.04	4.48	
	18 13	−0.79	+0.24	5.04	4.49	
3....	14 51	....	+0.48	4.03	4.51	
	15 49	....	+0.08	4.46	4.54	
	17 22	−0.79	+0.30	5.04	4.55	1 set
4....	14 55	....	+0.45	4.03	4.48	Poor sky
	15 22	....	−0.09	4.46	4.37	Poor sky
	15 42	....	+0.03	4.46	4.49	Poor sky
5....	15 40	....	+0.16	4.46	4.62	
	17 20	....	−0.48	5.04	4.56	
6....	14 42	....	−0.14	4.03	3.89	
	15 28	....	−0.60	4.46	3.86	Very poor sky
7....	14 52	....	−0.28	4.03	3.75	
	15 45	....	−0.75	4.46	3.71	1 set
	15 52	....	+0.10	3.64	3.74	
	16 45	−0.79	−0.48	5.04	3.77	
	18 09	−0.79	−0.48	5.04	3.77	Poor
9....	14 55	....	+0.02	4.03	4.05	
	15 20	....	+0.43	3.64	4.07	
	15 36	....	+0.38	3.64	4.02	
11....	14 39	....	+0.05	4.03	4.08	
	15 06	....	+0.43	3.64	4.07	
	15 30	....	+0.44	3.64	4.08	
	17 04	−0.79	−0.13	5.04	4.12	
12....	14 51	....	+0.37	4.03	4.40	
	15 20	....	−0.15	4.46	4.31	δ Aquilae faint
	15 43	....	−0.14	4.46	4.32	
	17 00	−0.79	+0.17	5.04	4.42	
14....	14 56	....	−0.04	4.46	4.42	
	15 24	....	−0.09	4.46	4.37	

TABLE IV—*Continued*

Date	G.M.T.	Shade	Difference of Magnitude	Magnitude of Comparison Star	Magnitude of Nova	Remarks
Aug. 21....	$14^h 24^m$	....	0.00	4.46	4.46	Moon δ faint
	14 48	....	+0.01	4.46	4.47	Moon
	16 22	−0.79	+0.26	5.04	4.51	Moon
22....	14 15	....	+0.51	4.03	4.54	Moon
	14 45	....	0.00	4.46	4.46	Moon δ faint
	15 07	....	0.00	4.46	4.46	Moon
	16 30	−0.79	+0.31	5.04	4.56	Moon
31....	14 17	....	+0.45	4.46	4.91	
	16 25	....	−0.12	5.04	4.92	
Sept. 1....	14 06	....	+0.47	4.46	4.93	
	14 36	....	−0.09	5.04	4.95	
5....	14 39	....	+0.03	4.46	4.49	
	15 19	....	−0.52	5.04	4.52	
6....	14 14	....	−0.08	4.46	4.38	
8....	14 12	....	+0.27	4.46	4.73	
	15 11	....	−0.32	5.04	4.72	
16....	13 49	....	−0.07	5.04	4.97	
	14 17	....	−0.06	5.04	4.98	
19....	14 23	....	−0.07	5.04	4.97	Nova changed
	14 40	....	−0.14	5.04	4.90	
21....	14 48	....	−0.27	5.04	4.77	
	15 13	....	−0.30	5.04	4.74	
22....	14 42	....	−0.13	5.04	4.91	
	15 03	....	−0.12	5.04	4.92	
26....	13 53	....	+0.04	5.04	5.08	
27....	14 48	....	−0.06	5.04	4.98	
28....	14 21	....	−0.04	5.04	5.00	
	14 42	....	−0.03	5.04	5.01	
29....	13 33	....	+0.04	5.04	5.08	
	13 52	....	+0.02	5.04	5.06	
Oct. 3....	13 20	....	+0.03	5.04	5.07	
	13 40	....	+0.03	5.04	5.07	
5....	14 33	....	+0.06	5.04	5.10	
6....	13 08	....	+0.08	5.04	5.12	
	13 34	....	+0.09	5.04	5.13	
12....	12 54	....	−0.23	5.04	4.81	
	13 15	....	−0.20	5.04	4.84	
14....	13 23	....	+0.24	5.04	5.28	
	13 47	....	+0.24	5.04	5.28	
15....	12 38	....	+0.26	5.04	5.30	Poor sky
16....	12 55	....	+0.24	5.04	5.28	Poor sky
	13 20	....	+0.25	5.04	5.29	Poor sky
20....	12 43	....	+0.16	5.04	5.20	
	13 01	....	+0.15	5.04	5.19	
21....	12 27	....	+0.27	5.04	5.31	
	12 48	....	+0.27	5.04	5.31	
27....	12 28	....	+0.36	5.04	5.40	
	12 56	....	+0.35	5.04	5.39	
28....	12 29	....	+0.34	5.04	5.38	
29....	12 11	....	+0.35	5.04	5.39	
	12 36	....	+0.36	5.04	5.40	

TABLE IV—*Continued*

Date	G.M.T.	Shade	Difference of Magnitude	Magnitude of Comparison Star	Magnitude of Nova	Remarks
Nov. 1....	12h52m	....	+0.17	5.04	5.21	
	13 09	....	+0.18	5.04	5.22	
2....	12 07	....	+0.35	5.04	5.39	
	12 30	....	+0.33	5.04	5.37	
Nov. 4....	12 51	....	+0.37	5.04	5.41	Smoke
5....	12 47	....	+0.39	5.04	5.43	
	12 47	....	−0.40	5.82	5.42	
10....	12 33	....	+0.40	5.04	5.44	
	12 33	....	−0.32	5.82	5.50	
11....	12 09	....	+0.44	5.04	5.48	
	12 09	....	−0.39	5.82	5.43	
12....	12 01	....	+0.44	5.04	5.48	Smoke
	12 01	....	−0.43	5.82	5.39	
12....	11 57	....	+0.40	5.04	5.44	
	11 57	....	−0.44	5.82	5.38	
24....	12 00	....	−0.18	5.82	5.64	1 set
	12 06	....	+0.54	5.04	5.58	
29....	11 39	....	−0.30	5.82	5.52	1 set
	11 42	....	+0.51	5.04	5.55	
30....	11 48	....	+0.54	5.04	5.58	
Dec. 10....	11 36	....	+0.63	5.04	5.67	Twilight

sets, but if less than two that fact is noted in the remarks. A set of measures normally includes two readings on the comparison star, four on the Nova, and then two on the comparison star again. Three sets may be secured in as short a time as fifteen minutes, but half an hour is a better average for all conditions. A half-set would mean that for some reason, usually because clouds came suddenly, the second pair of readings on the comparison star was not secured. It is our usual practice to reject all incomplete measures of this sort, but where the choice is between a perfect set or nothing at all we must take what we can get. The comparison star used is identified by its magnitude given in the fifth column. Each adopted magnitude of the Nova, as given in the sixth column, was obtained by adding the corresponding numbers in the three preceding columns.

An observation was marked poor when the readings of a set were discordant, which was usually due to a passing cloud or smoke. When the sky was called poor it was "thick" but apparently uniform, for only under extraordinary circumstances would measures be undertaken with an uneven sky. We observed the Nova on many occasions when conditions were unfit for our regular program, but it was felt that the results would be of value even with errors of several hundredths of a magnitude. The effect of the sky background at the time of full moon is usually negligible, as the light from a star passes through a diaphragm which has a field of only $2'.4$ diameter. However, when faint stars are compared and a shade is used for only one of them, the moon effect may be troublesome to determine, and these cases are noted in the remarks.

All of the measures have been corrected for atmospheric extinction, but in four-fifths of the cases this correction was less than $0^{\mathrm{M}}.03$, and only twice did it exceed $0^{\mathrm{M}}.07$, these occasions being noted in the remarks.

The accordance of the measures is shown by the residuals given by the several sets in each observation, from which we find,

$$\text{Probable error of one observation} = \pm 0^{\mathrm{M}}.009$$

This includes only the accidental errors at the time of the measures, and it is better to compute the probable error from the residuals formed by comparing each result in Table IV with the mean for the corresponding night. Using all measures after June 11 we find,

$$\text{Probable error of one observation} = \pm 0^{\mathrm{M}}.019$$

Included in this probable error are all poor and incomplete measures, all outstanding errors in the values for the shade glasses and in the magnitudes of the comparison stars, and all variations of light of the Nova and of the comparison stars. Both of the foregoing values of the probable error are larger than would be tolerated in regular work with this photometer, but the accuracy is quite sufficient to follow the changes of the Nova.

Our measures of Nova Aquilae were undertaken to detect any rapid changes of light, such as are sometimes reported in the case of a new star. In this we were quite disappointed, for although the measures often extended over four or five hours there was only one night, June 10, when a variation as large as 0.10 magnitude could be established. On other dates there were suspicions of progressive changes which will probably fit in with the course of the general light-curve of the Nova, but so far as these measures are concerned we may dismiss any notion of sudden and erratic variations in the course of an hour or so.

The measures on June 9 call perhaps for special mention, as there is some doubt from the estimates of visual observers as to how bright the Nova really became. Watch was kept here during most of that night, but it was only once when the sky cleared for a few minutes that some readings could be taken. Although we should have been glad to have had more measures, the resulting magnitude, -0.50, can scarcely be in error as much as a tenth of a magnitude. This photo-electric result cannot be reconciled with visual estimates of -1.0 magnitude or brighter, as the Nova was not a red star on that date.

In photometric observations it is always proper to suspect every comparison star of light-variation, and the measures in Table IV furnish good tests of the constancy of the various stars observed. The only star that we suspect strongly is δ Aquilae, which apparently was about a tenth of a magnitude faint on one or two nights, particularly on August 12. We do not overlook the fact that finger marks or a film of moisture on the shade glass would have produced just the same effect, but it is our practice to wipe off and examine both sides of a shade *every* time that it is used. It is therefore highly improbable that a dirty shade glass was the cause of these discordances.

Although we managed to observe the Nova on nearly half the nights during six months, the series is not extended enough to make a separate study of the light-curve, but it is probable that the magnitudes given will conform nearly with the visual scale, so that no great difficulty will be found in using these measures in the general discussion of the light-changes.

The development of the photo-electric photometer has been in collaboration with Dr. Jakob Kunz, and the work described in this paper is a part of that carried on with the aid of a grant from the Draper Fund of the National Academy of Sciences.

UNIVERSITY OF ILLINOIS OBSERVATORY
February 1919

REVIEWS

A Treatise on the Sun's Radiation and Other Solar Phenomena. By FRANK H. BIGELOW. New York: John Wiley & Sons, Inc., 1918. Figs. 44. 8vo, pp. ix+385. $5.00.

This book contains an account of the further developments of the thermodynamical theory of radiation which was described in the author's *Atmospheric Circulation and Radiation.*

It is well known that when an atmosphere is treated as an ideal gas stratified in such a way that there is indifferent or adiabatic equilibrium, the temperature-gradient is determined by the equation

$$C_p dT = -g dz, \tag{1}$$

where C_p is the specific heat at constant pressure, T the temperature, z the altitude, and g the acceleration of gravity. In the usual deduction[1] of this equation use is made of the equations

$$Pv = RT, \qquad C_p - C_v = R, \tag{2}$$

which are characteristic of an ideal gas.

Recognizing that actual temperature-gradients are often different from the adiabatic, Professor Bigelow proposes a non-adiabatic scheme of equations exactly like the foregoing except that C_p, C_v, and R are treated as quantities which vary with the altitude. The quantities C_p, C_v thus defined are still called specific heats, but it is not evident that this is justifiable, since the usual proof of equation (1) breaks down when the temperature-gradient is not adiabatic. If I understand the author correctly, he assumes that in a thermodynamical change, which takes place without a change in altitude, the variations in specific volume, v, pressure, P, and temperature, T, are connected by the relation

$$Pdv + vdP = RdT,$$

but that when in addition there is a change in altitude the corresponding relation should be

$$Pdv + vdP = RdT + TdR.$$

---

[1] Cf. W. J. Humphreys, "The Physics of the Air," *Journal of the Franklin Institute,* 184, 161, 1917.

The variation of R with altitude is attributed to changes in composition and motion of the atmosphere, to gravitation, to the emission and absorption of radiant heat, and perhaps also to the density and pressure of radiation. When the conditions are such that $dR=0$ the temperature-gradient is adiabatic.

It should be noticed that if, on the other hand, R is regarded as a function of P and v in the equation of state $Pv=RT$, the well-known equation[1]

$$C_p-C_v=T\left(\frac{\delta v}{\delta T}\right)_p\left(\frac{\delta P}{\delta T}\right)_v, \tag{3}$$

which is valid for any substance, gives, when combined with $C_p-C_v=R$, the following partial differential equation for R:

$$vR\frac{\delta R}{\delta v}+PR\frac{\delta R}{\delta P}=Pv\frac{\delta R}{\delta v}\frac{\delta R}{\delta P} \tag{4}$$

This equation evidently restricts the form of the equation of state. A complete integral is

$$R=cv^aP^b, \tag{5}$$

where a, b, and c are constants connected by the relation $ab=a+b$. Thus a possible equation of state is

$$P^xv^{\frac{1}{x}}=cT, \tag{6}$$

where x and c are constants, and a more general equation compatible with the foregoing conditions may be obtained by the method of generalization employed in the theory of partial differential equations.

The foregoing relation is not inconsistent with Mr. Bigelow's fundamental relations

$$\frac{P_1}{P_0}=\left(\frac{T_1}{T_0}\right)^{\frac{nk}{k-1}}, \qquad \frac{\rho_1}{\rho_0}=\left(\frac{T_1}{T_0}\right)^{\frac{n}{k-1}}, \tag{7}$$

which connect the pressures, temperatures, and densities at the top and bottom of a layer, for k is a constant, and n, the ratio of the adiabatic temperature-gradient to the actual gradient, is treated as practically constant within a layer, a mean value being actually used.

It should be mentioned that the equation $C_p-C_v=R$ is apparently simply a hypothesis. If, moreover, C_p is regarded as an adiabatic invariant defined by equation (1), and C_v as a second adiabatic invariant defined by the equations (2), it would seem that Bigelow's theory is essentially a development of the hypothesis that the ratio $\frac{C_p}{C_v}=K$ is constant, i.e., the same at all levels. This hypothesis is made, however, with some reservation, for in the computations it is found necessary to treat hydrogen as monatomic with $K=\frac{5}{3}$ below a certain level, but as diatomic H_2 with $K=\frac{7}{5}$ above that level.

---

[1] Cf. W. C. McC. Lewis, *Physical Chemistry*, 2, 74, 1916.

Computations are made in which the solar atmosphere is treated successively as consisting entirely of H_1, H_2, He, C, Ca, Zn, Cd, and Hg respectively. The partial pressures of the different substances in the actual solar atmosphere can of course be obtained from the total pressure, P, when the total amounts of the different substances are known.

A reference plane of thermal equilibrium at some unknown level in the sun is derived from the terrestrial data by multiplying P, v, T, R, C_p, and C_v by the factor $\gamma=28.028$, which is the ratio of the accelerations of gravity on the sun and the earth. The level of this reference plane ($P=28.028$ atmospheres) occurs at very different distances below the photosphere ($P=6$) for the different gases but is in each case in the isothermal region, which extends upward beyond the level of the photosphere and changes gradually into another non-adiabatic region, in which the temperature, pressure, and R decrease with the altitude down to zero values. Below the isothermal region there is an adiabatic region. The depth of the isothermal region is great for the light gases and small for the heavy ones. The temperature is estimated at $7686^{\circ}.7$ absolute, being nearly the same for all the gases. The level $P=6$ is also found to be the same for all.

Another interesting conclusion is that the sun's disk is not an optical effect but the vanishing level of the heavy metallic vapors under the operation of the prevailing thermodynamic conditions. The computations show in fact that the pressures of Hg, Cd, and Zn fall to zero at a comparatively small distance above the photosphere. The element Ca just floats comfortably upon the layers of the still heavier gases, and it is easily observed in the spectroheliograph as a surface phenomenon, having many configurations as in the faculae and flocculi. The lighter gases, especially He and H, rise to great heights, the H to the top of the inner corona. It is claimed that the distributions indicated by the computations conform well with the results of the spectroheliographic observations upon the heights of the chemical elements in the sun.

The author next attacks the problem of the solar radiation, basing his computations on the following expression for the mean value within a layer of the specific intensity of radiation:

$$K_{10} \equiv cT^a = \frac{U_1 - U_0}{v_1 - v_0}, \tag{8}$$

where U_1, U_0; v_1, v_0 are the values of the inner energy and specific volume respectively at the two boundaries of the layer. The quantities U_1 and U_0 are assumed to be connected by the relation

$$U_1 - U_0 = (C_{p_a} - R_{10})(T_a - T_0),$$

which is an "adaptation" of the thermodynamical relation which holds in the adiabatic case. As the derivation of this formula is not explained I cannot quite make out whether or not it involves the assumption that the non-adiabatic C_p and C_v are actually specific heats. The computations lead to some interesting conclusions. They indicate that the factor c in the formula (8) suffers a very large change in value in passing from the isothermal region to the adiabatic, while the exponent a changes from 4 to 2.4. It is concluded from this that the solar radiation originates in a saltum process at the bottom of the isothermal layer and continues throughout the isothermal region as black radiation, a result which is thought to be in agreement with Emden's conclusion that an isothermal atmosphere of sufficient thickness emits black radiation. It is found, moreover, that the gradient of the heat content does not change suddenly, and so it is concluded that radiation is not a common thermodynamic process, as Planck assumes. A theory of radiation is then developed and compared with those of Planck and Bohr.

The book is frankly revolutionary and challenges the results of previous investigators. Perhaps the most serious objections to the theory are that the number of assumptions seems to be large, and that many quantities which in everyday physics are reverently treated as constants are here regarded as variables. Among these we may mention the number of molecules per unit mass and the constant k in the Boltzmann-Planck relation between entropy and probability. This last assumption is in direct contradiction with the statement in Planck's *Theory of Heat Radiation* (translation by Masius, p. 119) that k is a universal constant, the same for a terrestrial as for a cosmic system. It should be mentioned, however, that according to P. Ehrenfest (*Phil. Mag.*, 33, 500, 1917) the relation $S = k \log W$ between entropy and probability has not been definitely proved and has been regarded in recent years simply as a postulate. Bigelow's hypothesis of a variable *k* may then be quite defensible.

As a work of imagination the book may be compared with Planck's *Theorie der Wärmestrahlung*, for it is a really remarkable attempt to develop a type of mathematical analysis which is capable of accounting for the complex conditions which obtain in the solar atmosphere. Just as in the case of Planck's theory there are many things which are hard to understand from a strictly logical point of view, but Planck's theory has been of undoubted value in the development of mathematical physics, and so it would be unwise to reject the present theory on account of these difficulties. As to the validity of the conclusions time must decide.

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February 27, 1919

Mirrors, Prisms and Lenses, a Text-book of Geometrical Optics. By JAMES P. C. SOUTHALL. New York: Macmillan, 1918. Pp. xix+579, Figs. 247. $3.25.

This book by Professor Southall ought to receive a wide welcome. The war has brought out too distinctly the error made by the English-speaking people in neglecting the optician's art, a neglect even more marked in this country than in England. It is to be hoped that this volume may do much to reduce this fault in the future.

The book has fifteen chapters, of which the first three deal with the elementary phenomena of optics. The following chapters, up to the last, give descriptions of the principal systems, with clear expositions of their elementary theories and of the principles of their actions. An admirable feature of the book is the collection of numerous problems at the ends of the chapters, which should yield a conscientious student a great deal of accurate and available knowledge when he shall have solved them.

The last chapter, which may be regarded as necessary for a complete view of the optical art, is hardly simple enough to attach logically to the book up to that point.

Singularly few important errors betray themselves to the reader, while such as were noticed would afford valuable exercise to the student in detection and emendation.

One feature of the presentation of the subject might be worthy of passing comment. Since it is practically impossible either to construct an optical system or to determine its physical constants after construction to a degree of precision greatly in excess of one part in one thousand, it seems unfortunate to suggest to the student that seven-place logarithms are a proper auxiliary to his work. The labor of practical application of the theory to the solution of problems of construction is sufficiently formidable without the drudgery of unnecessary labor of computations.

C. S. HASTINGS

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February 25, 1919

A CORRECTION

The following correction should be made in the article "On the Dark Markings of the Sky" in the number of the *Astrophysical Journal* for January, **49**, 1, 1919. In the last paragraph on page 4
for No. 160, in $\alpha = 21^h 34^m 47^s$, $\delta = +55°41'$
read No. 150, in $\alpha = 20^h 47^m 47^s$, $\delta = +59°50'$. The rest of the paragraph is correct.

On the same page, second paragraph: the dark object (No. 15) in $\alpha = 4^h 22^m 50^s$, $\delta = +46°21'$ is really in a dark part of the Milky Way.

E. E. BARNARD

May 8, 1919

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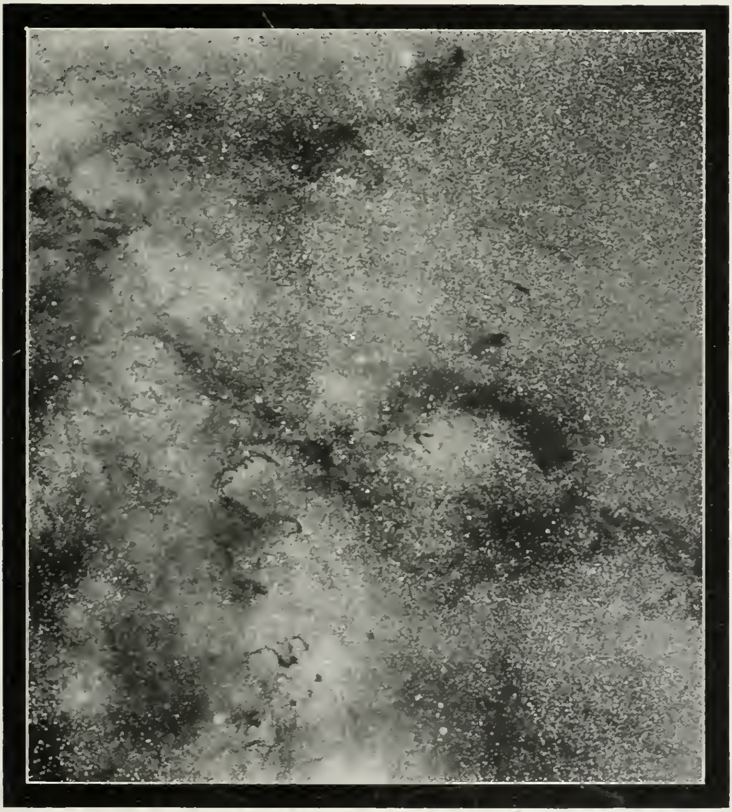
PLATE I

North

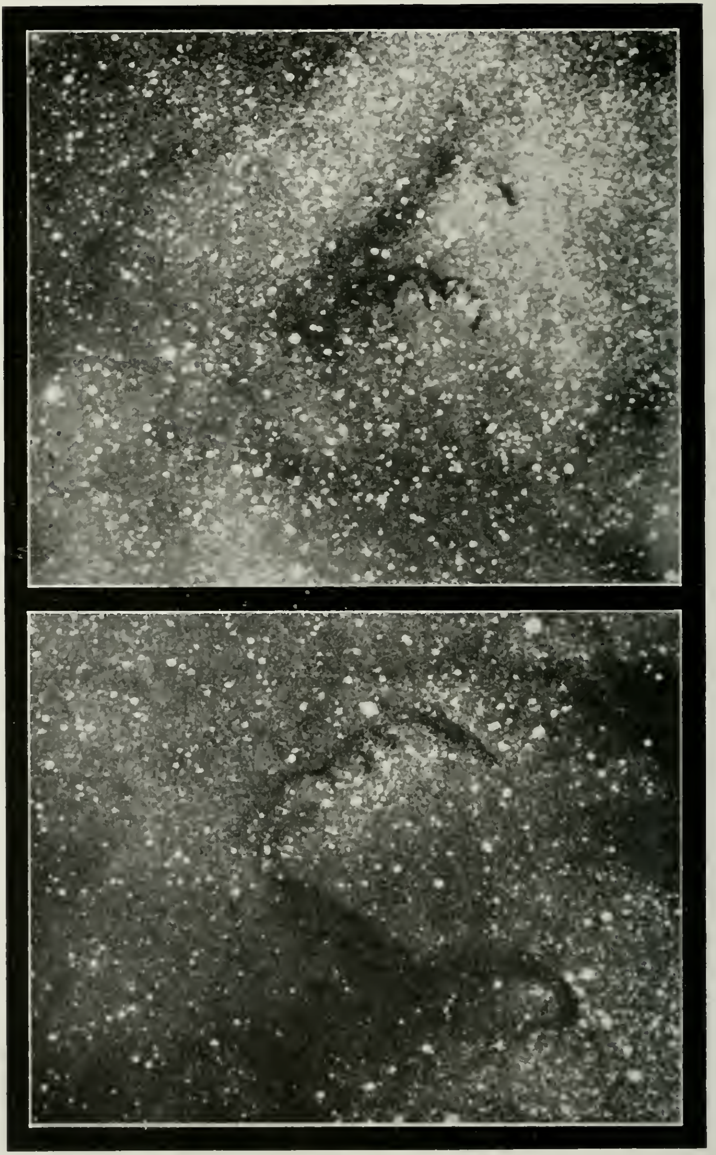


Region North of Theta Ophiuchi

$\alpha = 17^h\ 13^m,\ \delta = -21^\circ\ 0'$

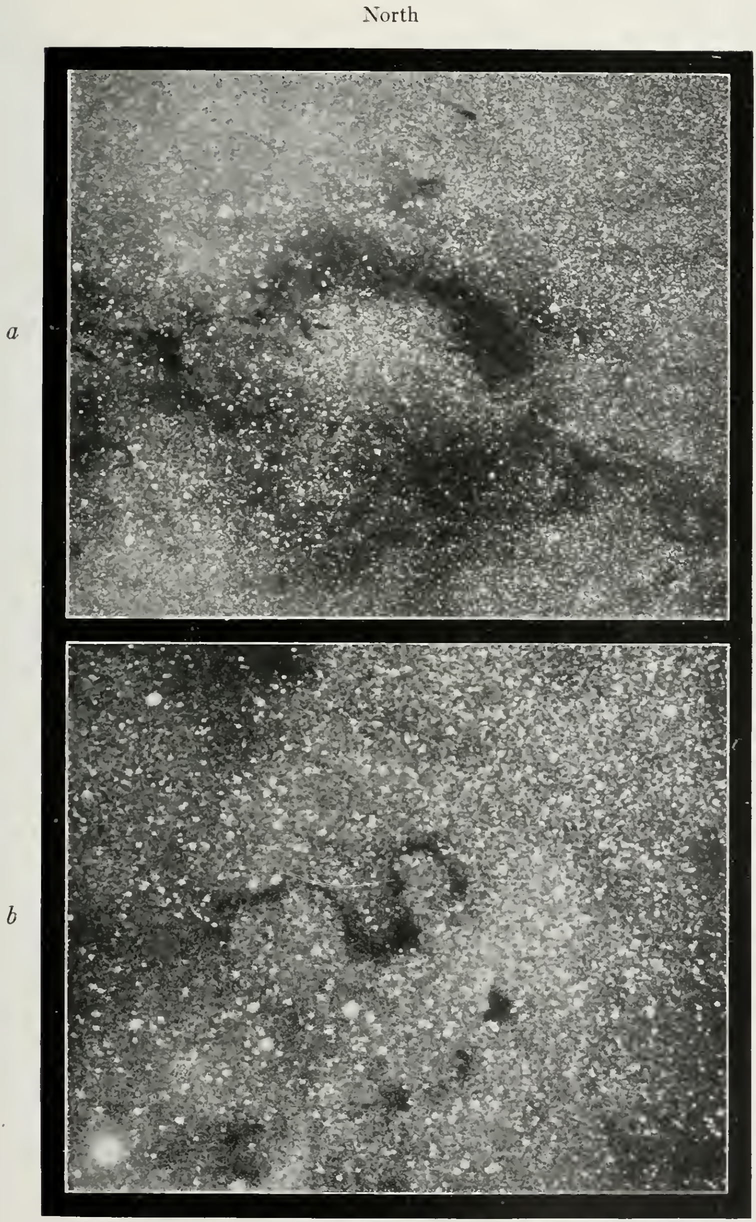
Scale: $1^{mm} = 234''$

PLATE II



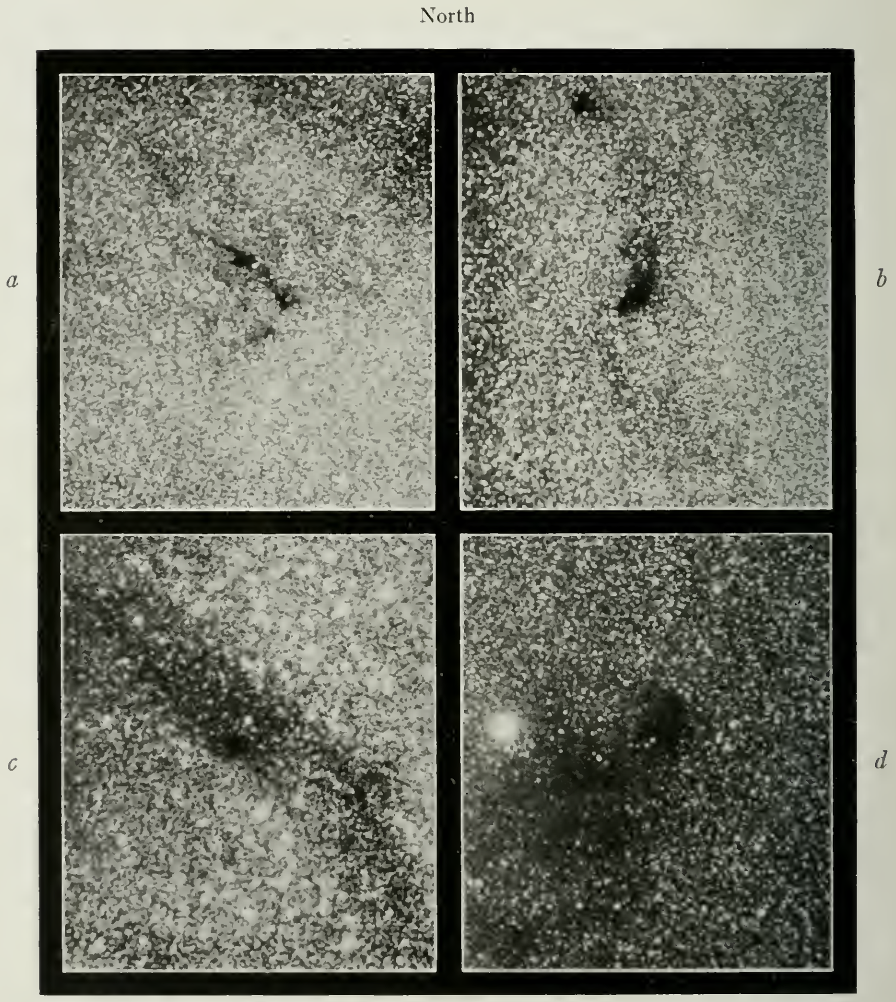
a) No. 84. $\alpha = 17^h 30^m$, $\delta = -20°12'$ Scale: $1^{mm} = 49''$
b) No. 75. $\alpha = 17^h 18^m$, $\delta = -21°55'$ Scale: $1^{mm} = 59''$

PLATE III



a) No. 63. $\alpha = 17^{h}\ 9^{m}$, $\delta = -21^{\circ}17'$ Scale: $1^{mm} = 160''$
b) No. 72. $\alpha = 17^{h}16^{m}$, $\delta = -23^{\circ}37'$ Scale: $1^{mm} = 60''$

PLATE IV



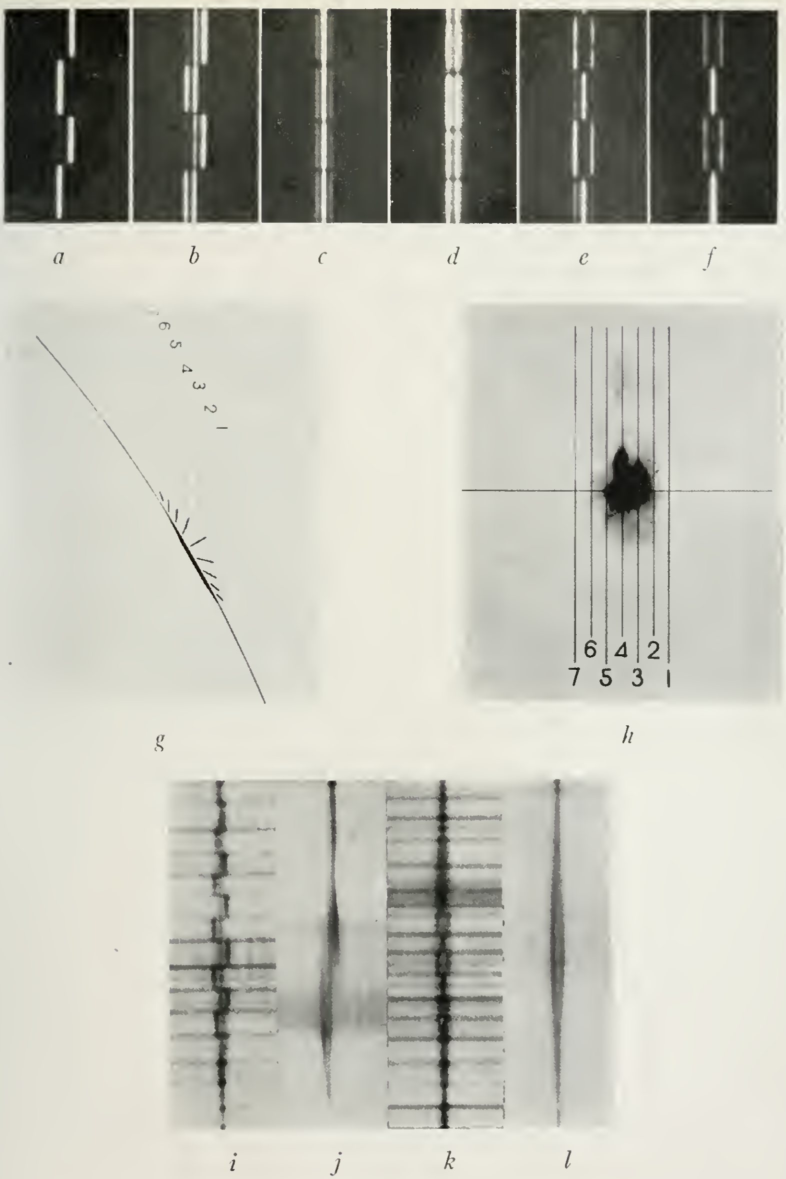
a) Nos. 127 and 129. $\alpha = 18^h 55^m$, $\delta = -5^\circ 37'$
Scale: $1^{mm} = 93''$

b) No. 133. $\alpha = 18^h 59^m$, $\delta = -\ 7^\circ 5'$
Scale: $1^{mm} = 89''$

c) No. 98. $\alpha = 18^h 26^m$, $\delta = -26^\circ 9'$
Scale: $1^{mm} = 63''.5$

d) No. 64. $\alpha = 17^h 10^m$, $\delta = -18^\circ 21'$
Scale: $1^{mm} = 78''$

PLATE V

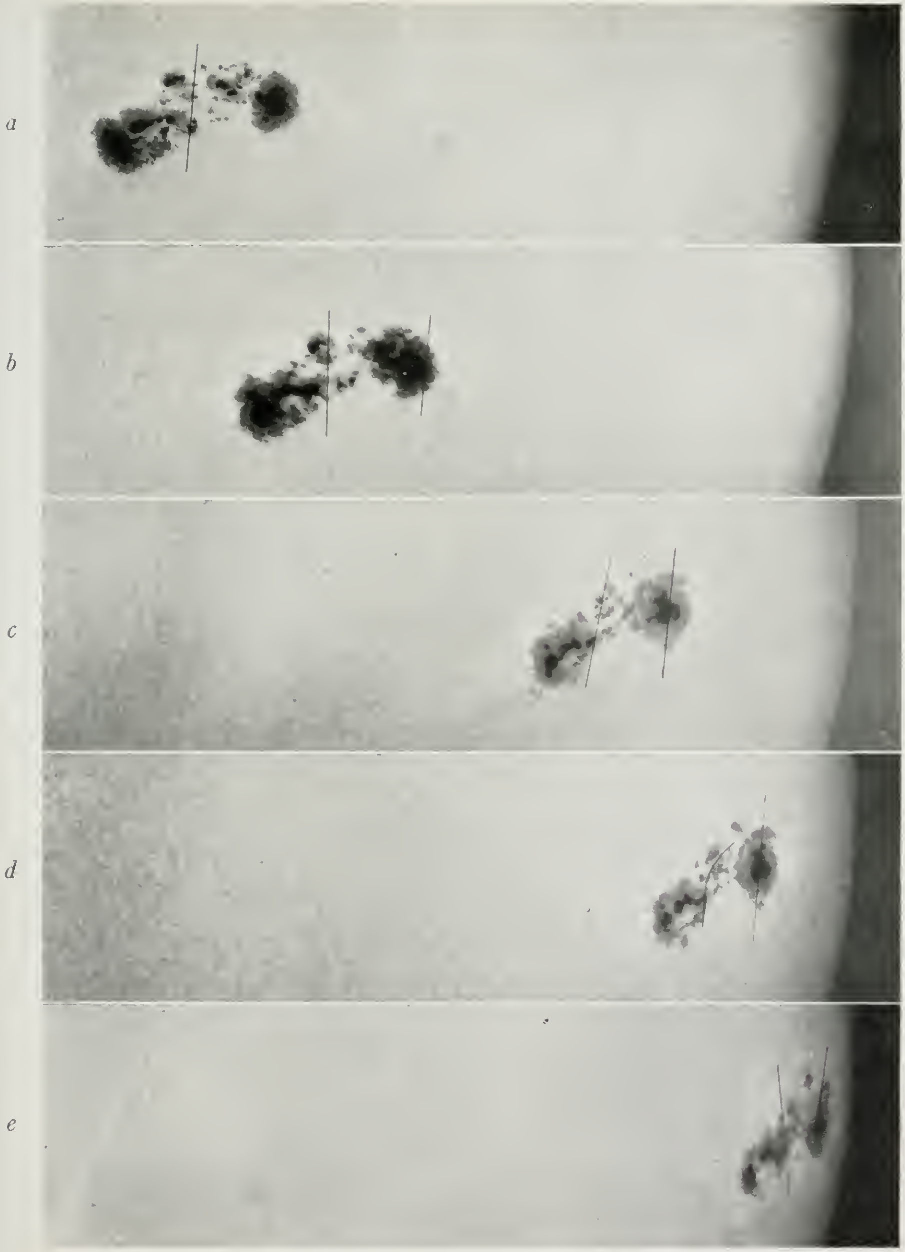


a, *b*, *c*, Observations of the zinc triplet λ4680 in the physical laboratory, with compound quarter-wave plate in front of the slit of the spectrograph; inclinations of line of sight to lines of force are 0°, 60°, 90°, respectively; *d*, *e*, *f*, the same, with compound half-wave plate substituted for the quarter-wave plate; compare with Fig. 3 and *i* and *k* of this plate.

g, Lines of force in a sun-spot near the west limb on October 1, 1915; *h*, photograph of the same spot; the numbered lines indicate successive positions of the slit of the spectrograph used to determine the inclination of the lines of force.

i, *j*, *k*, *l*, Iron triplet λ 6173 photographed in spectra of various sun-spots; *i*, *j*, circularly polarized light, spot near center of sun, observed with Nicol and (*i*, compound; *j*, single) quarter-wave plate; *k*, *l*, plane polarized light, spot near limb, with Nicol and (*k*, compound; *l*, single) half-wave plate; *j* shows reversal of polarity in two adjacent umbrae of large spot-group of August, 1917.

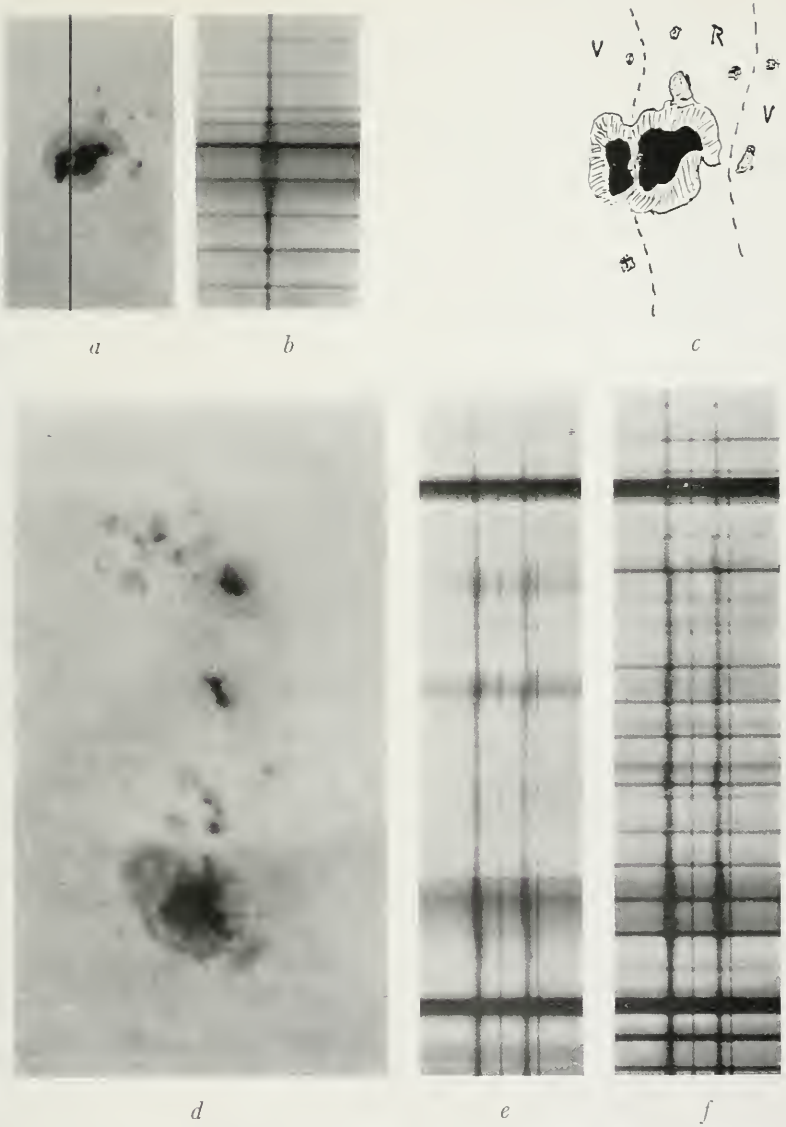
PLATE VI



Photographs of Large Bipolar Sun-Spot Group

a, February 9; *b*, February 10; *c*, February 12; *d*, February 13; *e*, February 14, 1917, showing change in position of the neutral line with change in longitude of spot.

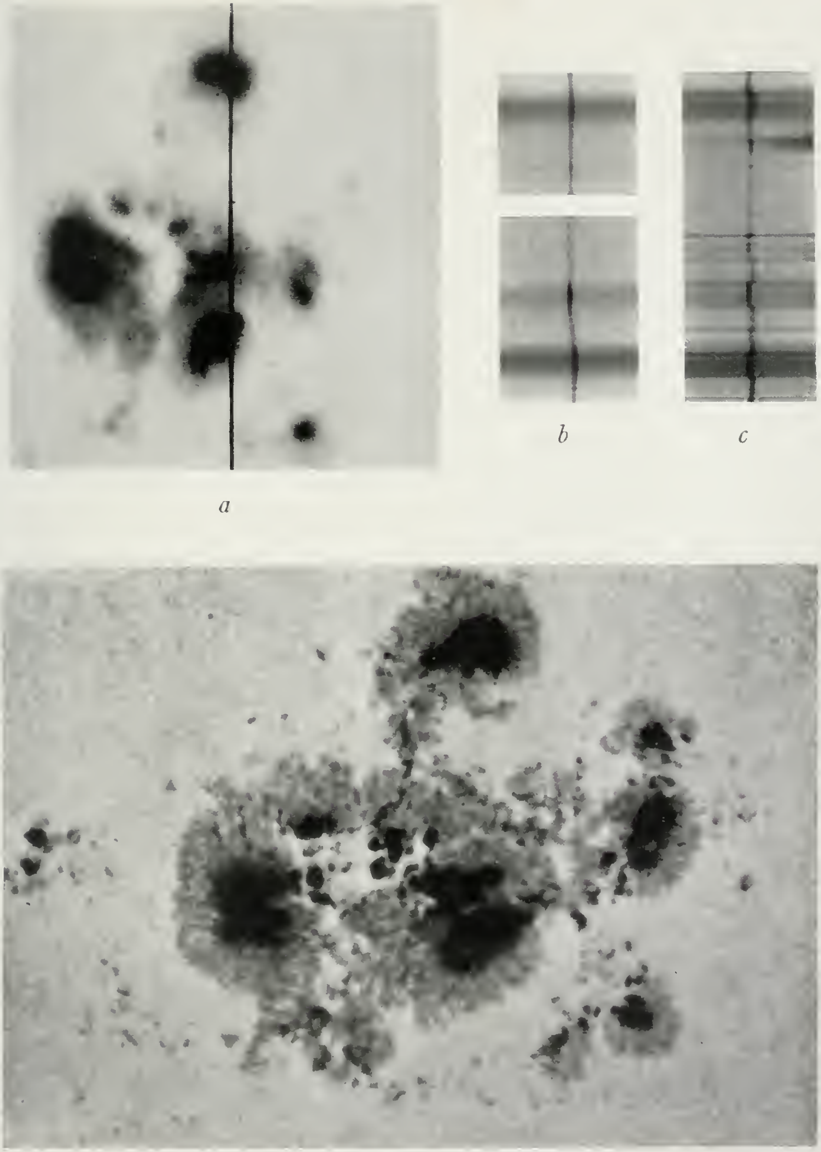
PLATE VII



a, Sun-spot photographed March 9, 1916, in longitude W. 23°, the umbra divided by a bridge; *b*, iron triplet λ6173 observed with Nicol and compound quarter-wave plate, the slit occupying the position shown by the line in *a;* the nearly equal intensities of the *n*-components in the *same* strip of spectrum indicate the presence of two fields of opposite polarities; compare with Plate IV *i*, which shows the appearance when only a single field is present; *c*, sketch showing the distribution of polarities for other members of the group.

d, Bipolar sun-spot group photographed June 19, 1914; *e*, *f*, the iron lines λ 6301.7 and λ6302.7 as shown in the spectra of the three principal members of the group, with Nicol and (*e*, single; *f*, compound) quarter-wave plate; the configuration of the lines indicates that the two smaller spots are of the same polarity, but opposite to that of the largest spot.

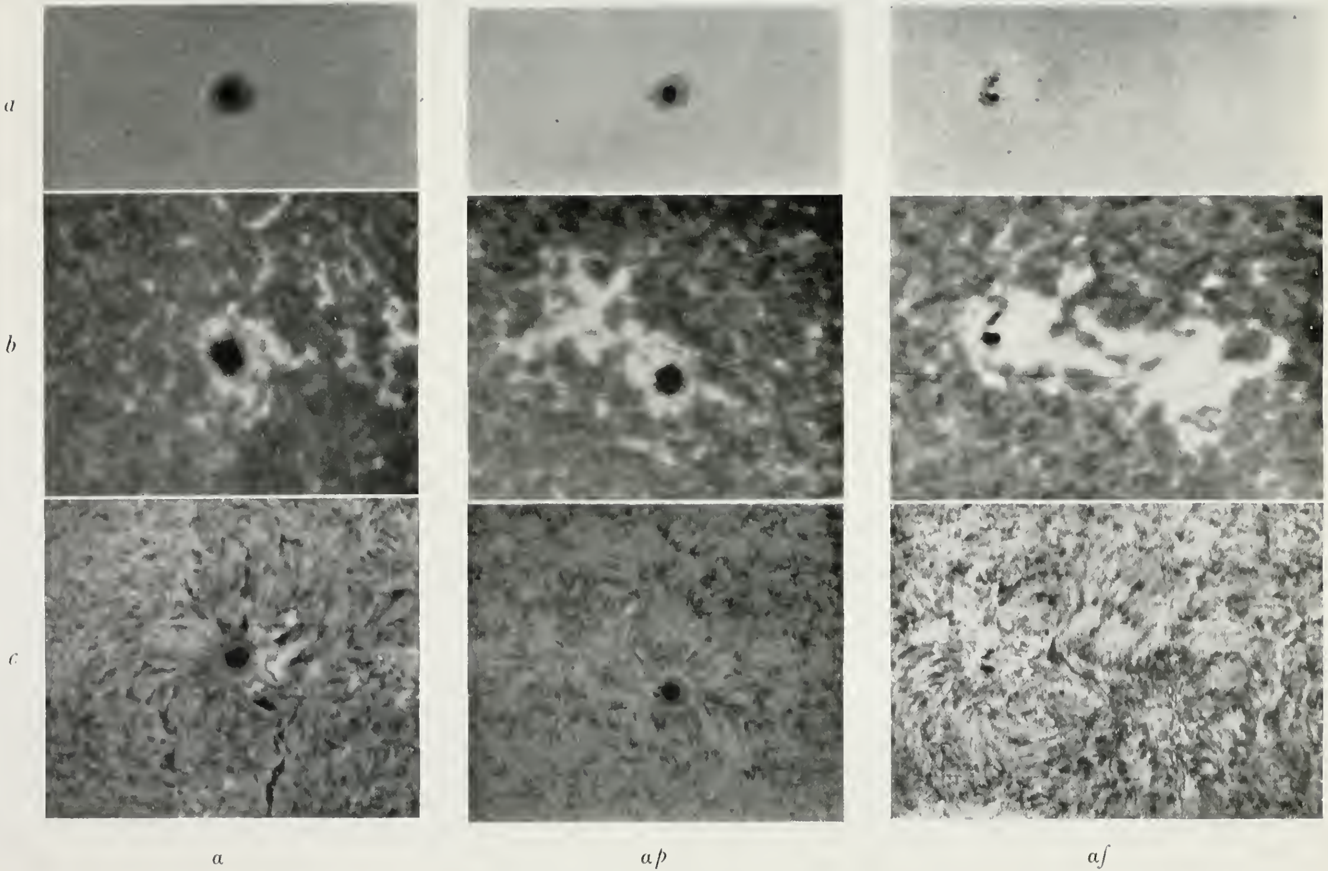
PLATE VIII



a, *b*, *c*, Photographic observations of a multipolar sun-spot group on August 8, 1917, similar to those described in Plate VII*d*, *e*, *f*, the spectral line in this case being the iron triplet λ6173; the lack of definition in *a* is due to poor seeing, together with the fact that it is a photograph of the image of the group formed by the 150-foot telescope, projected on a card mounted above the slit of the spectrograph.

d, An enlarged photoheliogram of the same group made with the 60-foot tower telescope on August 6, 1917, under good atmospheric conditions.

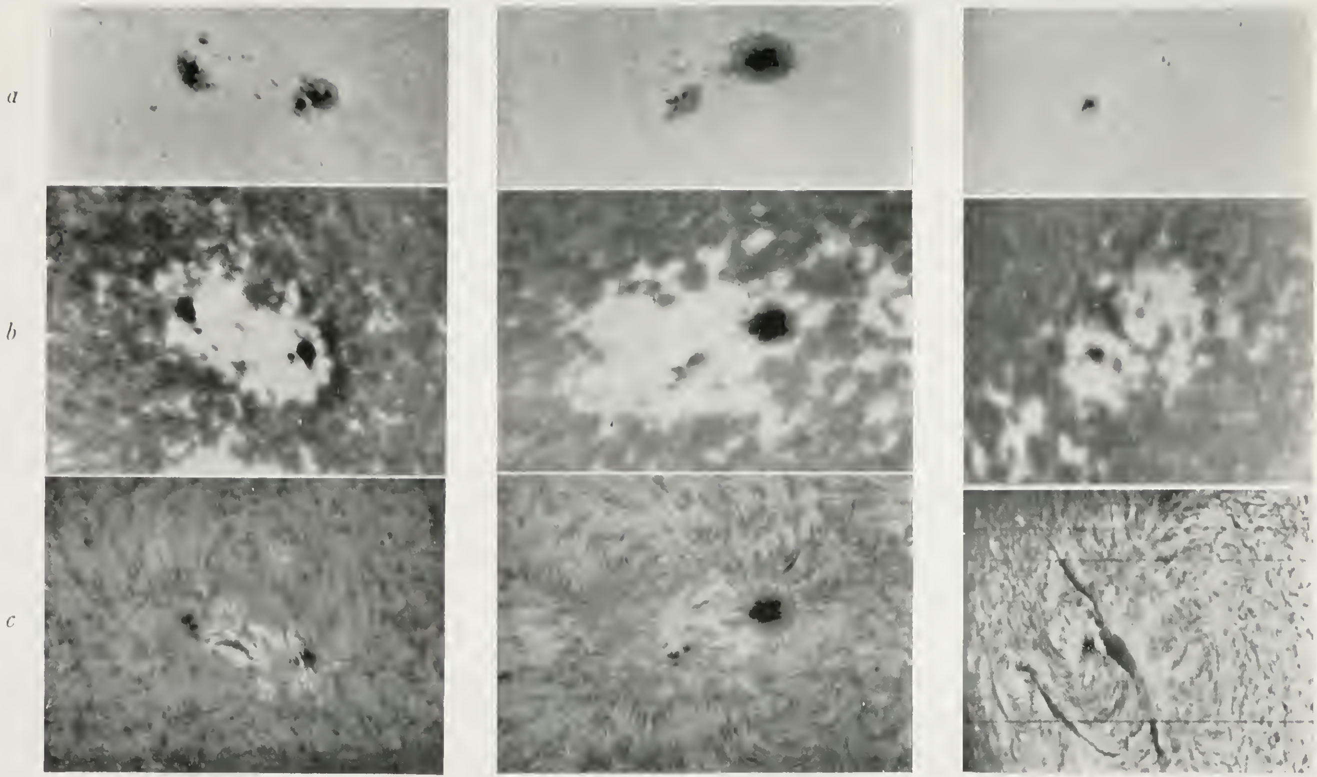
PLATE IX



Classification of Unipolar Sun Spots

a, Photoheliograms; *b*, K_2 spectroheliograms; *c*, Hα spectroheliograms; *α*, September 3, 1915; *αp*, September 15, 1914; *αf*, September 15, 1914.

PLATE X

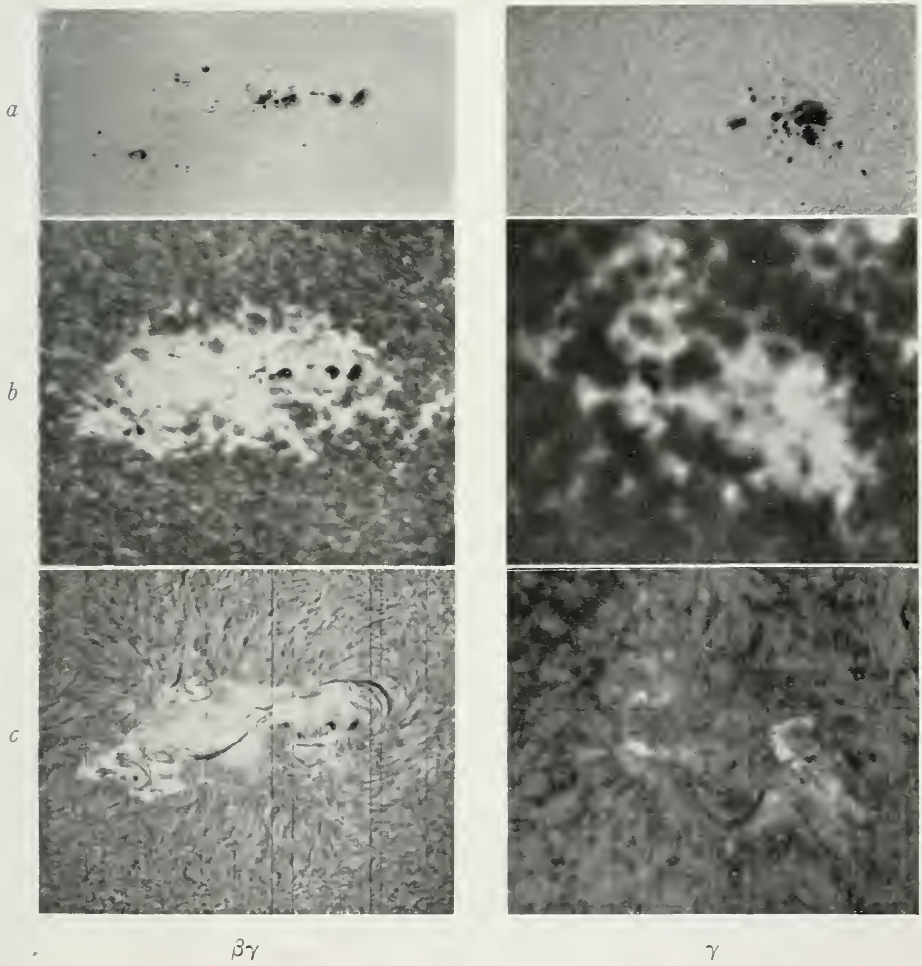


β βp βf

Classification of Bipolar Sun-Spots

a, Photoheliograms; *b*, K_2 spectroheliograms; *c*, Hα spectroheliograms; β, September 9, 10, 1909; βp, July 12, 1914; βf, April 22, 1916

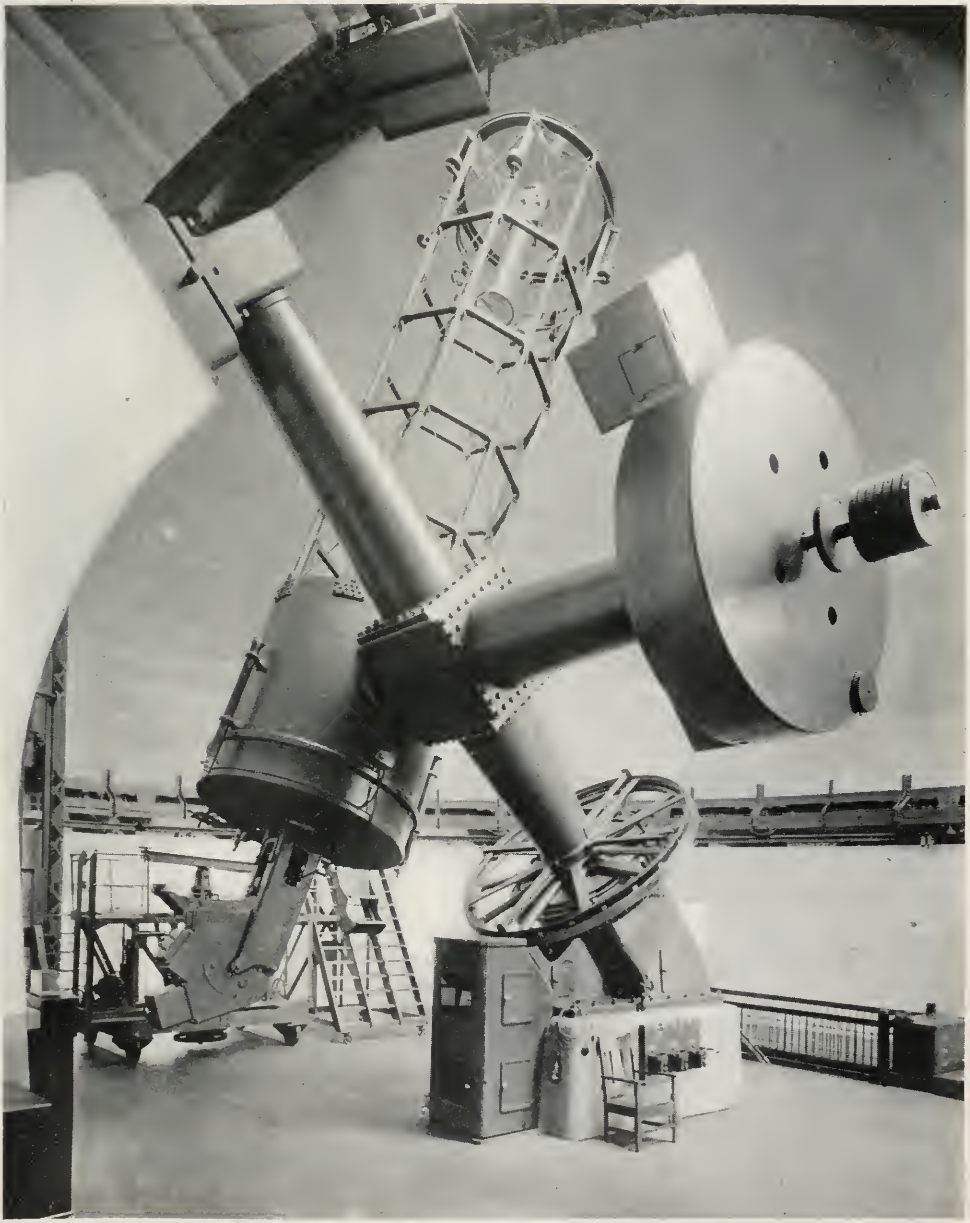
PLATE XI



CLASSIFICATION OF MULTIPOLAR SUN-SPOTS

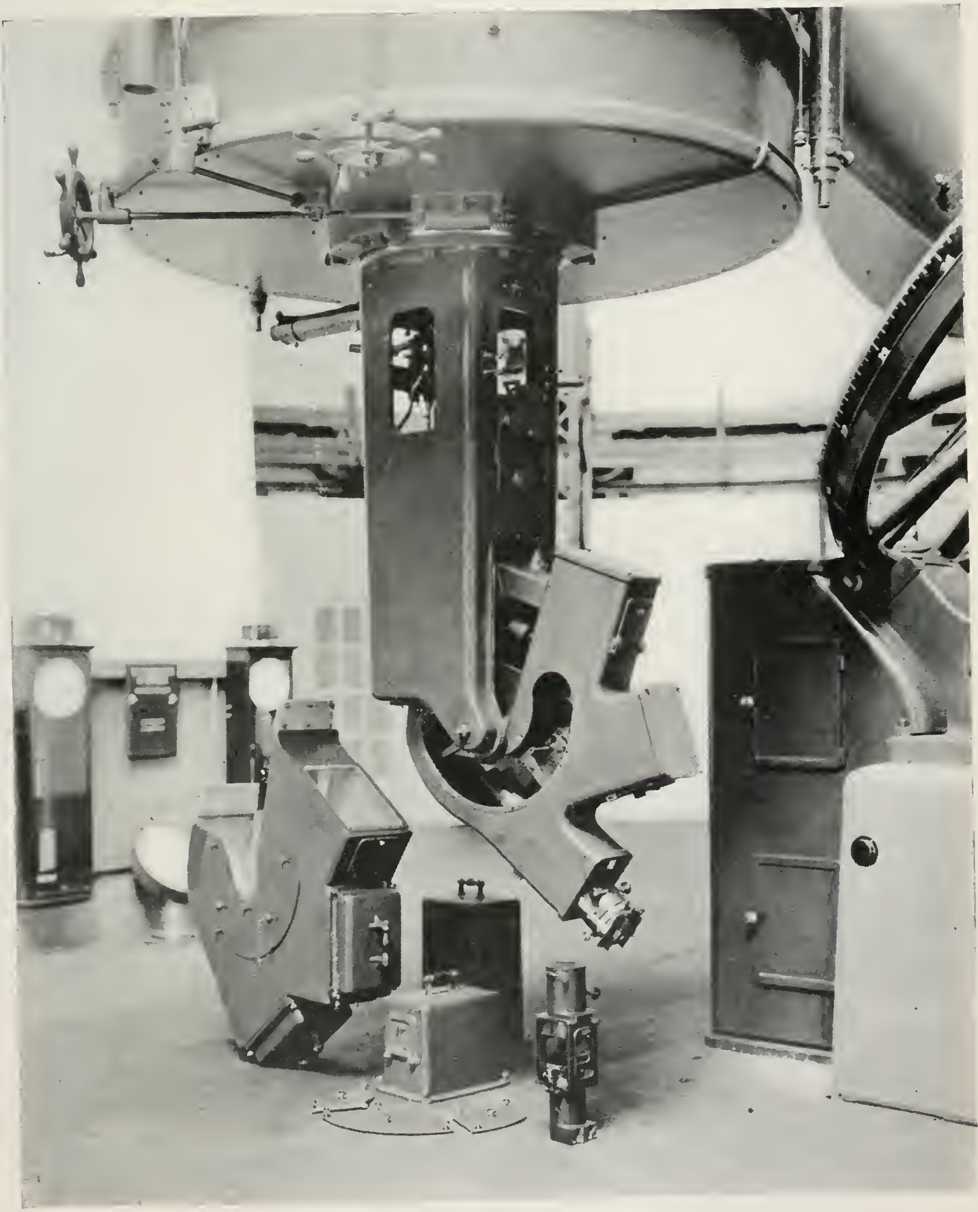
a, Photoheliograms; *b*, K_2 spectroheliograms; *c*, Hα spectroheliograms; $\beta\gamma$, an intermediate type, observed August 5, 1915; γ, July 25, 1917. Plate VIII*d* shows the photoheliogram of a large and unusually complicated γ group.

PLATE XII



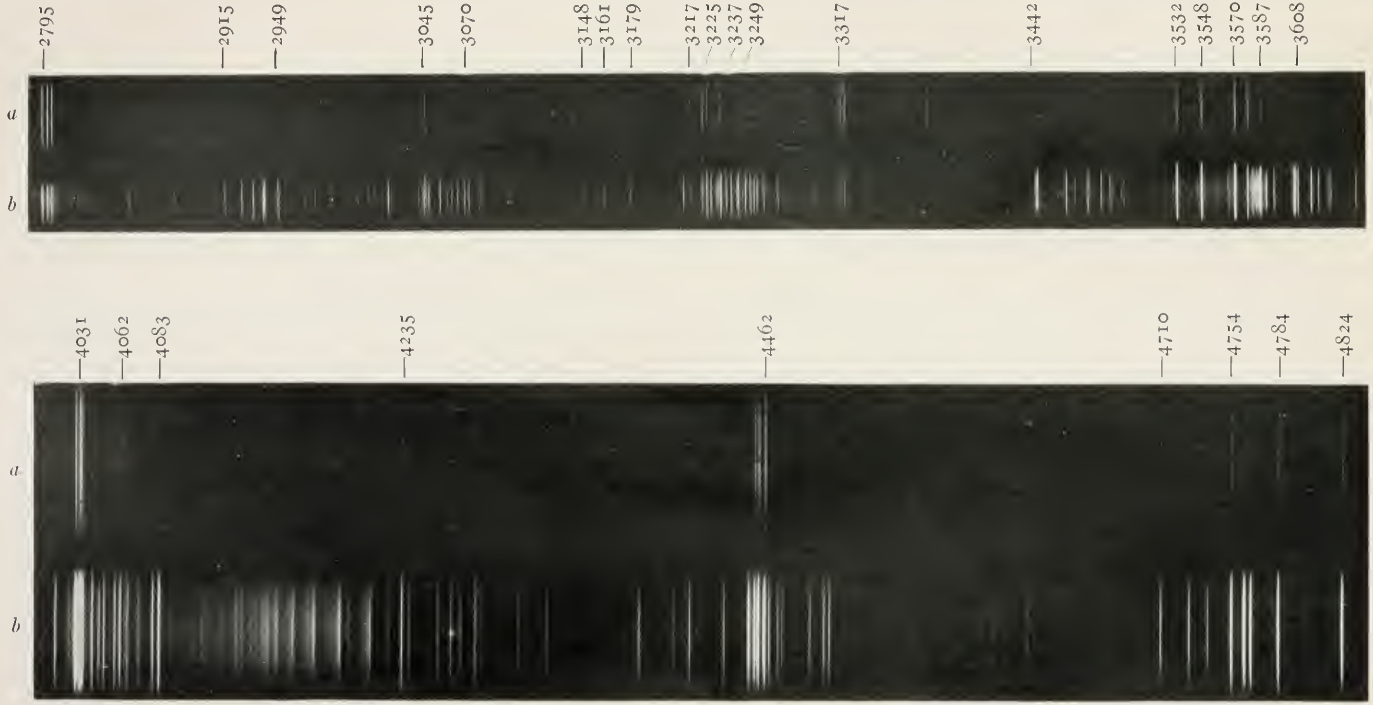
Seventy-two-Inch Reflecting Telescope with Spectrograph Attached

PLATE XIII



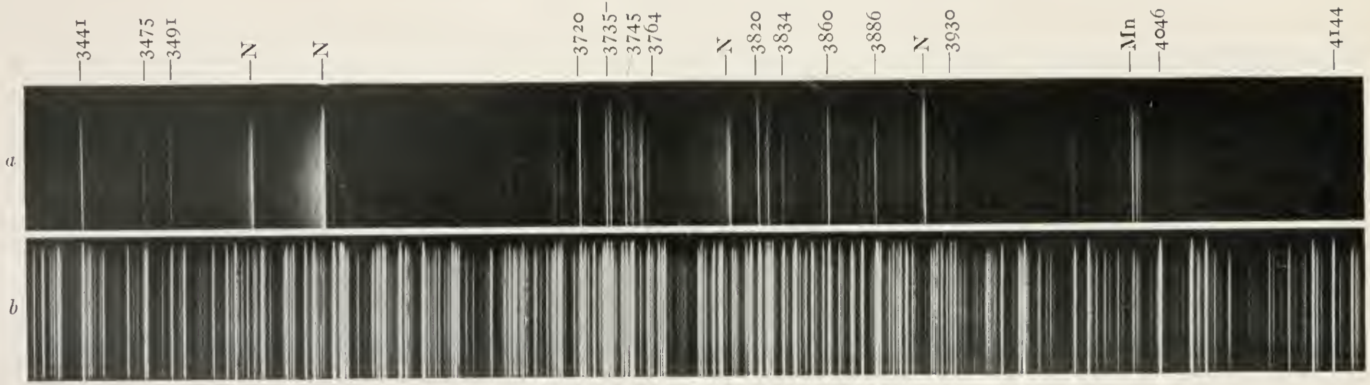
Spectrograph with Temperature Case Detached

PLATE XIV



(*a*) Luminescence and (*b*) arc spectra of manganese

PLATE XV



(*a*) Luminescence and (*b*) arc spectra of iron

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